Reciprocal Collision Avoidance for Quadrotor Helicopters using LQR-Obstacles

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Abstract

In this paper we present a formal approach to reciprocal collision avoidance for multiple mobile robots sharing a common 2-D or 3-D workspace whose dynamics are subject to linear differential constraints. Our approach defines a protocol for robots to select their control input independently (i.e. without coordination with other robots) while guaranteeing collision-free motion for all robots, assuming the robots can perfectly observe each other's state. To this end, we use the concept of *LQR-Obstacles* that define sets of forbidden control inputs that lead a robot to collision with obstacles, and extend it for reciprocal collision avoidance among multiple robots. We implemented and tested our approach in 3-D simulation environments for reciprocal collision avoidance of *quadrotor* helicopters, which have complex dynamics in 16-D state spaces. Our results suggest that our approach avoids collisions among over a hundred quadrotors in tight workspaces at real-time computation rates.

Introduction

Collision avoidance is a fundamental problem in (mobile) robotics. The problem can generally be defined in the context of an autonomous mobile robot navigating in an environment with obstacles and/or other robots, where the robot employs a continuous sensing-control cycle. In each cycle, the robot must compute an action based on its local observations, such that it stays free of collisions with the obstacles and the other robots and progresses towards a goal. Specifically accounting for the reactive nature of the other robots is called reciprocal collision avoidance.

In this paper, we present an approach for reciprocal collision avoidance for multiple robots whose dynamics are subject to linear differential constraints. Our approach is based on *LQR-Obstacles* (van den Berg et al. 2012), an extension of Velocity Obstacles (Fiorini and Shiller 1998), which define the set of control inputs to an LQR-controlled robot that will lead to a collision with an obstacle. We extend this concept for reciprocal collision avoidance among multiple robots.

LQR Control

Let the the dynamics of the robots be given by a deterministic discrete-time linear model:

$$\mathbf{x}^i_{t+1} = A\mathbf{x}^i_t + B\mathbf{u}^i_t. \quad (1)$$

Let $\mathbf{v}^\star_i \in \mathbb{R}^d$ denote a target velocity robot i wishes to reach. An infinite-horizon LQR feedback controller can optimally control the robot towards this target velocity given a quadratic cost function that trades-off reaching the target quickly versus not applying extreme control inputs:

$$\textstyle\sum_{t=0}^{\infty}((V\mathbf{x}^i_t - \mathbf{v}^\star_i)^T Q_v (V\mathbf{x}^i_t - \mathbf{v}^\star_i) + {\mathbf{u}^i_t}^T R\mathbf{u}^i_t), \quad (2)$$

where V maps the state to the velocity, which results in a feedback control policy of the following form:

$$\mathbf{u}^i_t = -L\mathbf{x}^i_t + E\mathbf{v}^*_i. \quad (3)$$

We can construct the *closed-loop* dynamics of the robots in terms of its target velocity rather than its low-level control input, by substituting Eq. (3) into (1):

$$\mathbf{x}^i_{t+1} = \tilde{A}\mathbf{x}^i_t + \tilde{B}\mathbf{v}^\star_i, \quad \tilde{A} = A - BL, \quad \tilde{B} = BE. \quad (4)$$

Given a current state $\mathbf{x}^i_0$ of robot i and a constant target velocity $\mathbf{v}^\star_i$, the state of the robot at a given time $t > 0$ is then given by solving the difference equation defining the closed-loop dynamics:

$$\mathbf{x}^i_t = F_t\mathbf{x}^i_0 + G_t\mathbf{v}^\star_i, \quad F_t = \tilde{A}^t, \quad G_t = \textstyle\sum_{k=0}^{t-1}\tilde{A}^k\tilde{B}. \quad (5)$$

LQR-Obstacles

For a pair of robots i and j, the state of robot i *relative* to the state of robot j is defined as $\mathbf{x}^{ij}_t = \mathbf{x}^i_t - \mathbf{x}^j_t$. The robots *collide* if their relative position $C\mathbf{x}^{ij}_t$ (C maps a state to a position) is contained within the Minkowski difference $\mathcal{O}_{ij} = \mathcal{O}_j \oplus -\mathcal{O}_i$ of the robot's geometries: $C\mathbf{x}^{ij}_t \in \mathcal{O}_{ij}$.

Substituting Eq. (5) given $\mathbf{x}^{ij}_0$ and the relative target velocity $\mathbf{v}^\star_{ij}$, the robots collide at a given time t if:

$$CF_t\mathbf{x}^{ij}_0 + CG_t\mathbf{v}^\star_{ij} \in \mathcal{O}_{ij}. \quad (6)$$

The relative *LQR-Obstacle* of the robots is then defined as the set of relative target velocities $\mathbf{v}^\star_{ij}$ that result in a collision within τ time into the future:

$$\mathcal{LQR}^\tau_{ij}(\mathbf{x}^{ij}_0) = \textstyle\bigcup_{t=1}^{\tau}(CG_t)^{-1}(\mathcal{O}_{ij} \oplus \{CF_t\mathbf{x}^{ij}_0\}). \quad (7)$$

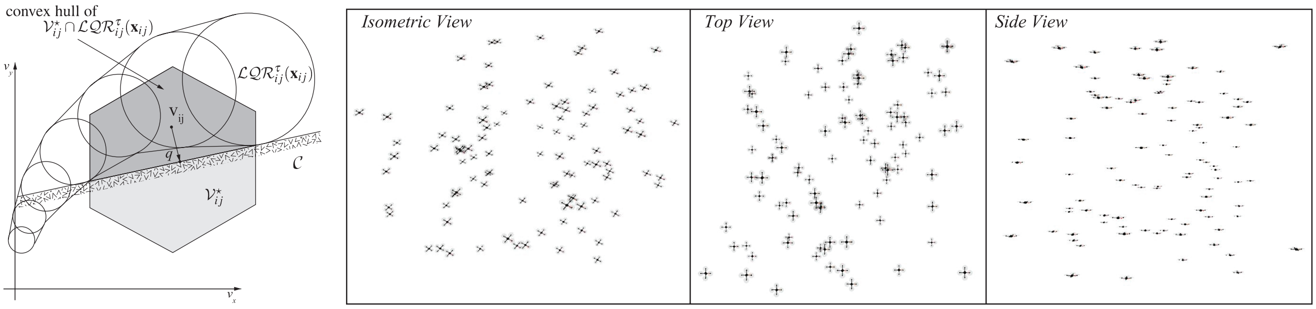

Figure 1: Left: the halfplane of valid target velocities for an example *LQR-Obstacle*. Right: Stills from a simulation of 100 quadrotors avoiding collisions with each other.

Hence, if the geometry of each of the robots is defined by a sphere (or an ellipsoid), the relative LQR-obstacle is a union of ellipsoids (see Fig. 1).

In order for a robot i to avoid collisions with a passive robot j it must choose a target velocity *outside* the relative LQR-Obstacle translated by j's target velocity within τ time:

$$\mathbf{v}^{\star}_i \notin \mathcal{LQR}^{\tau}_{ij}(\mathbf{x}^i_0 - \mathbf{x}^j_0) \oplus \{\mathbf{v}^{\star}_j\}. \quad (8)$$

Reciprocal Collision Avoidance

If robot j is not passive, but actively reacts to the presence of robot i in the same way robot i reacts to the presence of robot j, we must make sure that robot i and j choose their target velocities such that their relative target velocity $\mathbf{v}^{\star}_{ij}$ lies outside $\mathcal{LQR}^{\tau}_{ij}(\mathbf{x}^{ij}_0)$. Yet, we want each robot to make its control decision independently without coordination (van den Berg et al. 2011).

To this end, we look at the the smallest change $\mathbf{w}$ in the current relative velocity $\mathbf{v}^{ij}_0$ required to make sure that collisions are avoided, i.e. $\mathbf{v}^{ij} + \mathbf{w} \notin \mathcal{LQR}^{\tau}_{ij}(\mathbf{x}^{ij}_0)$, and divide the responsibility of avoiding collisions evenly among both robots. If the two robots are on a collision course robot i's velocity must change by at least $\frac{1}{2}\mathbf{w}$ and robot j's velocity must change by at least $-\frac{1}{2}\mathbf{w}$. More formally, robot i must choose its new target velocity anywhere in the half-plane through $\mathbf{v}^i_0 + \frac{1}{2}\mathbf{w}$ in the direction of $\mathbf{w}$, and robot j must choose its new target velocity anywhere in the half-plane through $\mathbf{v}^j_0 - \frac{1}{2}\mathbf{w}$ in the direction of $-\mathbf{w}$ (see Fig. 1).

In order for robot i to avoid collision with more than one robot, robot i determines its halfplane of valid target velocities with respect to each other robot j. The intersection of these halfplanes then determines the set of valid velocities with respect to all other robots, and robot i chooses its target velocity from this set closest to its *preferred* velocity.

Simulation Results

Simulations were run for a variety of configurations to demonstrate the performance of our algorithm. Videos of these simulations can be viewed at http://arl.cs.utah.edu/research/rca/.

Experiments were performed involving up to 128 quadrotors with random initial and target positions in a 10x10x10 meter space. As expected, the average computation time for each quadrotor is approximately linear in the number of other quadrotors it has to avoid collisions with. The limit on the number of quadrotors that can be avoided with the computations being performed in real-time ($33ms$ at a sensing-control cycle of 30Hz) is approximately 75. We note however, that for safe navigation it is not necessary to consider that many quadrotors in the collision avoidance but only a number of neighboring quadrotors.

Conclusions and Future Work

Our simulation results displayed that our approach is able to let a group of robots reach their target position from an initial position while smoothly avoiding collision with the other robots. In our approach each robot acts fully independently (no global coordination) and only need to continually observe each other's current state. Motivated by the promising simulation results, we are currently implementing our approach on real-world quadrotors.

Our approach has a number of limitations. Our approach requires that the state of the robot contains its position and its velocity, and that the geometry of the robot is fixed and only translates (does not change with rotation).While our approach works for avoiding collisions with any static and moving obstacles, the other robots with which collisions are reciprocally avoided must in our current formulation have exactly the same dynamics, in order to be able to formulate the dynamics model of the robots' *relative* motion. Robots of different dynamics could be handled by using an abstraction of their dynamics model.

References

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