Computing Logic Programs with Ordered Disjunction Using \textit{asprin}

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Abstract

Logic Programs with Ordered Disjunction (LPOD) is an extension of standard answer set programs to handle preference using the high-level construct of ordered disjunction whereas \textit{asprin} is a recently proposed, general, flexible, and extensible framework that provides low-level constructs for representing preference in answer set programming. We present an encoding of LPOD in the language of \textit{asprin} and the implementation of LPOD called LPOD2ASPRIN based on the encoding. Unlike the known method that applies only to a fragment of LPOD, our translation is general, direct, and simpler. It also leads to more efficient computation of LPOD using \textit{asprin}.

1 Introduction

Logic Programs with Ordered Disjunction (LPOD) (Brewka 2002) is an extension of standard answer set programs to handle preference using the high-level construct of ordered disjunction. \textit{asprin} (Brewka et al. 2015b) is a recently proposed, general, flexible, and extensible framework for expressing and computing preferences in answer set programming, and, as such, the preference specification in the language of \textit{asprin} is in a lower level than LPOD. Representing high-level preference constructs in the language of \textit{asprin} could be verbose, and end-users may find it complicated to use. To alleviate the problem, \textit{asprin} provides a library that implements several preference types, such as subset, less (weight), and ASO. However, LPOD preference types are not one of them.

In (Brewka, Niemelä, and Syrjänen 2004), LPOD is implemented using SMODELS by interleaving the execution of two ASP programs—a generator which produces candidate answer sets and a tester which checks whether a given candidate answer set is most preferred or produces a more preferred candidate answer set otherwise. In principle, the encodings in (Brewka, Niemelä, and Syrjänen 2004) can be used with \textit{asprin} to implement LPOD. However, this method introduces a large number of translation rules and auxiliary atoms since it does not utilize the main component of \textit{asprin}, preference statements.

In fact, it is known that using preference statements, some fragment of LPOD can be represented in the language of \textit{asprin} via the translation into Answer Set Optimization (ASO). Brewka, Niemelä, and Truszczynski (2003) show how to turn LPOD under Pareto-preference into ASO programs, and Brewka et al. (2015a) show that ASO programs can be represented in \textit{asprin}. By combining the two results, the fragment of LPOD can be represented in \textit{asprin}. It is also mentioned that LPOD under inclusion-preference can be turned into “ranked” ASO (Brewka et al. 2015a) but the representation appears quite complicated. Furthermore, it is not known how the results apply to the other LPOD preference criteria.

This paper presents a more direct and simpler translation from LPOD into the language of \textit{asprin}, handling all four preference criteria from (Brewka 2005) in a uniform way. Based on the translation, we implemented the system LPOD2ASPRIN, which translates LPOD programs into the input language of \textit{asprin} and internally invokes the \textit{asprin} system. Our experiments show that the system is more scalable than the other methods of computing LPOD.

2 Review of LPOD and \textit{asprin}

2.1 Review: LPOD

A (propositional) LPOD \(\Pi\) is \(\Pi_{\text{reg}} \cup \Pi_{\text{od}}\), where its regular part \(\Pi_{\text{reg}}\) consists of usual ASP rules \(\text{Head} \leftarrow \text{Body}\), and its ordered disjunction part \(\Pi_{\text{od}}\) consists of LPOD rules of the form

\[
C_1 \times \cdots \times C_n \leftarrow \text{Body}
\]

in which \(C_i\) are atoms, \(n\) is at least 2, and \(\text{Body}\) is a conjunction of atoms possibly preceded by \text{nor}. Rule (1) says “when \text{Body}\) is true, if possible then \(C_1\); if \(C_1\) is not possible then \(C_2, \ldots\); if all of \(C_1, \ldots, C_{n-1}\) are not possible then \(C_n\).”

Candidate answer sets are defined via the notion of split programs in (Brewka 2002). Based on the notion of a satisfaction degree of a rule by a candidate answer set, Brewka (2005) defined preferred answer sets under four preference criteria: cardinality-preferred (c), inclusion-preferred (i), Pareto-preferred (p), penalty-sum-preferred (ps) answer sets. We refer the reader to that paper for the details.

A set \(X\) of atoms is an \(s\)-preferred \((s \in \{c, i, p, ps\})\) answer set of an LPOD \(\Pi\) if \(X\) is a candidate answer set of \(\Pi\) and there is no candidate answer set \(X'\) of \(\Pi\) such that \(X'\) is preferred to \(X\) w.r.t. preference criterion \(s\).
2.2 Review: asprin

asprin computes the most preferred answer sets of an ASP program \( P \) according to a preference specification \( \hat{F}_s \) by repeated calls to CLINGO as in Figure 1. First, an arbitrary answer set \( X \) of \( P \) is generated as \( X' \). Second, asprin tries to find an answer set \( X \) of \( P \) that is better than \( X' \). Intuitively, each index \( i \) of the preference relation \( X \) gives a ranking of a literal in \( X \). The optimization directive \( (2) \) specifies which preference relation asprin uses for optimization.

![Figure 1: asprin Framework](image)

\( F_s \) is obtained from the preference specification \( \hat{F}_s \) by turning the optimization directive \( (2) \) into an ASP fact

\[
\text{optimize}(s)
\]

and turning the preference statement \( (3) \) into an ASP fact

\[
\text{preference}(s,t)
\]

along with

\[
\text{preference}(s,i,j,\text{for}(t_{\phi_j}),())
\]

for each \( j \)-th literal \( \phi_j \) in the \( i \)-th preference element \( e_i \) in \( (3) \). The term \( t_{\phi_j} \) is defined as \( a \) if the literal \( \phi_j \) is an atom \( a \), and is \( \text{not} \ a \) if the literal \( \phi_j \) is \( \text{not} \ a \). 3

4. \( E_{i,s} \) is the preference encoding for \( t_s \), where \( t_s \) is the type of the preference relation named \( s \). It defines a reserved predicate better\( (s) \), which is true iff there exists a candidate answer set \( X \) that is preferred to \( X' \) according to preference type \( t_s \) and the facts in \( F_s \). In Section 3.3, we show four preference encodings \( E_{\text{lpod}(c)}, E_{\text{lpod}(i)}, E_{\text{lpod}(p)}, \) and \( E_{\text{lpod(ps)}} \) for each of the four preference types (i.e., criteria) for LPOD.

5. \( H'_{X'} \) is the set of ASP facts

\[
\{\text{hold}'(a) \mid a \in X'\}
\]

which reifies the atoms in \( X' \) in the form of \( \text{hold}'(\cdot) \). 4

6. \( R_A \) is the set of ASP rules

\[
\{\text{hold}(a) \leftarrow a \mid a \text{ is an atom in } P\}
\]

which reifies the atoms in any candidate answer set \( X \) in the form of \( \text{hold}(\cdot) \).

7. \( C \) is a set of (domain-independent) ASP rules as follows.

\[
\bot \leftarrow \text{not better}(S), \text{optimize}(S).
\]

\[
\text{hold}(\text{neg}(A)) \leftarrow \text{not holds}(A),
\]

\[
\text{preference}(\cdot\cdot\cdot \text{for}(\text{neg}(A)),\cdot\cdot\cdot).
\]

\[
\text{hold}'(\text{neg}(A)) \leftarrow \text{not holds}'(A),
\]

\[
\text{preference}(\cdot\cdot\cdot \text{for}(\text{neg}(A)),\cdot\cdot\cdot).
\]

Rule (4) instructs the asprin system to find an answer set \( X \) that is better than \( X' \) according to the preference relation \( S \). Rule (5) is about \( X \), which is reified in the form of \( \text{hold}(\cdot) \); for the literal of the form “\( \text{not} \ A \)” in the preference statement \( (3) \), it says \( \text{hold}(\text{neg}(A)) \) is true if \( \text{hold}(A) \) is false in the reified \( X \) (i.e., \( X \neq A \)). Similarly, rule (6) is about \( X' \), which is reified in the form of \( \text{hold}'(\cdot) \).

Given a program \( P \) and a preference specification \( \hat{F}_s \), we say an answer set \( X \) of \( P \) is a preferred answer set of \( P \) w.r.t. \( \hat{F}_s \) if \( P \cup F_s \cup E_{t_s} \cup H'_{X'} \cup R_A \cup C \) has no answer set, where \( t \) is the type of the preference relation \( s \) declared in \( \hat{F}_s \).

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1. asprin in fact allows multiple preference statements in the input but for simplicity of the presentation we assume a single preference statement.

2. In general, asprin allows for a more general syntax of preference elements. For the purpose of this paper, it is sufficient to consider this simple fragment.

3. The last term is empty because we consider \( \phi_j \) as a non-weighted formula.

4. Note that this is based on the definition of \( H'_{X'} \), which is the set of ASP facts \( \{\text{hold}'(a) \mid a \in X'\} \).
3 Representing LPOD in asprin

Let $\Pi$ be an LPOD where $\Pi_{od}$ consists of $m$ propositional rules as follows.

$$C_1^1 \times \ldots \times C_{n_1}^1 \leftarrow Body_1$$

\[ \vdots \] (7)

$$m : \ C_m^1 \times \ldots \times C_{n_m}^m \leftarrow Body_m$$

where $1, \ldots, m$ are rule indices; $n_i \geq 2$ for $1 \leq i \leq m$.

In the following subsections, we present the component programs of asprin that encode LPOD $\Pi$, namely, $P$, $F_s$, $E_h$. The other components, $F_s$, $H_X$, $RA$ and $C$ are generated as described in Section 2.2.

3.1 Base Program $P$

For the LPOD program $\Pi = \Pi_{reg} \cup \Pi_{od}$, the base program $P$ contains all rules in $\Pi_{reg}$ and, for each LPOD rule

$$C_1^1 \times \ldots \times C_{n_i}^i \leftarrow Body_i$$

in $\Pi_{od}$, $P$ contains

$$body_i \leftarrow Body_i$$

$$\{C_1^i\} \leftarrow body_i$$

$$\ldots$$

$$\{C_{n_i}^i\} \leftarrow body_i, not C_1^i, \ldots, not C_{n_i-1}^i$$

$$C_{n_i}^i \leftarrow body_i, not C_1^i, \ldots, not C_{n_i-1}^i$$

Rule (8) defines the case when the body of rule $i$ is true. Rules (9)–(10) say that if the body of rule $i$ is true and each $C_j^i$ is false ($j \in \{1, \ldots, k-1\}$), then $C_k^i$ is possibly true. Rule (11) says that if the body of rule $i$ is true and $C_j^i$ is false for all $j \in \{1, \ldots, n_i-1\}$, then $C_{n_i}^i$ must be true.

The above method of generating candidate answer sets using choice rules is from (Cabalar 2011). It is not difficult to check that the answer sets of this program $P$ are the candidate answer sets of LPOD $\Pi$ (ignoring body, atoms).

Proposition 1 For any LPOD $\Pi$ and any set $X$ of atoms in $\Pi$, $X$ is a candidate answer set of $\Pi$ iff $X \cup \{body_i \mid X \text{ satisfies the body of rule } i \text{ in } \Pi_{od}\}$ is an answer set of $P$.

Example 1 Consider the following LPOD $\Pi$ about picking a hotel near the Grand Canyon. hotel(1) is a 2-star hotel but is close to the Grand Canyon, hotel(2) is a 3-star hotel and the distance is medium, and hotel(3) is a 4-star hotel but is too far.

$$close \times med \times far \times tooFar$$

$$star4 \times star3 \times star2$$

$$\{hotel(X) : X = 1..3\}$$

$$\leftarrow hotel(2), not med.$$  
$$\leftarrow hotel(2), not star3.$$  
$$\leftarrow hotel(3), not tooFar.$$  
$$\leftarrow hotel(1), not close.$$  
$$\leftarrow hotel(1), not star2.$$

The $P$-component of asprin obtained from this LPOD $\Pi$ is the following.

\begin{verbatim}
body_1.
(close) :- body_1.
(med) :- body_1, not close.
(far) :- body_1, not close, not med.
(tooFar) :- body_1, not close, not med, not far.
\end{verbatim}

$$\{hotel(1), close, star2, body_1, body_2\}$$

$$\{hotel(2), med, star3, body_1, body_2\}$$

$$\{hotel(3), tooFar, star4, body_1, body_2\}$$

which are exactly the unions of the candidate answer sets of $\Pi$ and $\{body_1, body_2\}$.

3.2 Preference Specification $\hat{F}_s$

$\hat{F}_s$ contains an optimization directive

\#optimize(s)

and a preference statement

\#preference(s, lpod(s)) {
  not body_1 > C_1^1 > \ldots > C_{n_1}^1;
  \ldots
  not body_m > C_1^m > \ldots > C_{n_m}^m
}

where $s \in \{c, i, p, ps\}$ denotes one of the four preference criteria for LPOD, and each line of (12) is associated with each LPOD rule to specify satisfaction degrees. Intuitively, to check the satisfaction degree of an LPOD rule $i$, we check the truth value of the literals in the order specified in the $i$-th preference element. We first check whether $not body_i$ is true. If $not body_i$ is true, the satisfaction degree is 1 and we do not have to check further; and if it is not the case, check whether $C_1^i$ is true, and so on.

Example 1 (Continued) For LPOD $\Pi$ which contains LPOD rules

$$close \times med \times far \times tooFar$$

$$star4 \times star3 \times star2$$

to find its cardinality-preferred answer sets, we set the preference criterion $s$ to $c$, and let $F_s$ be the following.

\#optimize(c).

\#preference(c, lpod(c)) {
  not body_1 >> close >> med >> far >> tooFar ;
  not body_2 >> star4 >> star3 >> star2
}.

asprin internally turns $\hat{F}_s$ into $F_s$ as follows.
optimize(c).

preference(c, lpod(c)).

preference(c, 1, 1, for(neg(body_1)), ()).
preference(c, 1, 2, for(close), ()).
preference(c, 1, 3, for(med), ()).
preference(c, 1, 4, for(far), ()).
preference(c, lpod(c)).

optimize(c).

The facts optimize(c) and preference(c, lpod(c)) assert that we optimize according to the preference statement c of type lpod(c) (inclusion preference). The fact preference(c, 2, 1, for(neg(body_2)), ()) asserts that the first literal of the second preference element of the preference statement c is “not body_2”.

3.3 Preference Encoding $E_{ts}$

The aim of $E_{ts}$ is to find an answer set $X$ (refined in the form of holds(·)) that is better than (i.e., preferred to) the current answer set $X'$ (refined in the form of holds'(·)) with respect to the preference type $ts$.

We introduce the preference encodings $E_{ts}$ for each $ts \in \{lpod(c), lpod(i), lpod(p), lpod(ps)\}$. Each $E_{ts}$ contains the common rules $Deg$ as defined below.

**Degree** The aim of $Deg$ is to find the satisfaction degree to which each LPOD rule $R$ is satisfied by $X$ or $X'$.

$Deg$ consists of the following two rules.

$$\text{degree}(R, D) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(\_)), \text{preference}(S, R, I, \_).$$
$$D = \#\text{max}(1; I - 1),$$
$$I = \#\text{min}\{J : \text{holds}(A), \text{preference}(S, R, J, \_), J\}. \quad (13)$$

$$\text{degree'}(R, D) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(\_)), \text{preference}(S, R, I, \_).$$
$$D = \#\text{max}(1; I - 1),$$
$$I = \#\text{min}\{J : \text{holds'}(A), \text{preference}(S, R, J, \_), J\}. \quad (14)$$

Rule (13) records the degree $D$ to which rule $R$ is satisfied by $X$ ($X$ is refined in the form of holds(·)). It asserts that if we want to optimize according to preference relation $S$ whose type is one of the four $\text{lpod}(\cdot)$ types, then we need to calculate the satisfaction degree $D$ for each rule $R$: $D$ is the maximum value of $1$ and $I - 1$ where $I$ is the index of the first literal in the preference element $R$ that is true in $X$. Rule (14) is similar to rule (13) except that it finds the satisfaction degree $D$ of rule $R$ for $X'$.

**Cardinality-Preferred** $E_{lpod(c)}$ contains $Deg$ and the following two rules:

$$\text{worse2degree}(S, D) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(c)), \text{preference}(S, \text{lpod}(\_)), \text{degree'}(\_).$$
$$\#\text{sum}\{1, R : \text{degree}(R, D)\} - 1, R : \text{degree'}(R, D)\} < 0. \quad (15)$$

$$\text{better}(S) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(c)), \text{degree'}(\_).$$
$$\#\text{sum}\{1, R : \text{degree}(R, D)\} - 1, R : \text{degree'}(R, D)\} > 0,$$
$$\text{not worse2degree}(S, J) : J = 1..D - 1. \quad (16)$$

Rule (15) defines the case when $X$ is worse than, i.e., less preferred to, $X'$ at degree $D$: $X$ satisfies less LPOD rules to degree $D$ than $X'$. In this case, there must be at least one LPOD rule that is satisfied to degree $D$ by $X'$, which is guaranteed by $\text{degree'}(\_).$ Rule (16) says that $X$ is better than $X'$ according to the preference type $\text{lpod}(c)$ if there exists a degree $D$ such that $X$ is preferred to $X'$ at degree $D$ (i.e., $X$ satisfies more rules to degree $D$ than $X'$) and $X$ is not worse than $X'$ at all lower degrees. Note that “$\text{not worse2degree}(S, J) : J = 1..D - 1$” is a conditional literal, and is equivalent to the conjunction of literals “$\text{not worse2degree}(S, J')$” for all $J \in \{1, \ldots, D - 1\}$.

**Inclusion-Preferred** $E_{lpod(i)}$ contains $Deg$ and two rules:

$$\text{prf2degree}(S, D) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(i)), \text{degree'}(\_).$$
$$\#\text{count}\{J : \text{degree}(J, D), \text{not degree'}(J, D)\} > 0,$$
$$\text{degree}(J, D) : \text{degree'}(J, D). \quad (17)$$

$$\text{better}(S) \leftarrow \text{preference}(S, \text{lpod}(i)), \text{prf2degree}(S, D), \text{degree}(R, J) : \text{degree'}(R, J), J < D. \quad (18)$$

Rule (17) defines the case when $X$ is preferred to $X'$ at degree $D$: (i) $X$ satisfies at least one rule to degree $D$; (ii) there is a rule $J$ that is satisfied by $X$, but not by $X'$, to degree $D$; and (iii) all rules $J$ that are satisfied by $X'$ to degree $D$ are also satisfied by $X$ to the same degree. Rule (18) says that $X$ is better than $X'$ according to preference type $\text{lpod}(i)$ if there exists a degree $D$ such that $X$ is preferred to $X'$ at degree $D$, and any rule $R$ that is satisfied by $X'$ to a lower degree than $D$ should also be satisfied by $X$ to the same degree.

**Pareto-Preferred** $E_{lpod(p)}$ contains $Deg$ and two rules:

$$\text{equ}(S) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(p)), \text{degree'}(R, D1), \text{degree'}(R, D2).$$
$$D1 = D2 : \text{degree}(R, D1), \text{degree'}(R, D2). \quad (19)$$

$$\text{better}(S) \leftarrow \text{optimize}(S), \text{preference}(S, \text{lpod}(p)), \text{not equ}(S),$$
$$D1 \leq D2 : \text{degree}(R, D1), \text{degree'}(R, D2). \quad (20)$$

Rule (19) defines that $X$ and $X'$ are “equivalent” if they satisfy each LPOD rule to the same degree. Rule (20) says that $X$ is better than $X'$ according to preference type $\text{lpod}(p)$ if $X$ is not “equivalent” to $X'$, and $X$ satisfies each LPOD rule $R$ to a degree that is the same or lower than the degree to which $X'$ satisfies $R.$
We already mentioned the method of (Brewka et al. 2015a) works under the Pareto preference. However, the reduction under inclusion preference requires a translation from LPOD to “ranked” ASO programs, which further requires a more complex reduction to asprin. Besides, the reductions from LPOD to ASO programs under cardinality and penalty-sum preferences were not shown. In comparison, our method reduces LPOD directly to asprin, which yields a simpler and uniform method that applies to all preference criteria for LPOD.

Asuncion et al. (2014) present a first-order semantics of logic programs with ordered disjunction by a translation into second-order logic.

Lee and Yang (2018) show a reduction from LPOD to answer set programs, where the semantics of each preference type is also represented by standard ASP rules. Their reduction is one-pass: the preferred answer sets are computed by calling an answer set solver one time by generating all candidate answer sets and then applying preference criteria unlike the multiple calls to CLINGO in asprin.

asprin has a library of built-in preference types, but LPOD preference is not one of them. Our preference encodings may be included in the asprin library to benefit the end-users.

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References


