Reasoning with Justifiable Exceptions in Contextual Hierarchies

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Abstract

The problem of representing and reasoning with context dependent knowledge has been of certain interest since the beginning of AI. Among the available solutions, we consider the Contextualized Knowledge Repository (CKR) framework. In CKR applications it is often useful to reason over a hierarchical organization of contexts: however, the CKR model is not able to represent exception handling in the inheritance of knowledge across contexts. In this paper we develop a proposal, based on a recent principle for exception handling for inheritance in description logics, that allows CKRs with context dependent defeasible axioms which can be overridden by more specific local knowledge. We provide an alternative semantics for a core (simple) version of CKR that copes with contextual defeasible axioms, and we define a datalog translation generating programs that are complete w.r.t. instance checking under the proposed semantics in the case of ranked contextual hierarchies.

Introduction

In Knowledge Representation and Reasoning, the problem of dealing with context dependent knowledge is a well-known area of study and proposals for its formalization date back to the works of McCarthy (1993), Lenat (1998), and Giunchiglia et al. (1994). Recently, the interest in representing and reasoning with contexts has been recognized as relevant in the area of Semantic Web due to the need to interpret the knowledge of data sets in the right context: this lead to a number of (description) logic based approaches e.g. (Brewka, Roelofsen, and Serafini 2007; Brewka and Eiter 2007; Krähenbühl and Terziyan 2006; Klarkman 2013; Serafini and Homola 2012; Straccia et al. 2010; Tanca 2007; Udrea, Recupero, and Subrahmanian 2010).

We consider one of the most recent DL based formalisms, the Contextualized Knowledge Repository (CKR) framework (Serafini and Homola 2012; Bozzato and Serafini 2013), with its latest formulation in (Bozzato, Eiter, and Serafini 2018). A CKR knowledge base is a two-layer structure where the higher level consists of a global context and the lower level consists of a set of local contexts. The global context contains two types of knowledge: (i) context facts about the domain of discourse, accessible by all the local contexts. (ii) meta-knowledge specifying the properties and structure of local contexts. Local contexts contain knowledge that holds under specific situations (e.g. during a certain period of time, region in space) and thus they represent different partial and perspective views of the domain. Knowledge in different contexts is not completely independent: the context independent knowledge is propagated from the global to the local contexts and it is used to constrain local knowledge in different contexts. In (Bozzato, Eiter, and Serafini 2018) an extension to the CKR model was proposed by introducing a notion of justifiable exceptions. Axioms in the global context may be specified as defeasible: in general they can be applied to instances in the local contexts, but these can be locally “overridden” on some exceptional instance if they would cause a local contradiction.

A limitation of the latter proposal is that propagation (and overriding) of defeasible axioms is bounded to the direction from the global to the local contexts: in other words, the context organization is only limited to a two-level hierarchy and further refinements are not directly expressible. Thus, specialization is limited to two levels and it is not possible to use defeasible axioms in local contexts and propagate knowledge across such contexts. In general, one may want to specify more complex structures of contexts and control the knowledge propagation across such structures: in particular, it is the case of hierarchies of contexts specified by a context coverage relation (Serafini and Homola 2012). Serafini and Homola defined this structure implicitly by a combination of contextual dimensions (e.g. time, location, topic). E.g., consider the structure defined by the subdivision of a state in regions and provinces. Then, the interpretation of defeasible axioms must be extended for knowledge propagation in such a hierarchical structure: e.g. while most regulations and laws propagate to the national laws down to the ones applied in regions and provinces, there might be some local exceptions in regulations (e.g. use of heating, mandatory use of snow chains, limits on public water supply etc.).

In this paper, we generalize the approach in (Bozzato, Eiter, and Serafini 2018) by allowing for local defeasible axioms. Intuitively, a (non-strict) preference on such axioms is derived from their position in the coverage hierarchy: in the possible models, we prefer those which override the axioms in the higher levels of the hierarchy, in order to prefer the most
specific axioms in the lower levels. In this way, the order on defeasible axioms preserves the natural ordering of contexts and the flow of information defined by the coverage relation is implicitly defined by such organization.

In particular, for the definitions in this work we concentrate on ranked contextual hierarchies, namely hierarchies that can be divided in a linear order of levels: this restriction allows to define a simple model preference relation based on the level of the overridden defeasible axioms. We note that this kind of hierarchies are typical in CKR applications e.g. obtained by the combination of contextual dimensions in (Serafini and Homola 2012). Moreover, in the case of ranked hierarchies, the translation to datalog presented in (Bozzato, Eiter, and Serafini 2018) can be naturally extended to perform instance checking and conjunctive query answering only on preferred models which enforce the most specific defeasible axioms.

The contributions of this paper are briefly summarized as follows:

- We provide an extension of the CKR semantics with justifiable exceptions from (Bozzato, Eiter, and Serafini 2018) where defeasible axioms can appear in local contexts. Inheritance and overriding of such axioms is defined over an explicit coverage relation (Serafini and Homola 2012) across local contexts. In particular, in order to concentrate on the context coverage structure, we work on a simplified definition of CKR that we call simple CKR (sCKR). For the definition of the semantics, we concentrate on ranked hierarchies which allow us to define a simple notion of model preference based on the global number and level of its overrideings.

- We investigate the computational complexity of major reasoning tasks, and characterize it for axiom entailment and conjunctive query (CQ) answering. It turns out that under this preference, axiom entailment is \( \Delta^p \)-complete and CQ-answering is \( \Pi^p \)-complete; thus while preference increases the complexity of entailment, it does not so for CQ answering.

- Over such new definition of local defeasible axioms, we extend the translation to datalog (with negation under answer set semantics) initially proposed in (Bozzato, Eiter, and Serafini 2018; 2014) to simple CKRs in SROIQ-RL with ranked contextual hierarchies. Additional rules allow to determine (and constrain) the level of the defeasible axioms in order to prefer the models in which the most specific axioms hold. We show that the translation provides a sound and complete materialization calculus for instance checking and conjunctive query answering on such sCKR.

While this kind of hierarchical organization of knowledge is not new (Serafini and Homola 2012), in this work we show that combining the kind of defeasible reasoning introduced in (Bozzato, Eiter, and Serafini 2018) and inheritance in contextual hierarchies is non-trivial and introduces a serious degree of complexity in dealing with the conflicting exceptions on a hierarchical branch. The choice to restrict to ranked hierarchies is supported by the real-world usages of CKR and it allows us to provide an intuitive definition of the datalog translation by means of weak constraints. Complexity has also a role in these restrictions: our goal is a balance between computational cost and intuitive modelling. The paper is organized as follows. After a brief summary of the preliminaries in the next section, we define the syntax and semantics of simple CKRs with ranked contextual hierarchies; we then address the complexity of reasoning in the defined framework and we briefly discuss alternative definitions for the proposed model ordering. In the subsequent section we present the datalog translation and show its correctness for instance checking with respect to the semantics of simple CKRs on ranked hierarchies. We then provide a comparison of our approach with related works for defining non-monotonicity in description logics and context representation and we conclude by providing some directions for future research.

Further details of the language, the complexity results and the datalog translation are provided in an external appendix (Bozzato, Serafini, and Eiter 2018).

### Preliminaries

#### Description Logics and SROIQ-RL Language

We assume the usual concepts of description logics (Baader et al. 2003) and the definition of the logic SROIQ (Horrocks, Kutz, and Sattler 2006): we summarize in the following the basic definitions needed in this paper.

A DL vocabulary \( \Sigma \) consists of the mutually disjoint countably infinite sets NC of atomic concepts, NR of atomic roles, and NI of individual constants. Complex concepts (complex roles) are recursively defined as the smallest sets containing all concepts and roles that can be inductively constructed using the constructors of the considered DL language (see (Horrocks, Kutz, and Sattler 2006) for SROIQ).\(^1\) A SROIQ knowledge base \( \mathcal{K} = (\mathcal{T}, \mathcal{R}, \mathcal{A}) \) consists of: a TBox \( \mathcal{T} \) containing general concept inclusion (GCI) axioms \( \mathcal{C} \subseteq \mathcal{D} \) and concept equivalence axioms \( \mathcal{C} \equiv \mathcal{D} \), where \( \mathcal{C}, \mathcal{D} \) are concepts; an RBox \( \mathcal{R} \) containing role inclusion (RIA) axioms \( \mathcal{S} \subseteq \mathcal{R} \), reflexivity, and role disjointness axioms, where \( \mathcal{S}, \mathcal{R} \) are roles; an ABox \( \mathcal{A} \) composed of assertions of the forms \( \mathcal{D}(a), \mathcal{R}(a,b), \neg \mathcal{R}(a,b), a = b, \) and \( a \neq b \), with \( a, b \in \mathcal{NI} \). In \( \mathcal{K} \) each role in min-cardinality and self restrictions as well as in (ir)reflexivity, asymmetry, and disjointness axioms must be simple and the RBox is required to be regular (Horrocks, Kutz, and Sattler 2006).

A DL interpretation is a pair \( I = (\Delta^t, \Delta^r) \) where \( \Delta^t \) is a non-empty set called domain and \( \Delta^r \) is the interpretation function which assigns denotations for language elements: \( a^t \in \Delta^t \), for \( a \in \mathcal{NI} \); \( \Delta^r \subseteq \Delta^t \), for \( \mathcal{A} \in \mathcal{NC} \); \( R^t \subseteq \Delta^t \times \Delta^t \), for \( R \in \mathcal{NR} \). The interpretation of non-atomic concepts and roles is defined by the evaluation of their description logic operators (see (Horrocks, Kutz, and Sattler 2006) for SROIQ). An interpretation \( I \) satisfies an axiom \( \phi \), denoted \( I \models I \phi \), if it verifies the respective semantic condition, in particular: for \( \phi = D(a) \), \( a^t \in \Delta^t \); for \( \phi = R(a,b) \), \( \langle a^t, b^t \rangle \in R^t \); for \( \phi = (a = b) \), \( a^t = b^t \) (and similarly for \( \neq \)); for \( \phi = C \subseteq D \), \( C^t \subseteq \Delta^t \) (and similarly for RIA's). \( I \) is a model of \( \mathcal{K} \), denoted \( I \models I \mathcal{K} \), if it satisfies all axioms of \( \mathcal{K} \).

\(^1\)For ease of reference, we summarize the syntax and semantics of SROIQ constructors and axioms as a table in (Bozzato, Serafini, and Eiter 2018).
Without loss of generality, we adopt the standard name assumption (SNA) in the DL context (Eiter et al. 2008). We consider an infinite subset \( \mathbb{N}_I \subseteq \mathbb{N} \) of individual constants, called standard names s.t. in every interpretation \( I \) we have (i) \( \Delta' = \mathbb{N}_I = \{ c \mid c \in \mathbb{N} \} \); (ii) \( c^2 \neq d^2 \), for every distinct \( c,d \in \mathbb{N}_I \). Thus, we may assume that \( \Delta' = \mathbb{N}_I \) and \( c^2 = c \) for each \( c \in \mathbb{N}_I \). The unique name assumption (UNA) can be enforced by assertions \( c \neq d \) for all constants in \( \mathbb{N} \setminus \mathbb{N}_I \) resp. occurring in the knowledge base.\(^2\)

As in (Bozzato, Eiter, and Serafini 2018), we work on a restriction of the \( SROIQ \) syntax called \( SROIQ-RL \) (which corresponds to OWL-RL (Motik et al. 2009)). We restrict as follows left-side concepts \( C \) and right-side concepts \( D \):

\[
C := A \mid \{ a \} \mid C \sqcap C \mid C \sqcup C \mid \exists R.C \mid \exists R.\top \tag{1}
\]

\[
D := A \mid \neg C \mid D \sqcap D \mid \exists R.\{ a \} \mid \forall R.D \mid \leq nR.\top \tag{2}
\]

where \( A \in \mathbb{NC}, R \in \mathbb{NR} \) and \( n \in \{ 0, 1 \} \). A both-side concept is an expression that is both a left- and right-side concept. \( SROIQ-RL \) TBox axioms can only take the form \( C \sqsubseteq D \), where \( C \) is a left-side and \( D \) is a right-side or \( E \not\equiv F \), where \( E \) and \( F \) are both-side concepts. A \( SROIQ-RL \) RBox can contain all role axioms of \( SROIQ \) except Ref\((R)\), \( SROIQ-RL \) ABox concept assertions can only be of form \( D(a) \), where \( D \) is a right-side concept.

**Datalog Programs and Answer Sets**

We express our rules in datalog with (default) negation under answer sets semantics (Gelfond and Lifschitz 1991): however, in order to represent preferences across different overridings we use the mechanism of weak constraints (Leone et al. 2002).

**Syntax.** A signature is a tuple \( \langle \mathbb{C}, \mathbb{P} \rangle \) of a finite set \( \mathbb{C} \) of constants and a finite set \( \mathbb{P} \) of predicates. We assume a set \( \mathbb{V} \) of variables; the elements of \( \mathbb{C} \cup \mathbb{V} \) are terms. An atom is of the form \( p(t_1,...,t_n) \) where \( p \in \mathbb{P} \) and \( t_1,...,t_n \) are terms.

A (datalog) rule \( r \) is an expression of the form

\[
a \leftarrow b_1,...,b_k, \text{not } b_{k+1},...,\text{not } b_m. \quad \tag{3}
\]

where \( a,b_1,...,b_m \) are atoms and \( \text{not} \) is the negation as failure symbol (NAF). We denote with \( \text{Head}(r) \) the head \( a \) of rule \( r \) and with \( \text{Body}(r) = \{ b_1,...,b_k, \text{not } b_{k+1},...,\text{not } b_m \} \) the body of \( r \), respectively. We allow that \( a \) is missing (constraint), viewing \( a \) as logical constant for falsity. A weak constraint is an expression of the form:

\[
\leftarrow b_1,...,b_k, \text{not } b_{k+1},...,\text{not } b_m. \quad [w : l] \quad \tag{4}
\]

where \( b_1,...,b_m \) are atoms, while \( w \) (weight) and \( l \) (level) are variables of positive integer constants. A (datalog) program \( P \) is a finite set of rules; in case \( P \) contains weak constraints, we denote with \( \text{WC}(P) \) the set of its weak constraints and with \( \text{R}(P) \) the set of other rules. An atom (rule etc.) is ground, if no variables occur in it. A ground substitution \( \sigma \) for \( \langle \mathbb{C}, \mathbb{P} \rangle \) is any function \( \sigma : \mathbb{V} \rightarrow \mathbb{C} \); the ground instance of an atom (rule, etc.) \( \chi \) from \( \sigma \), denoted \( \chi^\sigma \), is obtained by replacing in \( \chi \) each occurrence of variable \( v \in \mathbb{V} \) with \( \sigma(v) \). A fact \( H \) is a ground rule \( r \), \( grnd(r) \), is the set of all ground instances of \( r \), and the grounding of a program \( P \) is \( grnd(P) = \bigcup_{r \in P} grnd(r) \).

**Semantics.** Given a program \( P \), the (Herbrand) universe \( \mathcal{U}_P \) of \( P \) is the set of all constants occurring in \( P \) and the (Herbrand) base \( B_P \) of \( P \) is the set of all the ground atoms constructable from the predicates in \( P \) and the constants in \( \mathcal{U}_P \). An interpretation \( I \subseteq B_P \) is any subset of \( B_P \). An atom \( I \) is true in \( I \), denoted \( I \models I \), if \( I \models I \).

Given a rule \( r \in grnd(P) \), we say that \( \text{Body}(r) \) is true in \( I \), denoted \( I \models \text{Body}(r) \), if (i) \( I \models b \) for each atom \( b \in \text{Body}(r) \) and (ii) \( I \not\models b \) for each atom \( \text{not } b \in \text{Body}(r) \). A rule \( r \) is satisfied in \( I \), denoted \( I \models r \), if either \( I \models \text{Head}(r) \) or \( I \not\models \text{Body}(r) \). An interpretation \( I \) is a model of \( P \), denoted \( I \models P \), if \( I \models r \) for each \( r \in grnd(P) \); moreover, \( I \) is minimal, if \( I' \not\models P \) for each subset \( I' \subset I \).

Given an interpretation \( I \) for \( P \), the (Gelfond-Lifschitz) reduct of \( P \) w.r.t. \( I \), denoted by \( Grd(I) \), is the set of rules obtained from \( grnd(P) \) by (i) removing every rule \( r \) such that \( I \models l \) for some \( l \in \text{Body}(r) \); and (ii) removing the NAF part from the bodies of the remaining rules. Then \( I \) is an answer set of \( P \), if \( I \models \text{Facts} \) is a minimal model of \( Grd(I) \); the minimal model is unique and exists iff \( Grd(I) \) has some model. Moreover, if \( M \) is an answer set for \( P \), then \( M \) is a minimal model of \( P \). In presence of weak constraints, we want to compute the answer sets of \( R(P) \) that minimizes the weight of the violated weak constraints at the higher priority levels: this is formally achieved by minimizing an objective function \( H^\alpha(S) \) for \( S \) answer set of \( P \) (see (Leone et al. 2002)). For a program \( P \) with weak constraints, \( S \) is an optimal answer set of \( P \) iff: (1) \( S \) is an answer set for \( R(P) \); (2) \( H^\alpha(S) \) is minimal over all answer sets of \( R(P) \). We say that an atom \( a \in B_P \) is a consequence of \( P \) and we write \( P \models a \) iff for every (optimal) answer set \( M \) of \( P \) we have that \( M \models a \).

**Simple CKR with Justifiable Exceptions**

In this section we provide a simplified definition of CKR w.r.t. the original formulation presented in (Bozzato, Eiter, and Serafini 2018; Bozzato and Serafini 2013; Bozzato, Eiter, and Serafini 2014). Intuitively, in this version of the framework, a CKR is still a two layered structure where the upper layer describes the contextual structure and the lower layer contains the local contextualized knowledge. However, in order to emphasize the role of the coverage relation, we simplify the upper layer to be a poset based on such relation.

**Syntax**

The upper layer of our simplification of CKR is defined by a poset of context names over a (strict) coverage relation. Consider a non empty-set of context names \( \mathbb{N} \subseteq \mathbb{N} \). We define a coverage relation \( \ll \mathbb{N} \times \mathbb{N} \). Given context names \( c_1, c_2 \in \mathbb{N} \), we say that \( c_2 \) covers \( c_1 \) if \( c_1 \ll c_2 \). The coverage relation \( \ll \) is a strict partial order relation on \( \mathbb{N} \), i.e. it is irreflexive and transitive. Intuitively, \( c_1 \ll c_2 \) means that \( c_2 \) is a more general context w.r.t. \( c_1 \), in the sense that \( c_2 \) refers
to a portion of the world that covers the portion of the world described by c₁ (Serafini and Homola 2012). We may use the non-strict relation c₁ ≤ c₂ to indicate that c₁ can be covered by c₂ or is the same context. Moreover, we write c₁ ≃ c₂ to denote the direct coverage relation, i.e. if c₁ < c₂ and there does not exist a c₃ s.t. c₁ < c₃ < c₂.

We can now define the language used in the local contexts to express their knowledge.

**Definition 1** (Contextual language). Given a set of context names N, for every description language Lₓ we define $L_{x,N}$ as the description language $L$ with the following additional rule for concept and role formation: eval($X$, c) is a concept (resp. role) of $L_{x,N}$ if $X$ is a concept (resp. role) of $L_x$ and $c ∈ N$.

**Definition 2** (defeasible axiom). A defeasible axiom is any expression of the form $\alpha \in L_x$ where $\alpha$ is an axiom of $L_x$.

The DL language $L^D_{x,N}$ extends $L_{x,N}$ with the set of defeasible axioms in $L_x$.

Notice that no eval nesting is allowed and eval can not appear in defeasible axioms. Using these definitions, we are ready to formulate our definition of contextualized repository.

**Definition 3** (Simple Contextualized Knowledge Repository, sCKR). A Simple Contextualized Knowledge Repository (sCKR) over $Σ$ and N is a structure $R = (Σ, K_N)$ where:

- $Σ$ is a poset $(N, <)$ and
- $K_N = \{K_c|c ∈ N\}$ for each context name $c ∈ N$. $K_c$ is a DL knowledge base over $L^D_{x,N}$.

**Example 1.** In this example, we show that the coverage structure $Σ$ can be defined as the product of different contextual dimensions as in (Serafini and Homola 2012), which define different aspects of the situations described by local contexts.

Let us consider a sCKR $R_{org} = (Σ, K_N)$ describing the organization of a large international corporation: the organization wants to define different policies with respect to its local branches (location dimension) and time period (time dimension). The dimensional values of these dimensions are organized in ranked hierarchies as shown in Figure 1. By considering the combination of such dimensional values, the resulting sCKR hierarchy of contexts $Σ$ is shown in Figure 2.

By using defeasible axioms in different contexts, the corporation can enforce policies that hold in a particular area or time period. In particular, the corporation is active in different fields: Musical instruments (M), Electronics (E), Robotics (R) and Vehicles (V). A supervisor (S) can be assigned to manage only one of these fields. By putting defeasible axioms in

![Figure 1: Dimensional values of example sCKR.](image)

the correct contexts in $K_N$, we can assign local supervisors to their field:

$C_{2018, EU}^1 : \{D(S ⊑ E), D(S ⊑ R)\}$

For example, in $C_{2018, EU}$ we say that supervisors are assigned to Electronics, while in the sub-context for $C_{2018, DE}$ we contradict this by assigning all local supervisors to the Musical instruments area. The situation is similar in the Asian branch.

In the most local contexts we have information about the instances: for the first semester S1, both in the German and Japanese branch the supervisors i₁ and i₂ override all upper defeasible axioms. In the second semester S2, however, different assignments of areas is possible by instantiating the defeasible axioms in one branch: intuitively, we want to prefer the interpretations that override the higher defeasible axioms in $C_{2018, EU}$ and $C_{2018, DE}$.

Note that sCKRs restrict the notion of CKR in (Bozzato, Eiter, and Serafini 2018; Bozzato and Serafini 2013; Bozzato, Eiter, and Serafini 2014): (i) global metaknowledge is restricted only to a poset of context names: in particular, no context classes and relations are allowed (other than the coverage context relation); (ii) there is no global object knowledge: intuitively, this can be however represented by adding a context at the root of poset $Σ$; (iii) each context has only one associated knowledge module; (iv) eval expressions can be only defined on single contexts (as no context classes are available). This restrictions serve to concentrate on the aspects of defeasibility across contexts: any sCKR can be easily translated in a CKR using the intuitive transformations above (and by suitably adapting the interpretation of coverage context relation like in the following definition of semantics).

In this work, we concentrate on simple CKRs defined on ranked contextual hierarchies: intuitively, ranked hierarchies identify context posets in which a notion of level can be defined. While this restriction limits the form of the contextual

![Figure 2: Context hierarchy and contents of example sCKR.](image)
structures, it allows us to simplify the notion of preference on defeasible axioms: basically, the preference abstracts from the partial order defined by the coverage structure and only considers the linear order of its levels.

Given a sCKR $\mathcal{R} = (\mathcal{C}, \mathcal{K}_\mathcal{R})$, we say that a context $c \in \mathcal{C}$ is a root context if there does not exist any other $c' \in \mathcal{C}$ s.t. $c < c'$. Given $c_a, c_b \in \mathcal{N}$ with $c_a \prec c_b$, a path from $c_a$ to $c_b$ is a sequence of contexts $\pi = (c_a, \ldots, c_b)$ such that $c_a \prec c_0 \prec \ldots \prec c_n = c_b$. The length of a path is $|\pi|$. We say that a contextual hierarchy (poset) $\mathcal{C} = (\mathcal{N}, \prec)$ is ranked if, for every root context $c \in \mathcal{C}$ and for every context $c' \in \mathcal{C}$ with $c \prec c'$, all of the paths from $c$ to $c'$ have the same length.

We call level function the function $l : \mathcal{N} \rightarrow \mathbb{N}$ (i.e. mapping context names to natural numbers) defined as:

$$l(c) = \begin{cases} 0, & \text{if } c \text{ is root} \\ 1 + \max(l(c'), |c < c'|), & \text{otherwise} \end{cases}$$

While the choice of ranked hierarchies limits the scope of the possible contextual structures, we note that this case covers the form of context structures relevant in the real application of the CKR framework: the context hierarchies generated by the combination of ranked hierarchies of contextual dimensions (Serafini and Homola 2012).

**Example 2.** We note that the hierarchy $\mathcal{C}$ of previous example is clearly a ranked hierarchy by the above definition. In particular, it is easy to verify that all products of ranked hierarchies (in our case, of dimensional values hierarchies) produce a ranked hierarchy. The level $l(c)$ associated to each context $c$ of the structure is shown on the left part of Figure 2.

We can define an ordering on the defeasible axioms based on the level of the contexts in which they appear: formally, given $D(\alpha) \in \mathcal{K}_\mathcal{R}$, we say that the level $l(\alpha)$ of $D(\alpha)$ is $l(\alpha) = n$. Note that this allows us to abstract from the ordering on defeasible axioms defined by the coverage structure and map it to the linear order defined by the level ordering.

**Semantics**

An interpretation for a sCKR is simply a set of interpretations for each local context.

**Definition 4 (sCKR interpretation).** An interpretation for $\mathcal{L}_{\mathcal{S}, \mathcal{N}}$ is a family $\mathcal{I} = \{I(c)\}_{c \in \mathcal{N}}$ of $\mathcal{L}_{\mathcal{S}}$ interpretations, such that $\Delta^{I(\cdot)} = \Delta^{I(\cdot)}$ and $a^{I(\cdot)} = a^{I(\cdot)}$, for every $a \in \text{NI}_\mathcal{S}$ and $c, c' \in \mathcal{N}$.

The interpretation of concepts and role expressions in $\mathcal{L}_{\mathcal{S}, \mathcal{N}}$ is obtained by extending the standard interpretation with the following clause: for every $c \in \mathcal{N}$, eval$X, c'\Delta^{I(\cdot)}$ = $X^{I(\cdot)}$.

We consider the notion of axiom instantiation and clashing assumption as defined in (Bozzato, Eiter, and Serafini 2018).

**Definition 5 (axiom instantiation).** Given an axiom $\alpha \in \mathcal{L}_{\mathcal{S}}$ with FO-translation $\forall x, \phi_\alpha(x)$, the instantiation of $\alpha$ with a tuple $e$ of individuals in NI, written $a(\alpha)$, is the specialization of $\alpha$ to $e$, i.e., $\phi_\alpha(e)$, depending on the type of $\alpha$.

As shown in (Bozzato, Eiter, and Serafini 2018), for SROIQ-RL this translation can be given so that it is semantically equivalent to a conjunction of universal Horn clauses.

**Definition 6** (clashing assumptions and sets). A clashing assumption for a context $c$ is a pair $(\alpha, e)$ such that $a(\alpha)$ is an axiom instantiation of $\alpha$, and $D(\alpha) \in \mathcal{K}_\mathcal{R}$ is a defeasible axiom of some $\alpha' > c$. A clashing set for $(\alpha, e)$ is a satisfiable set $S$ of ABox assertions such that $S \cup \{a(\alpha)\}$ is unsatisfiable.

A clashing assumption $(\alpha, e)$ represents that $a(\alpha)$ is not (DL-)satisfiable, and a clashing set $S$ provides an assertional “justification” for the assumption of local overriding of $a$ on $e$. We account for this and extend our interpretations.

**Definition 7** (CAS-interpretation). A CAS-interpretation is a pair $\mathcal{I}_{\text{CAS}} = (\mathcal{I}, \chi)$ where $\mathcal{I}$ is an interpretation and $\chi$ maps every $c \in \mathcal{N}$ to a set $\chi(c)$ of clashing assumptions for $c$.

**Definition 8** (CAS-model). Given a sCKR $\mathcal{R}$, a CAS-interpretation $\mathcal{I}_{\text{CAS}} = (\mathcal{I}, \chi)$ is a CAS-model for $\mathcal{R}$ (denoted $\mathcal{R} \models_{\text{CAS}} \chi$), if the following holds:

(i) for every $\alpha \in \mathcal{K}_c$ (strict axiom), and $c' \leq c$, $I(c') \models \alpha$;
(ii) for every $D(\alpha) \in \mathcal{K}_c$, for every $c' < c$, if $d \notin \{e \mid \langle \alpha, e \rangle \in \chi(c')\}$, then $I(c') \models \phi_\alpha(d)$.

In order to define an order across clashing assumption sets, we define the notion of their profile. Let $n$ be the maximum level of an sCKR $\mathcal{R}$, the global profile of a clashing assumption map $\chi$ denoted $p(\chi)$ is a vector $(l_0, \ldots, l_n)$, where each $l_i$ is the number of clashing assumptions $(\alpha, e) \in \chi(c)$ for all the contexts $c$ such that $l(\alpha) = i$. We define a lexicographic order over such profiles as follows. For $p(\chi_1) = (l_{i_1}, \ldots, l_{i_n})$ and $p(\chi_2) = (l_{j_1}, \ldots, l_{j_n})$, we have $p(\chi_1) < p(\chi_2)$ if some $j_k \neq 0$. We order profiles as $p(\chi_1) < p(\chi_2)$ if $l_{j_k} < l_{j_k}$ for the maximum $k$. We consider the case $p(\chi) = (0, \ldots, 0)$ corresponding to an interpretation with no overridings is always minor (thus “preferred”) than any profile having an overriding in one of the levels. Intuitively, this definition of ordering on the overridings (clashing assumptions) is only dependent on the order of defeasible axioms in the level structure: defeasible axioms at the lower levels are preferred (not overridden) with respect to the ones higher in the covering ordering.

Two DL interpretations $I_1$ and $I_2$ are NI-congruent, if for $c^{I_1} = c^{I_2}$ holds for every $c \in \mathcal{N}$. This naturally extends to a (CAS) interpretation $\mathcal{I} = (\mathcal{I}, \chi)$ is justified by considering all context interpretations $\mathcal{I}(c)$ in $\mathcal{I}$. We say that a clashing assumption $(\alpha, e) \in \chi(c)$ is justified for a CAS model $\mathcal{I} = (\mathcal{I}, \chi)$, if some clashing set $S$ exists such that, for every CAS-model $\mathcal{I} = (\mathcal{I}, \chi)$ of $\mathcal{R}$ that is NI-congruent with $\mathcal{I}$, it holds that $I(c) \models S(\alpha, e)$.

**Definition 9** (justified CAS model). A CAS model $\mathcal{I}_{\text{CAS}}$ of a sCKR $\mathcal{R}$ is justified, if every $(\alpha, e) \in \mathcal{I} = (\mathcal{I}, \chi)$ is justified in $\mathcal{R}$.
Definition 10 (CKR model). An interpretation $\mathcal{I}$ is a CKR model of a $s$CKR $\mathcal{R}$ (in symbols, $\mathcal{R} \models \mathcal{I}$) if:

- $\mathcal{R}$ has some justified CAS model $\mathcal{I}_{\text{CAS}} = (\mathcal{I}, \chi)$;
- there exists no justified CAS model $\mathcal{I}'_{\text{CAS}} = (\mathcal{I}', \chi')$ that is preferred to $\mathcal{I}_{\text{CAS}}$.

Example 3. Considering the example $s$CKR $\mathcal{R}_{\text{org}}$ of previous examples, we note that different justified CAS models are possible, corresponding to the different assignments of $i_1$ and $i_2$ to the alternative classes of products denoted by the defeasible axioms in their continent and country of reference.

By local assertions, in $S1$ contexts $c_{S1,DE}$ and $c_{S1,JP}$ the two supervisors are already locally assigned to R and E areas, thus contradicting all of the upper defeasible axioms: this implies that, in all CAS-models that are justified, their local clashing assumptions contain:

$$\chi(c_{S1,DE}) = (S \subseteq E, i_1), (S \subseteq M, i_2)$$
$$\chi(c_{S1,JP}) = (S \subseteq R, i_2), (S \subseteq V, i_2)$$

However, different solutions are possible for the $S2$ contexts $c_{S2,DE}$ and $c_{S2,JP}$, in which the two supervisors can obey the rules of the country or continent of reference. We have four possible assignments, corresponding to four different clashing assumptions maps for the two contexts:

$$\chi_1(c_{S2,DE}) = (S \subseteq E, i_1), (S \subseteq M, i_1)$$
$$\chi_1(c_{S2,JP}) = (S \subseteq R, i_2), (S \subseteq V, i_2)$$

$$\chi_2(c_{S2,DE}) = (S \subseteq M, i_1), (S \subseteq E, i_1)$$
$$\chi_2(c_{S2,JP}) = (S \subseteq V, i_2), (S \subseteq R, i_2)$$

$$\chi_3(c_{S2,DE}) = (S \subseteq M, i_1), (S \subseteq E, i_2)$$
$$\chi_3(c_{S2,JP}) = (S \subseteq V, i_2), (S \subseteq R, i_2)$$

$$\chi_4(c_{S2,DE}) = (S \subseteq E, i_2), (S \subseteq M, i_2)$$
$$\chi_4(c_{S2,JP}) = (S \subseteq R, i_2), (S \subseteq V, i_2)$$

In order to decide the preferred model(s) across these alternative solutions, we compare these clashing assumption sets by their global profile:

$$p(\chi_1) = (0, 2, 4, 0)$$
$$p(\chi_2) = (0, 4, 2, 0)$$
$$p(\chi_3) = (0, 3, 3, 0)$$
$$p(\chi_4) = (0, 3, 3, 0)$$

Thus, following the definition of preference, there is one preferred model for our $s$CKR which corresponds to $\chi_1$: note that it corresponds to the intended interpretation in which the defeasible rules associated to the continents win over the more general rules asserted for the continents.

Discussion: Preference Relation

We consider in this section some major reasoning tasks similar to those in (Bozzato, Eiter, and Serafini 2018), viz.

- $c$-entailment, where $\mathcal{R} \models c : \alpha$ denotes for an axiom $\alpha$ that $\alpha$ is entailed in every CKR-model of $\mathcal{R}$ at context $c$ (i.e., $I(c) \models \alpha$), and

- the problem of (Boolean) conjunctive query (CQ) answering $\mathcal{R} \models \exists y_1 \ldots \exists y_m \gamma$, where $\gamma = y_1 \wedge \ldots \wedge y_m$ is a conjunction of atoms $\gamma_i = c_i : \alpha_i(t_i)$, where each $c_i$ is a context name and $\alpha_i(t_i)$ is an assertion in which variables occur, which is existentially closed.

It has been shown in (Bozzato, Eiter, and Serafini 2018) that justified CAS-model checking, i.e. deciding whether a given CAS-interpretation is a justified CAS-model of a given CKR $\mathcal{R}$ is feasible in polynomial time, and that satisfiability (existence of a CKR-model) is NP-complete. Furthermore, $c$-entailment testing and (Boolean) CQ-answering were shown to be coNP- and $\Pi_2^p$-complete problems, respectively.

For contextual hierarchies, while the complexity of satisfiability trivially remains unchanged, model checking is intractable, already for ranked hierarchies. As a consequence, the complexity of $c$-entailment increases, while CQ answering remains unchanged. In what follows, we assume the setting of (Bozzato, Eiter, and Serafini 2018) for the complexity analysis.

Proposition 1. Deciding whether a CAS-interpretation $\mathcal{I}_{\text{CAS}}$ of a $s$CKR $\mathcal{R}$ is a CKR-model is coNP-complete.

Informally, $\mathcal{I}_{\text{CAS}}$ can be refuted if it is not a justified CAS-model of $\mathcal{R}$, which can be checked in polynomial time using the techniques in (Bozzato, Eiter, and Serafini 2018), or some preferred model $\mathcal{I}_{\text{CAS}}$ exists; the latter can be guessed and checked in polynomial time. The coNP-hardness can be shown by a reduction from a variant of UNSAT.

Theorem 1. Given an $s$CKR $\mathcal{R}$, a context name $c$ and an axiom $\alpha$, whether $\mathcal{R} \models c : \alpha$ is $\Delta_2^p$-complete for profile-based preference.

Proof (Sketch). In the general case, we can guess a CKR-model $\mathcal{I}_{\text{CAS}}$ of $\mathcal{R}$ such that $\mathcal{I}_{\text{CAS}} \not\models c : \alpha$ and verify the guess with an NP oracle. The $\Pi_2^p$ hardness is shown by a reduction from QBF evaluation.

For profile based comparison, we can compute the lexicographic maximum profile $p'$ of a CKR-model by extending a partial profile $(p_n, p_{n-1}, \ldots, p_i, \ldots, 0)$ using an NP oracle in polynomial time. We then can check with the NP oracle whether every justified CAS-model $\mathcal{I}_{\text{CAS}}$ having this profile fulfills $\mathcal{I}_{\text{CAS}} \not\models c : \alpha$.

The $\Delta_2^p$-hardness is shown by a reduction from deciding whether for the lexicographic maximum satisfying assignment of a SAT instance $E(x_1, \ldots, x_n)$ it holds that $x_n = 1$. \hfill \Box

Theorem 2. Deciding where an $s$CKR $\mathcal{R}$ entails a Boolean CQ $\gamma$ is $\Pi_2^p$-complete for profile-based preference.

Proof (Sketch). Similarly as for $c$-entailment, a CKR-model $\mathcal{I}_{\text{CAS}}$ that does not entail $\gamma$ can be guessed and checked with the help of an NP oracle (ask whether no preferred $\mathcal{I}_{\text{CAS}}$ exists and whether $\gamma$ is entailed), and likewise for profile-based preference. The $\Pi_2^p$-hardness is inherited from ordinary CKR. \hfill \Box

Data Complexity. Concerning the data complexity, i.e., the CKR $\mathcal{R}$ is fixed and only the assertions in the knowledge modules vary, we note that while CKR model checking remains coNP-complete, the complexity of $c$-entailment drops to $\Delta_2^p (O(\log n)) = \text{P}^\text{NP}$ [cf. (Eiter and Gottlob 1997)], as the problem can be decided with a constant number $k$ of rounds of parallel NP oracle queries; it is complete for this class. CQ entailment instead remains $\Pi_2^p$-complete.

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defeat can be studied (e.g. in the spirit of argumentation approaches (Bikakis and Antoniou 2010)), by also accounting for the feasibility in terms of complexity and implementation.

In particular, the current preference relation is based on a global comparison across clashing assumptions and the level of their defeasible axioms. In order to generalize the preference to non-ranked hierarchies, we can consider a local preference defined directly along the coverage relation: in particular, we can state that \( \chi_1(c) > \chi_2(c) \), if for every \( \langle \alpha_1, e \rangle \in \chi_1(c) \) with \( D(\alpha_1) \) at a context \( c_1 \), there exists an \( \langle \alpha_2, f \rangle \in \chi_2(c) \) \( \setminus \chi_1(c) \) with \( e_2 \) at context \( c_2 \) such that \( c_1 > c_2 \). This definition reflects the intuition that if we make in \( \chi_1(c) \) an exception at \( c_1 \), then a “more costly” exception should be made at a context \( c_2 \) below \( c_1 \) by \( \chi_2(c) \) that is not made by \( \chi_1(c) \). Model preference then can be defined from such local priority: \( \exists^{\text{CAS}}_1 = \langle \exists, \chi_1 \rangle \) is preferred to \( \exists^{\text{CAS}}_2 = \langle \exists, \chi_2 \rangle \) iff there exists some \( c \in N \) s.t. \( \chi_1(c) > \chi_2(c) \) and for no context \( c' \neq c \in N \) it holds that \( \chi_1(c') < \chi_2(c') \).

Some comments on the complexity of this characterization can be derived. As for local preference at a context \( c \), let for any context \( c'' \) above \( c \) denote \( X(c') \) the set of all clashing assumptions \( \langle \alpha, e \rangle \) for defeasible axioms at \( c' \) made at \( c \) in some CRK-model of \( S \). Call a context \( c' \) a connector for \( c \), if it directly covers \( c \) and for every \( c'' \), if \( c'' \) covers \( e' \), then \( c'' \) covers \( e \) (i.e., every path in the covers-graph from a node \( c'' \) above \( c \) must pass through \( c'' \)). Consider the following property of the local preference >:

(CP) If \( c' \) is a connector for \( c \) and (i) \( X(c') \subseteq \chi_1(c) \), (ii) \( X(c') \not\subseteq \chi_3(c) \), and (iii) \( \chi_1(c''') = \chi_2(c''') \), for each \( c'' \) such that \( c'' \neq c' \), then \( \chi_1(c) > \chi_2(c) \).

That is, the worst possible overriding at a connector for \( c \) is always less preferred, if the clashing assumptions agree on the contexts that are not above \( c' \) or not reachable from such contexts; this condition seems to be plausible for local preference. Then the following can be shown.

**Theorem 3.** Suppose \( S \) is a sCRK with global preference induced by a local preference > that is polynomial-time decidable and satisfies (CP). Then deciding \( \sigma \)-entailment \( \models \sigma \) is \( \Pi^2_1 \)-complete.

In fact, the \( \Pi^2_1 \)-hardness even holds for ranked hierarchies with three levels (for two levels, it remains coNP-complete).

### Datalog Translation with Preferences

In this section we present a datalog translation for reasoning on sCRK in SROIQ-RL. The translation directly builds on the definitions of the datalog translation proposed in (Bozzato, Eiter, and Serafini 2018), but introduce several modifications needed for the interpretation of coverage and local defeasible axioms.

We show that, under the assumption of a ranked context hierarchy, the reasoning process proposed in (Bozzato, Eiter, and Serafini 2018) can be naturally extended for reasoning with simple CRKs and local defeasible axioms: while in the original calculus inference is obtained from all answer sets of the resulting program (i.e. cautious reasoning), in this case we need to select the preferred models accordingly to the model ordering defined by sCRK models.

In the case of ranked hierarchies, by their linear level abstraction to the coverage structure, we can use the interpretation of levels provided by weak constraints (Leone et al. 2002) to compute the models that are preferred w.r.t. the ordering of their clashing assumptions (cf. Definition 10).

### Language and normal form.

As in the original version of the translation, we limit the defeasible axioms to the language of SROIQ-RLD: i.e. in defeasible axioms, \( D \cap D \) can not appear as a right-side concept and each right-side concept \( VR.D \) has \( D \in NC \). Also, we consider the normal form and normal form transformation proposed in (Bozzato, Eiter, and Serafini 2018) for the formulation of the rules (considering the formulas that can appear in the simple CRKs) and we assume again the Unique Name Assumption.

### Translation overview.

In the following we provide an overview of the components of the translation by showing some examples of rules.\(^3\) This translation naturally extends the one presented in (Bozzato, Eiter, and Serafini 2018) the non-trivial use of non monotonic negation and inference on negative literals by contradiction for the interpretation of exceptions is here extended to reason on local defeasible inheritance. As previously mentioned, in order to recognize the models minimizing the level of the overridden axioms, we use a set of weak constraints that define such preference over answer sets.

As in the original formulation (inspired by the materialization calculus in (Krötzsch 2010)), the translation includes sets of input rules (which encode DL axioms and signature in datalog), deduction rules (datalog rules providing instance level inference) and output rules (that encode in terms of a datalog fact the ABox assertion to be proved). The translation is composed by the following sets of rules:

**SROIQ-RL input rules:** rules in \( I_{g} \) translate to datalog facts (in a given context \( c \) ) SROIQ-RL axioms and signature. For example, atomic concept inclusions are translated with the rule \( A \sqsubseteq B \rightarrow \{ \text{subClass}(A, B, c) \} \). Since these rules refer to the interpretation of local knowledge, they are analogous to the original translation.

**SROIQ-RL deduction rules:** rules in \( P_{\ell} \) encode the inference rules for SROIQ-RL axioms: e.g., for atomic concept inclusions:

\[
\text{instd}(x, z, c, t) \leftarrow \text{subClass}(y, z, c), \text{instd}(x, y, c, t).
\]

As for the previous set of rules, since the interpretation of local knowledge is not modified, this set of rules is analogous to their original formulation.

**Global and local translations:** global input rules \( I_{gub}(\exists) \) encode the interpretation of the contextual structure and levels. E.g., \( c_1 < c_2 \rightarrow \{ \text{prec}(c_1, c_2) \} \) translates coverage across contexts as instances of the \( \text{prec} \) predicate, while \( l(c_1) = n \rightarrow \{ \text{level}(c_1, n + 1) \} \) represents the association between context and its level as level atom in the final program. Local input and deduction rules implement the interpretation of \( \text{eval} \): note that differently from (Bozzato, Eiter, and Serafini 2018), \( \text{eval} \) can be only defined over single contexts instead of context classes.

\(^3\)For completeness, the full set of rules can be found in (Bozzato, Serafini, and Eiter 2018).
Defeasible axioms input translations: defeasible input rules $I_D(S, c)$ declare that a local axiom is defeasible: differently from the original translation, the resulting atoms also contain the context in which the axiom has been introduced. For example, $D(A \subseteq B)$ in context $c$ translates to $\text{def\_subclass}(A, B, c)$.

Overriding rules: overriding rules in $P_D$ provide rules defining when an axiom of a certain form is locally overridden. W.r.t. original rules, this version of overriding rules has to consider the context in which the defeasible axiom has been declared (in order to determine the level of the overridden axiom). For example, for axioms of the form $D(A \subseteq B)$ in context $c$:

$$\text{ovr}(\text{subclass}, x, y, z, c_1, c_2) \leftarrow \text{def\_subclass}(y, z, c_1),$$

$$\text{prec}(c, c_1), \text{instd}(x, y, c, \text{main}),$$

$$\text{not\_test\_fails}(\text{nlit}(x, z, c)).$$

Note that $\text{test\_fails}$ is used to encode a proof by contradiction for the negative literals in the rule.

Inheritance rules: $P_D$ provides the rules for defeasible inheritance of axioms from the higher to the lower local contexts in the coverage structure. The definition of rules is similar to the original translation, but also considers the $\text{prec}$ relation across contexts to define the direction of the inheritance. E.g., the following rule propagates a (possibly defeasible) atomic concept inclusion axiom:

$$\text{instd}(x, z, c, t) \leftarrow \text{subclass}(y, z, c_1),$$

$$\text{instd}(x, y, c, t),$$

$$\text{prec}(c, c_1), \text{not\_ovr}(\text{subclass}, x, y, z, c_1, c).$$

Note that this rule propagates also to instances of strict axioms, since their overriding is never verified.

Test rules: the test rules in $P_D$ are used to instantiate and define the “environments” for the tests for negative literals in overriding rules. The rules are similar to the original translation, but need to be adapted to the new definition of $\text{ovr}$ atoms and $\text{prec}$ (i.e., they need to consider the context in which the axiom has been declared).

Overriding level preference rules and weak constraints: this new set of rules in $P_D$ defines and constrains (for each possible kind of defeasible axiom) the level at which overriding is preferred. Intuitively, by following the definition of the semantics, our goal is to prefer models in which the overridden axioms are the most general ones (i.e. defined in contexts with the highest level). For example, in the case of a defeasible atomic concept inclusions of the form $D(A \subseteq B)$, the overriding level of the axiom is defined by the rule:

$$\text{ovr\_level\_subclass}(x, y, z, c, n) \leftarrow \text{ovr}(\text{subclass}, x, y, z, c_1, c),$$

$$\text{level}(c_1, n).$$

The preference to overrides with higher level are then constrained using weak constraints:

$$\omega \leftarrow \text{ovr\_level\_subclass}(x, y, z, c, n). [1 : n]$$

Namely, answer sets that minimize violations of such constraints (i.e. overrides) at the higher levels of the hierarchy (i.e. with smaller values of $n$) are preferred.

Translation process. By the simplification of the CKR structure (and in particular with no definition of knowledge modules), the translation process is now straightforward. Given a sCKR $\mathcal{R} = (\mathcal{E}, K_0)$ in SROIQ-RLD normal form, a program $PK(\mathcal{R})$ that encodes query answering for $\mathcal{R}$ is obtained as:

1. the global program for $\mathcal{E}$ is built as:

$$PG(\mathcal{E}) = I_{\text{glob}}(\mathcal{E})$$

2. for each $c \in \mathbb{N}$, we define each local program $PC(c, \mathcal{R})$ as:

$$PC(c, \mathcal{R}) = P_I(\mathcal{R}) \cup P_{\text{bool}} \cup P_D(\mathcal{R}) \cup I_{\text{def}}(K_c, c) \cup I_{\text{ovr}}(K_c, c) \cup I_{\text{ld}}(K_c, c)$$

where $K_c^D = \{ \alpha \in L_S | D(\alpha) \in K_c \}$.

3. The CKR program $PK(\mathcal{R})$ is defined as:

$$PK(\mathcal{R}) = PG(\mathcal{E}) \cup \bigcup_{c \in \mathbb{N}} PC(c, \mathcal{R})$$

Query answering $R \models \alpha$ is then obtained by testing whether the (instance) query, translated to datalog by $O(\alpha, c)$, is a consequence of $PK(\mathcal{R})$, i.e., whether $PK(\mathcal{R}) \models O(\alpha, c)$ holds. Analogously, this can be extended to conjunctive queries as in (Bozzato, Eiter, and Serafini 2018).

Correctness. We can show that the presented translation provides a sound and complete materialization calculus for instance checking (with respect to c-entailment) and conjunctive query answering for normal form SROIQ-RLD simple CKRs with ranked context hierarchies. In presence of weak constraints, the result is shown by extending the correspondence between minimal justified CKR-models of $\mathcal{R}$ and answer sets of $PK(\mathcal{R})$ from (Bozzato, Eiter, and Serafini 2018) to optimal answer sets. As in the original formulation, the adoption of UNA and named models (i.e. restricting to models having a $\mathcal{N} \subseteq \mathcal{N}_1 \cup \mathcal{N}_2$ s.t. the interpretation of atomic concepts and roles belongs to $\mathcal{N}^I$) allows to concentrate on Herbrand models for $\mathcal{R}$, denoted as $\mathcal{S}_c$.

Let $\mathcal{S}_{\text{CAS}}$ be a justified named CAS-model: its set of corresponding overriding assumptions is defined as:

$$OVR(\mathcal{S}_{\text{CAS}}) = \{ ovr(p(\epsilon)) \mid (\alpha, \epsilon) \in \chi(\mathcal{R}), I_{\text{rl}}(\alpha, c) = p \}.$$ Then, given a CAS-interpretation $\mathcal{S}_{\text{CAS}} = (\mathcal{S}, \chi)$, we can build from its components a corresponding Herbrand interpretation $I(\mathcal{S}_{\text{CAS}})$ of the program $PK(\mathcal{R})$: the construction is similar to the one in (Bozzato, Eiter, and Serafini 2018), by suitably interpreting the newly added $\text{ovr\_level}$ facts.

It is then possible to show that the answer sets of the final program $PK(\mathcal{R})$ correspond with the least justified models of $\mathcal{R}$ by the following result:

**Lemma 1.** Let $\mathcal{R}$ be a sCKR in SROIQ-RLD normal form, then:

1. for every (named) justified clashing assumption $\chi$, the interpretation $S = I(\mathcal{S}(\chi)))$ is an answer set of $PK(\mathcal{R})$;
2. every answer set $S$ of $PK(\mathcal{R})$ is of the form $S = I(\mathcal{S}(\chi))$ with $\chi$ a (named) justified clashing assumption for $\mathcal{R}$.

**Proof (Sketch).** Intuitively, as the newly added weak constraint rules for model preference do not play a role in the computation of such answer sets, the result can be proved along the lines of Lemma 6 in (Bozzato, Eiter, and Serafini 2018) by showing that the answer sets of (the rules part of) $PK(\mathcal{R})$ coincide with the sets $S = I(\mathcal{S}(\chi))$ where $\chi$ is a justified clashing assumption of $\mathcal{R}$. 

In the case of a sCKR with a ranked context hierarchy, the correspondence with sCKR models is obtained by considering the interpretation of preference on answer sets defined by weak constraints on overrides and the notion of preference on justified CAS-models in the semantics.
Theorem 5. Let $\mathfrak{R}$ be a sCKR in SROIQ-RLD normal form with ranked context hierarchy. Then, $\hat{\mathfrak{S}}$ is a CKR model of $\mathfrak{R}$ iff there exists a (named) justified clashing assumption $\chi$ s.t. $I(\mathfrak{S}(\chi))$ is an optimal answer set of $PK(\mathfrak{R})$.

Proof (Sketch). The existence of a justified $\chi$ corresponds to the first condition of Definition 10 and is derived from Lemma 1. Then, $I(\mathfrak{S}(\chi))$ is optimal iff there does not exist a justified $\chi'$ s.t. $p(\chi') < p(\chi)$: this is verified by the correspondence of the lexicographic order on profiles with the order induced by objective function $H^p(\mathfrak{S})$ on answer sets. $\square$

The correctness result for instance checking is then obtained as consequence of previous results:

Theorem 4. Let $\mathfrak{R}$ be a sCKR in SROIQ-RLD normal form with ranked context hierarchy, and let $\alpha$ and $\mathfrak{C}$ such that $O(\alpha, \mathfrak{C})$ is defined. Then $\mathfrak{S} \models \alpha : a$ iff $PK(\mathfrak{R}) \models O(\alpha, \mathfrak{C})$.

Similarly to the original translation, this result can be extended to answering of a conjunctive query $\mathcal{Q}$, by constructing its translation $O(\mathcal{Q})$ by applying output rules to its atoms.

Theorem 5. Let $\mathfrak{R}$ be a sCKR in SROIQ-RLD normal form with ranked context hierarchy, and let $\mathcal{Q} = \exists y(\gamma)$. Then $\mathfrak{S} \models \mathcal{Q}$ iff $PK(\mathfrak{R}) \models O(\mathcal{Q})$.

Related Work

We provide a brief summary of the works related to CKR that include notions of defeasibility in contextual systems and in DLs: we refer to (Bazzanato, Eiter, and Serafini 2018) for an extended comparison of the framework with related work.

The first analogy we consider is with non-monotonic multi-context systems (MCS) (Brewka and Eiter 2007): the idea of multi-context systems is to align knowledge from different contexts (locally based on possibly different monotonic and non-monotonic logics) in a single system using (possibly non-monotonic) bridge rules. The semantics of nonmonotonic MCS is defined in terms of equilibria, a collection of local models that verifies the context knowledge content and the knowledge propagated through bridge rules. As noted in (Bazzanato, Eiter, and Serafini 2018), CKRs with defeasible inheritance may be realized in the MCS framework by controlling knowledge propagation by bridge rules. On the other hand, in sCKR the knowledge propagation is directly defined by the coverage semantics and not by explicitly asserted bridge rules. A different non-monotonic semantics for MCS was proposed in (Bikakis and Antoniou 2010). Their semantics is based on the common argumentation semantics of Defeasible Logic extended with distribution of knowledge and preferences across contexts. Intuitively, if consequences are derived using “external” knowledge by mapping rules, conflicts over a literal are resolved using a local context preference, where clashes across arguments are considered. In comparison, in CKR preference is global and defined by the interpretation of coverage; moreover, as above, the coverage structure also defines the knowledge propagation, that would need to be explicitly stated in MCS using bridge rules. Our notion of overriding compares to a “conflict” among two arguments for conflicting literals: differently from MCS, we allow reasoning by cases in multiple preferred models.

In description logics, different proposals have been made to incorporate notions of “normal” concepts and defeasible subsumptions. For example, (Giordano et al. 2013) formalize in their logic $\mathcal{ALC}+\mathcal{T}_{\text{min}}$ the intuition that a prototypical element of a concept $C$ is a “typical element” of $C$. Formally, each concept $C$ in the language of $\mathcal{ALC}$ is associated with an extended concept $\text{T}(C)$ representing its “typical” instances. The typicality operator $\text{T}$ is interpreted by extending DL interpretations with a preference relation on the domain: each element in $\text{T}(C)$ is a member of $C$ minimal w.r.t. such preference. In order to enforce that elements of a concept $C$ must belong to its typical subclass $\text{T}(C)$, the models are restricted to the ones which minimize the set of exceptional instances: the resulting logic is $\mathcal{ALC}+\mathcal{T}_{\text{min}}$. Similarly to our approach, in this work membership of an element in a concept must be blocked; however, instead of using model minimization, in CKR exceptions have to be justified in terms of a semantic consequence. In CKR preference is not used to decide the membership to a concept, but the applicability of defeasible axioms and it is implicitly defined by the contextual structure. Another approach to represent overriding in DLs is (Bonatti et al. 2015): it proposes a family $\mathcal{DL}^\chi$ of non monotonic DLs defined by extending a generic base DL $\mathcal{DL}$ with an operator $\mathcal{NC}$ for normality concepts (i.e. “normal” instances of type $C$) and with defeasible inclusions (DIs) $C \sqsubseteq_1 D$, interpreted as “normally, instances of $C$ are instances of $D$, unless stated otherwise”. The semantics of a defeasible inclusion $C \sqsubseteq_1 D$ w.r.t. normal individuals $NE$ is defined to manage the conflicts of inclusions on $NE$: to decide which DIs should be overridden, a priority relation $\prec$ is defined on DIs. In particular, Bonatti et al. consider specificity relation, i.e., $(C_1 \sqsubseteq_1 D_1) \prec (C_2 \sqsubseteq_1 D_2)$ iff $C_1 \subseteq C_2$ holds and not $C_2 \not\subseteq C_1$. To reason in presence of DIs, Bonatti et al. provide a translation that compiles defeasible inclusions away into a $\mathcal{DL}$ based KB. The idea of individual exceptions and axiom overriding is similar in spirit to our approach. A difference can be noted in the definition of precedence between defeasible axioms: in our formalism, precedence is defined by the CKR contextual hierarchy. Moreover, our formalism has no notion of “normal” concepts (i.e. every individual is “normal” w.r.t. all axioms, but can be exceptional w.r.t. given defeasible axioms). As shown in (Bazzanato, Eiter, and Serafini 2018), similarly to this approach we can deal with property inheritance at the instance level: however, in case of clashing inheritances that can not be resolved using preference, our semantics allows to reason by cases on all alternative models.

Conclusions

In this paper we presented an extension to the CKR contextual framework to introduce a definition of defeasible axioms in local contexts and knowledge propagation (and overriding) along a coverage hierarchy on contexts. Considering the case of ranked hierarchies, we defined a preference relation across CKR models which prioritize the validity of defeasible axioms at more specific contexts by minimizing the level and number of overrides. Moreover, we provided an extension to the datalog translation proposed in (Bazzanato, Eiter, and Serafini 2018) based on the use weak constraints that is complete for instance checking with respect to the
presented semantics.

There are some possible directions for extending this work: first of all, as discussed above, we aim to study different notion of preference on defeasible axioms, both to reflect the intended reading of knowledge propagation and to restrict the complexity of reasoning. In this direction, it is interesting to extend the work to the case of general contextual hierarchies and other kinds of restricted hierarchies. Another aspect which needs further study is the interpretation of a defeasible eval operator, which would introduce a mean for defeasible propagation along local contexts and can interact with knowledge propagation along the hierarchy. Similarly, we can consider to introduce different contextual relations other than coverage (with different reading for context propagation) and study their interaction. With respect to the datalog translation, we are interested in considering different reasoning approaches or translations, both to extend the work to more expressive DLs and to identify CKR fragments in which reasoning is more efficient. Moreover, we plan to implement the presented translation in the CKRew (CKR datalog rewrite) prototype.

References


\*http://ckrew.fbk.eu/