

# Towards Explainable Inference about Object Motion Using Qualitative Reasoning

**Xiaoyu Ge, Jochen Renz, Hua Hua**  
{xiaoyu.ge, jochen.renz, hua.hua}@anu.edu.au  
Research School of Computer Science and Engineering  
The Australian National University  
Canberra, Australia

## Abstract

The capability of making explainable inferences regarding physical processes has long been desired. One fundamental physical process is object motion. Inferring what causes the motion of a group of objects can even be a challenging task for experts, e.g., in forensic science. Most of the work in the literature rely on physics simulation to draw such inferences. The simulation requires a precise model of the underlying domain to work well and is essentially a black-box from which one can hardly obtain any useful explanation.

By contrast, qualitative reasoning methods have the advantage in making transparent inferences with ambiguous information, which makes it suitable for this task. However, there has been no suitable qualitative theory proposed for object motion in three-dimensional space. We take this challenge and develop a qualitative theory for the motion of rigid objects. Based on this theory, we develop a reasoning method to solve a very interesting problem: Assuming there are several objects that were initially at rest and now have started to move. We want to infer what action causes the movement of these objects.

## Introduction

We are living in an era where an increasing number of AI agents entering into our daily lives and helping us with daily tasks such as household chores. To successfully perform these tasks, an AI agent needs to understand its surrounding environment and to be able to draw useful inferences based on their perceptual input. Living in a physical world requires AI be capable of inferring physical behaviours of everyday objects. This capability not only involves predicting what behaviours an object can have but also being able to figure out what causes their behaviours. In this paper, we focus on reasoning about object motion which is a most common physical behaviour of an object. Making an inference about object motion can be a challenging task for AI.

For example, Fig. 1a shows a scene where a set of blocks were initially at rest. At a certain time, there was an action made to exert an impulse at one of the blocks, which caused the movement of the blocks as depicted in Fig. 1b. When we observe this change, one natural question to ask is where and in which direction the impulse has been made. We humans can make such inference rapidly given only the information

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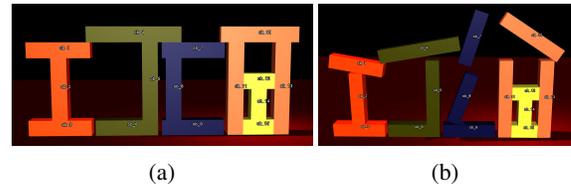


Figure 1: An example scenario (simulated in Mujoco) where we want to infer what action has been made to cause the illustrated movement of the objects.

obtained from our visual perception. The knowledge we have about the scene is ambiguous, e.g., we do not know exact physical parameters of the blocks, precise shapes or coordinates of their locations. However, we can still draw useful inferences based on this piece of knowledge and we can provide clear explanations of how the inference is derived. What human does in making the physical inference is conceptually similar (Hegarty 2010) to the methodology adopted by the qualitative reasoning community where the entities in that problem domain are characterised by a spatial representation, and the inference is drawn by reasoning about the constraints or relations between the entities.

As object motion in 3D space can be complex, it is critical to ensure that the qualitative representation is expressive enough and can capture all possible motions of an object in the space. Hence, we develop our theory according to a well-established physics modelling approach (Baraff 1997) that is also widely used in nowadays physics engines. We devise a qualitative representation for spatial entities and constraints that the modelling approach uses for motion prediction. We show that our theory can cover all the possibilities of the motion of objects in a system that can be described by the modelling approach.

## A Qualitative Theory of Object Motion

We develop a qualitative representation to describe forces and their effects on object motion based on sign calculus (De Kleer and Brown 1984). We use sign calculus to represent a vector of numbers with each component of the vector is replaced by a sign that indicates whether the component is positive (+), negative(-), or zero(0). A force is then represented based on sign vectors.

**Definition 1 (Qualitative Force)** A qualitative force on an object  $O$  is a 3-tuple  $\langle \mathbf{qd}, \mathbf{qr}, O \rangle$  where  $\mathbf{qd}$  is a sign vector representing the qualitative direction of the force,  $\mathbf{qr}$  is a sign vector of the direction pointed from the mass centre of  $O$  to the point where the force is acted upon. The qualitative force of gravity on  $O$  is  $\langle \mathbf{qd} = (0, 0, -), \mathbf{qr} = (0, 0, 0), O \rangle$ .

Given a qualitative force, its components  $\langle \mathbf{qr}, O \rangle$  refer to a qualitative location where the actual force is acted upon. We can obtain a more accurate region when the shape of an object is given. To model contact forces between objects and the physical constraints between the forces, we refer to the theory (Baraff 1997) of rigid body dynamics that is widely applied in physics engines. The goal of our theory is to capture all the possible motions of a group of rigid objects when the qualitative representation of their forces is known. Given an observed change in the motion of an object, the formalisation should also allow inferring what forces have caused the change.

**Definition 2 (Object State)** The state of an object  $O$  at time  $t$ , denoted  $O_t$ , is a tuple  $\langle \mathbf{qv}_t, \mathbf{q}\omega_t \rangle$  where  $\mathbf{qv}_t, \mathbf{q}\omega_t$  are the sign vectors of  $O$ 's linear and angular velocity, respectively. There are  $27 \times 27 = 729$  possible qualitative states.

**Definition 3 (State Change)** The state change  $\Delta(O_{t_1}, O_{t_2})$  of an object  $O$  from  $t_1$  to  $t_2$  is defined as follows.

$$\Delta(O_{t_1}, O_{t_2}) = \langle Q(\mathbf{v}_{t_2} - \mathbf{v}_{t_1}), Q(\boldsymbol{\omega}_{t_2} - \boldsymbol{\omega}_{t_1}) \rangle$$

where  $Q(\cdot)$  is a procedure converting numbers to signs.

**Definition 4 (Qualitative Action)** An action exerts a impulse at a point location  $p$  on the exterior boundary of an object  $O$ . A qualitative action is a qualitative force representing the impulse force exerted by that action.

**Lemma 1** Given a state change  $\Delta(O_{t_1}, O_{t_2})$  resulted from a set of actual forces  $\{\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_n\}$  acted upon  $O$  between  $t_1$  and  $t_2$ , let  $\mathbf{QF}$  be a set of qualitative forces obtained by converting each actual force to its qualitative form, and let  $\mathbf{D}$  be another set of qualitative forces. If  $\mathbf{QF} \subset \mathbf{D}$ , then  $\Delta(O_{t_1}, O_{t_2}) \in \Delta_{\mathbf{D}}$ .

Ideally, for every object, we want to obtain a small-sized  $\mathbf{D}$  that contains all the actual qualitative forces on the object. The size of  $\mathbf{D}$  is up to  $27 \times 27 = 729$ , which is the number of possible combinations of two sign vectors. We developed several rules according to the standard constraints in physics simulations (Baraff 1997) that can help to reduce the size of  $\mathbf{D}$  without discarding any solutions. Below is an example of a rule derived from the simulation routine.

**Rule 1 (Vanishing Point)** A contact point  $p$  is a vanishing point when  $O_i$  and  $O_j$  are moving away at  $p$ . There is no contact force at any vanishing point.

**Lemma 2** Given a rule that is satisfied by a set of actual forces  $\{\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_n\}$ , the qualitative version of the rule is also satisfied by  $\{Q(\mathbf{f}_1), Q(\mathbf{f}_2), \dots, Q(\mathbf{f}_n)\}$ .

Lemma. 1-2 can be proved by the theory of the rigid dynamics (Baraff 1997) and the definition of sign operations.

## Solving Action Inference Problem

We now formally define the problem based on the qualitative theory.

**Definition 5 (Action Inference Problem)** An action inference problem  $AIP\langle O_{t_1}, O_{t_2} \rangle$  is, given a set  $\mathbf{O}$  of objects and their qualitative states  $O_{t_1}$  at time  $t_1$  and a set of their qualitative states  $O_{t_2}$  at later time  $t_2$ , assuming there is an action made between  $t_1$  and  $t_2$ , what is the qualitative representation of the action?

We formalise  $AIP$  as a constraint satisfaction problem  $AIP-SAT\langle \mathbf{X}, \mathbf{D}, \mathbf{C} \rangle$  where  $\mathbf{X}$  is the set of variables with each variable can be assigned with a value from its non-empty domain  $\mathfrak{D}_i \in \mathbf{D}$ .  $\mathbf{C}$  is set of constraints with each constraint specifies some relations that must be held between a subset of variables. The goal is to find an assignment of qualitative forces that can cause the observed change from  $O_{t_1}$  to  $O_{t_2}$ .

**Definition 6 (AIP-SAT)** Given an  $AIP$  problem  $AIP\langle O_{t_1}, O_{t_2} \rangle$ , and let  $n$  be the number of force variables, we obtain the following  $AIP-SAT$  problem:

- $\mathbf{X} = \{x_{action}, x_1, x_2, \dots, x_n\}$ : Each variable  $x_{1:n}$  corresponds to a force at a contact point or the force of gravity at the mass centre, and  $x_{action}$  is the variable of the action. Given an object  $O_i$ ,  $\mathbf{X}_i$  denotes a subset of variables whose forces are on  $O_i$ .
- $\mathbf{D} = \{\mathfrak{D}_{action}, \mathfrak{D}_1 \dots \mathfrak{D}_n\}$ : Each domain  $\mathfrak{D}_{1:n} = \{\langle \mathbf{qd}, \mathbf{qr}, O \rangle : \mathbf{qd} \in \mathfrak{S}\}$  contains a set of qualitative forces that  $x_{1:n}$  can be assigned with;  $\mathbf{qr}$  and  $O$  are fixed except for the action variable  $\mathfrak{D}_a = \{\langle \mathbf{qd}, \mathbf{qr}, O \rangle : \mathbf{qd}, \mathbf{qr} \in \mathfrak{S}, O \in \mathbf{O}\}$  as we need to infer the location upon which the action is exerted.
- $\mathbf{C}$ : There are two constraints, namely,
  - $C_1 : \forall O_i \in \mathbf{O}, \Delta(O_{it_1}, O_{it_2}) \in \Delta_{\mathbf{D}_{\mathbf{X}_i}}$  where  $\mathbf{D}_{\mathbf{X}_i}$  is a set of assigned values of the variables in  $\mathbf{X}_i$ .
  - $C_2 : \forall x \in \mathbf{X}$ , value of  $x$  is consistent with the rules.

We developed a graph-based solver that follows a standard routine of constraint satisfaction. Its completeness can be proved based on Lemma. 1-2. In the given example (Fig. 1) where there are 15 objects, there are  $26 \times 27 \times 15 = 10530$  possible qualitative actions. The solver detected a complete set of 48 distinct qualitative forces that can lead to the observed change.

## Conclusion

We proposed a qualitative theory for the motion of rigid objects based on the modelling approaches from qualitative reasoning and physics simulations. Based on the formulation we solved an interesting action inference problem.

## References

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