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Studies in Credibility-Limited Base Revision

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Abstract

In this paper we present axiomatic characterizations for several classes of credibility-limited base revision functions and establish the interrelation among those classes. We also propose and axiomatically characterize two new base revision functions.

1 Introduction

The logic of theory change became a major subject in artificial intelligence in the middle of the 1980's. The most important model that is now known as the AGM model of belief change, was proposed by Alchourrón, Gärdenfors and Makinson in (Alchourrón, Gärdenfors, and Makinson 1985). The AGM model is a formal framework to characterize the dynamics and state of belief of a rational agent. In that framework, each *belief* of an agent is represented by a sentence (of a propositional language $\mathcal{L}$) and the *belief state* of an agent is represented by a logically closed set of (belief-representing) sentences. These sets are called *belief sets*. A change consists in adding or removing a specific sentence from a belief set to obtain a new belief set. The AGM model has acquired the status of a standard model of belief change. The AGM model inspired many researchers to propose extensions and generalizations as well as applications and connections with other fields. Regarding extensions we can mention:

Belief Base Dynamics: Instead of belief sets, a belief base is a set of sentences that is not (except as a limiting case) closed under logical consequence. A belief base has a fundamental property: it can distinguish between explicit beliefs (element of the belief base) and derived belief, *i.e.*, elements that are logical consequence of the belief base, but that are not (explicitly) in the belief base. In order to represent real cognitive agents belief bases are a more suitable representation than belief sets. As Gärdenfors and Rott pointed out "when we perform revisions or contractions, it seems that we never do it to the belief set itself (...) but rather on some typically finite base for the belief set" (Gärdenfors and Rott 1995). On the other hand, because belief sets are often too big (eventually even infinite), they are not adequate for computational implementations of belief change models. A set A is a *base* for a belief set $\mathbf{K}$ if and only if $Cn(A) = \mathbf{K}$. A sentence α is believed if and only if $\alpha \in Cn(A)$. For an overview see (Hansson 1999a) and (Fermé and Hansson 2011).

Non-prioritized change: In AGM Belief Revision the new information is always accepted, following the principle of primacy of new information. In some scenarios this behaviour can be inappropriate and one could require that some new pieces of information can be rejected by the agent because, for instance, of insufficient plausibility. This has given rise to several models of non-prioritized Belief Revision. For an overview see (Hansson 1998; 1999; Ferme and Hansson 2011).

In this paper we study operators that combine the extensions mentioned, namely credibility-limited revision operators for belief bases. Credibility-limited revision (CL revision for short) was defined in (Hansson et al. 2001). It is based on the assumption that some inputs are accepted, others not. If α is credible, then α is accepted in the revision process, otherwise no change is made to the belief set. This model was proposed and characterized for a single revision step in (Hansson et al. 2001) and extended to cover iterated revision in (Booth et al. 2012). Fermé *et al.* have considered, in (Fermé, Mikalef, and Taboada 2003), an operator of CL base revision induced by a partial meet revision operator and a set C, the associated set of credible sentences, satisfying certain properties. In this paper we present axiomatic characterizations for operators of CL base revision induced by other kinds of standard base revision functions and by sets of credible sentences satisfying other properties.

CL revision can be seen as a modified version of Makinson's Screened revision (Makinson 1997). In Screened revision instead of a set of credible sentences it is considered a set A of sentences that are immune to revision and a revision of a belief set $\mathbf{K}$ by a given sentence α only gives rise to a new (appropriately changed) belief set if the input sentence α is consistent with $A \cap \mathbf{K}$ (otherwise the belief set $\mathbf{K}$ is left unchanged).

We note that the AGM belief change operators are not intended to force an agent to remove or incorporate beliefs that he/she is unwilling to remove or incorporate, but that such change operations are only applied for those beliefs that the agent decides to remove or incorporate after performing some type of previous processing of the information.[1] In the AGM model, this information preprocessing is left implicit. In the case of the CL operators studied in this paper this prior information processing is made explicit by means of a specification of the set of those beliefs that the agent is willing to incorporate in terms of properties.

This paper is organized as follows: In Section 2 we present axiomatic characterizations of some base revision functions (two of which are new). In Section 3 we present a formal definition of CL base revision. We recall some desirable properties that the set of credible sentences should satisfy and introduce some new ones. We also recall and present some new postulates for CL revision and establish interrelations between postulates. In Section 4 we axiomatically characterize several classes of CL base revisions and establish the relations between them. In Section 5 we summarize the main results of this paper. Throughout the text we provide proof sketches for the main results. Full proofs for all the original results are available at http://www.cee.uma.pt/ferme/papers/cred_limit_KR18.pdf.

1.1 Formal preliminaries

Beliefs are expressed in a language $\mathcal{L}$ that is called the object language. We assume that the language contains the usual truth functional connectives: negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), implication ($\rightarrow$) and equivalence ($\leftrightarrow$). $\perp$ denotes an arbitrary contradiction and $\top$ an arbitrary tautology. The letters $\alpha, \alpha_i, \beta, \ldots$ will be used to denote sentences of $\mathcal{L}$ and $p, q, r, \ldots$ will be used to denote atomic sentences of $\mathcal{L}$. $A, A', B, \ldots$ denote subsets of sentences of $\mathcal{L}$. Cn denotes a consequence operation that satisfies the standard Tarskian properties, namely inclusion, monotony and iteration. We also assume it is supraclassical and compact, and satisfies deduction. $A \vdash \alpha$ will be used as an alternative notation for $\alpha \in Cn(A)$, $\vdash \alpha$ for $\alpha \in Cn(\emptyset)$ and $Cn(\alpha)$ for $Cn(\{\alpha\})$.

2 Base Revision

The following postulates are well known postulates for belief base revision[2]:

Success $\alpha \in A * \alpha$.
Inclusion $A * \alpha \subseteq A \cup \{\alpha\}$.
Vacuity If $A \not\vdash \neg\alpha$, then $A \cup \{\alpha\} \subseteq A * \alpha$.
Consistency If $\alpha \not\vdash \perp$, then $A * \alpha \not\vdash \perp$.
Consistency preservation If $A \not\vdash \perp$, then $A * \alpha \not\vdash \perp$.
Uniformity If for all subsets $A' \subseteq A$, $A' \cup \{\alpha\} \vdash \perp$ if and only if $A' \cup \{\beta\} \vdash \perp$, then $A \cap A * \alpha = A \cap A * \beta$.
Relevance If $\beta \in A$ and $\beta \notin A * \alpha$, then there is some A' such that $A * \alpha \subseteq A' \subseteq A \cup \{\alpha\}$, $A' \not\vdash \perp$ but $A' \cup \{\beta\} \vdash \perp$.
Core-retainment If $\beta \in A$ and $\beta \notin A * \alpha$, then there is some $A' \subseteq A$ such that $A' \not\vdash \neg\alpha$ and $A' \cup \{\beta\} \vdash \neg\alpha$.

Additionally in this paper we propose the following three postulates for belief base revision:[3]

Disjunctive Elimination If $\beta \in A$ and $\beta \notin A * \alpha$, then $A * \alpha \not\vdash \neg\alpha \vee \beta$.
Weak Relative Closure $A \cap Cn(A * \alpha \cap A) \subseteq A * \alpha$.
Relative Extensionality If $\vdash \alpha \leftrightarrow \beta$, then $A \cap A * \alpha = A \cap A * \beta$.

Disjunctive elimination states that if β is removed when revising a set A by α, then from the revision of A by α we can not deduce that α implies β. *Weak relative closure* states that the set formed by the elements of A that are kept on the outcome of revising A by α is logically closed relative to A.[4,5] *Relative extensionality* states that if α and β are two logically equivalent beliefs then everything that is kept when revising by α is also kept when revising by β. We note that *relative extensionality* is a weaker version of *extensionality*: If $\vdash \alpha \leftrightarrow \beta$, then $A*\alpha = A*\beta$. We also note that *extensionality* in general does not hold in belief bases. The following example illustrates this fact: Let α and β be two distinct sentences such that $\vdash \alpha \leftrightarrow \beta$. Let A be a belief base such that $A \cap \{\alpha, \beta\} = \emptyset$. Let $*$ be a revision operator on A that satisfies *success* and *inclusion*. Then $\alpha \in A * \alpha$ but $\alpha \notin A * \beta$, thus $A * \alpha \neq A * \beta$.

2.1 Base Revision Functions

In this subsection we recall the axiomatic characterizations of partial meet base revision and of kernel base revision. We also introduce and axiomatically characterize new kinds of base revision functions.

Partial Meet Base Revision In order to revise A by a sentence α we consider the set $A \perp \neg\alpha$ of all maximal subsets of A not implying $\neg\alpha$. A selection function γ selects the most plausible elements of $A \perp \neg\alpha$. It satisfies the two following properties: (i) if $A \perp \neg\alpha \neq \emptyset$, then $\emptyset \neq \gamma(A \perp \neg\alpha) \subseteq A \perp \neg\alpha$, and (ii) if $A \perp \neg\alpha = \emptyset$, then $\gamma(A \perp \neg\alpha) = \{A\}$. The *partial meet revision* $*_\gamma$ based on γ is defined as $A *_\gamma \alpha = \bigcap \gamma(A \perp \neg\alpha) \cup \{\alpha\}$ (Alchourrón and Makinson 1982; Alchourrón, Gärdenfors, and Makinson 1985; Hansson 1991a).

Observation 1 (Hansson 1991a) Let A be a belief base. An operator $*$ on A is a partial meet revision function for A if and only if $*$ satisfies success, consistency, inclusion, relevance and uniformity.

[1]This was made clear, for example, in (Makinson 1997).

[2]For an overview of these postulates see (Hansson 1999b; Fermé and Hansson 2011; 2018).

[3]These three postulates are adaptations, for revision, of the contraction postulates: *Disjunctive Elimination* (Fermé, Krevneris, and Reis 2008), *Relative Closure* (Hansson 1994) and *Extensionality* (Gärdenfors 1982).

[4]A set A is logically closed relative to B if and only if $Cn(A) \cap B \subseteq A$ ((Hansson 1991b)).

[5]We note that the intersection with the set A that appears in the argument of consequence operator Cn is not irrelevant as one might think. To see this consider the following example: Let $A = \{\alpha \rightarrow \beta, \beta, \beta \rightarrow \neg\alpha\}$ and $A * \alpha = \{\alpha \rightarrow \beta, \alpha\}$. Hence $\beta \in A \cap Cn(A * \alpha)$ but $\beta \notin A * \alpha$. On the other hand, $\alpha \rightarrow \beta$ is the only element of A that can be deduced from $A \cap A * \alpha$. It holds that $\alpha \rightarrow \beta \in A * \alpha$. Thus, $*$ satisfies *relative closure* but not the property $A \cap Cn(A * \alpha) \subseteq A * \alpha$.

Kernel Base Revision *Kernel revision*, introduced in (Hansson 1994; Wassermann 2000) can be viewed as a dual vision of partial meet revision. Let $A \perp\!\!\!\perp \neg\alpha$ be the set of minimal $\neg\alpha$-implying subsets of A. An incision function is a function σ that selects sentences to be discarded. It satisfies the two basic properties (i) $\sigma(A \perp\!\!\!\perp \neg\alpha) \subseteq \bigcup(A \perp\!\!\!\perp \neg\alpha)$, and (ii) if $\emptyset \neq X \in A \perp\!\!\!\perp \neg\alpha$, then $X \cap \sigma(A \perp\!\!\!\perp \neg\alpha) \neq \emptyset$. The kernel revision $*_\sigma$ based on σ is defined by the relationship $A *_\sigma \alpha = (A \setminus \sigma(A \perp\!\!\!\perp \neg\alpha)) \cup \{\alpha\}$.

Observation 2 (Wassermann 2000) Let A be a belief base. An operator $*$ on A is a kernel revision function for A if and only if $*$ satisfies consistency, success, inclusion, uniformity and core-retainment.[6]

Smooth Kernel Base Revision Sometimes, when revising a set by a kernel revision function, some beliefs are removed for no reason. For example, if $A = \{p, q, p \vee q, p \rightarrow q\}$, then $A \perp\!\!\!\perp q = \{\{q\}, \{p, p \rightarrow q\}, \{p \vee q, p \rightarrow q\}\}$. Thus if $\sigma(A \perp\!\!\!\perp \alpha) = \{q, p \vee q, p \rightarrow q\}$, then $A *_\sigma (\neg q) = \{p, \neg q\}$. Since p is kept when revising A by $\neg q$, then $p \vee q$ is implied by $A \cap A * (\neg q)$. Thus it seems that the removal of $p \vee q$ was unnecessary and violates the *principle of minimal change*. Such a loss can be prevented if we ensure that the operator of kernel contraction satisfies *weak relative closure*. This can be done if we impose that the incision functions satisfies the following condition: if it holds for all subsets A' of A that if $A' \vdash \beta$ and $\beta \in \sigma(A \perp\!\!\!\perp \alpha)$ then $A' \cap \sigma(A \perp\!\!\!\perp \alpha) \neq \emptyset$. Such incision functions are called smooth. A kernel revision is smooth if and only if it is based on a smooth incision function.[7]

Observation 3 Let A be a belief base. An operator $*$ on A is a smooth kernel revision if and only if it satisfies consistency, success, inclusion, uniformity, core-retainment and weak relative closure.

Proof Sketch:
(Construction-to-postulates) Let $*$ be a smooth kernel revision on A. It follows from Observation 2 that $*$ satisfies *consistency, success, inclusion, uniformity* and *core-retainment*. Since $*$ is a smooth kernel revision operator it follows that $*$ is based on a smooth incision function σ such that for all sentences α: $A * \alpha = (A \setminus \sigma(A \perp\!\!\!\perp \neg\alpha)) \cup \{\alpha\}$. It can be shown that $*$ satisfies *weak relative closure*.
(Postulates-to-construction) Let $*$ be an operator that satisfies all the postulates listed in the statement of the observation. Let $\sigma(A \perp\!\!\!\perp \neg\alpha) = A \setminus (A \cap (A * \alpha))$. σ is an incision function for A and $A * \alpha = (A \setminus \sigma(A \perp\!\!\!\perp \neg\alpha)) \cup \{\alpha\}$ (see (Wassermann 2000, Proof of Theorem 5.2.14)). Furthermore, it holds that σ is smooth. ∎

[6] To be more precise we note that this axiomatic characterization is equivalent to the one actually presented in (Wassermann 2000), which uses the postulate of *non-contradiction* (if $\not\vdash \neg\alpha$, then $A * \alpha \not\vdash \neg\alpha$) instead of *consistency*.

[7] In (Hansson 1994) the concept of smooth incision function was used in the definition of the smooth kernel contraction. Here we use it to define the smooth kernel revision.

Basic AGM-generated Base Revision Below we define and axiomatically characterize the basic AGM-generated base revisions, which consist of base revision functions that are defined from an AGM (belief set) revision function.

Definition 4 Let $\mathbf{K}$ be a belief set. Let $\star$ be an operator on $\mathbf{K}$. If $\star$ satisfies the six basic AGM (Gärdenfors) postulates for revision, namely: success; consistency; extensionality and
Inclusion: $\mathbf{K} \star \alpha \subseteq Cn(\mathbf{K} \cup \{\alpha\})$;
Vacuity: If $\neg\alpha \notin \mathbf{K}$, then $Cn(\mathbf{K} \cup \{\alpha\}) \subseteq \mathbf{K} \star \alpha$;
Closure: $\mathbf{K} \star \alpha$ is a belief set;
then $\star$ is a basic AGM revision for $\mathbf{K}$.

A natural way to define a revision operator $*$ on a set A from a basic AGM revision $\star$ for the belief set $Cn(A)$ is through the condition $A * \alpha = (Cn(A) \star \alpha) \cap (A \cup \{\alpha\})$ (for all α). Note that the intersection on the right side of the previous equality ensures that the operator $*$ satisfies *inclusion* and *success*. Note also that, regarding the *success* postulate, this would not be the case if the intersection of $Cn(A) \star \alpha$ were with A instead of with $A \cup \{\alpha\}$. An operator $*$ defined as in the last equality is designated by basic AGM-generated base revision.

Definition 5 Let A be a belief base. An operator $*$ for A is a basic AGM-generated base revision if and only if there exists some basic AGM revision $\star$ for $Cn(A)$ such that:

$$A * \alpha = (Cn(A) \star \alpha) \cap (A \cup \{\alpha\})$$

Observation 6 Let A be a belief base. An operator $*$ on A is a basic AGM-generated base revision if and only if it satisfies consistency, success, inclusion, vacuity, relative extensionality and disjunctive elimination.

Proof Sketch:
(Construction-to-postulates) Let $A * \alpha = (Cn(A) \star \alpha) \cap (A \cup \{\alpha\})$ and $\star$ be a basic AGM revision for $Cn(A)$. Hence $\star$ satisfies *success, inclusion, vacuity, consistency, extensionality* and *closure*. That $*$ satisfies *inclusion* follows from the definition of $*$. That $*$ satisfies *consistency, vacuity, disjunctive elimination, relative extensionality* and *success* follows by the definition of $*$ and, respectively $\star$ *consistency*, *vacuity* and *success*, *success* and *closure*, *extensionality* and *success*.
(Postulates-to-construction) Let $*$ be an operator on A that satisfies the postulates listed in the statement of the observation. Let $\star$ be an operator on $Cn(A)$ defined, for all $\alpha \in \mathcal{L}$, as follows: $Cn(A) \star \alpha = Cn(A * \alpha)$. It holds that:
a) $\star$ is a basic AGM revision for $Cn(A)$.
b) $A * \alpha = (Cn(A) \star \alpha) \cap (A \cup \{\alpha\})$. ∎

3 CL Revision

In standard belief revision, a revision is always successful, *i.e.* a given belief is always incorporated when revising by it. But this is not a realistic feature of belief revision. There are beliefs that an agent is not willing to incorporate independently of the revision to be performed. The basic idea of CL revision is to define a function in two steps. In the first step, one needs to define which sentences are credible, *i.e.*,

the sentences that an agent is willing to incorporate when performing a revision. Afterwards the function should:
- leave the revised set of beliefs unchanged when revising by a non-credible sentence;
- work as a "standard" revision when revising by a credible sentence.
The following definition formalizes this concept:

Definition 7 Let $*$ be a revision operator on a belief base A. Let C be a set of sentences (the associated set of credible sentences). Then $\circledast$ is a CL base revision induced by $*$ and C if and only if:

$$A \circledast \alpha = \begin{cases} A * \alpha & \text{if } \alpha \in C \\ A & \text{otherwise} \end{cases}.$$

The following example illustrates the outcome of a CL base revision of a set A by credible and non-credible sentences.

Example 8 Let $A = \{p, p \vee \neg q, p \rightarrow \neg q\}$, Cn be purely truth-functional and $*$ be a partial meet base revision on A. It holds that $A \perp \neg q = \{\{p, p \vee \neg q\}, \{p \rightarrow \neg q\}\}$. Let γ be a selection function for A such that $\gamma(A \perp \neg q) = \{\{p, p \vee \neg q\}\}$. Hence $A * q = \{p, p \vee \neg q, q\}$. Let $C = Cn(q) \cup Cn(p) \cup Cn(\neg q)$ and $\circledast$ be the CL base revision induced by $*$ and C. It holds that $p \wedge q \notin C$, hence $A \circledast (p \wedge q) = A$. On the other hand $q \in C$. Hence $A \circledast q = A * q = \{p, p \vee \neg q, q\}$.

3.1 Credible sentences

In the CL model the set of sentences that an agent is willing to accept when a revision is performed is called *set of credible sentences*. This set will be denoted by C.

In (Hansson et al. 2001) some desirable properties for the set C were presented.

Element consistency If $\alpha \in C$, then $\alpha \nvdash \perp$.
Single sentence closure If $\alpha \in C$, then $Cn(\alpha) \subseteq C$.
Disjunctive completeness If $\alpha \vee \beta \in C$, then either $\alpha \in C$ or $\beta \in C$.
Expansive credibility If $A \nvdash \alpha$, then $\neg\alpha \in C$.
Revision credibility If $\alpha \in C$, then $Cn(A \circledast \alpha) \subseteq C$.
Credibility of logical equivalents If $\vdash \alpha \leftrightarrow \beta$, then $\alpha \in C$ if and only if $\beta \in C$.[8]

Given a belief base A:
Element consistency states that contradictions are not credible. *Single sentence closure* says that if a sentence is credible then all its logical consequences are also credible. *Disjunctive completeness* states that if two sentences are not credible, then their disjunction is not credible. *Expansive credibility* informally states that sentences that are consistent with A are credible. *Revision credibility* states that sentences in the outcome of a revision by a credible sentence are credible. *Credibility of logical equivalents* states that logically equivalent sentences should be both elements of C or of $\mathcal{L} \setminus C$.

[8]This property was named *closure under logical equivalence* in (Hansson et al. 2001).

We note that some of the properties of the set C are independent from the belief base A, namely *element consistency*, *single sentence closure*, *disjunctive completeness* and *credibility of logical equivalents*. Nevertheless, it is natural to consider also properties for the set C which are sensitive to the set A, since this set represents the belief state of the agent (under consideration).

In this paper we propose three new properties for C (the set of credible sentences).

Strong expansive credibility If $\alpha \notin C$, then $A \cap A \circledast \beta \vdash \neg\alpha$.
Credibility lower bounding If A is consistent, then $Cn(A) \subseteq C$.
Uniform credibility If it holds for all subsets A' of A that $A' \cup \{\alpha\} \vdash \perp$ if and only if $A' \cup \{\beta\} \vdash \perp$, then $\alpha \in C$ if and only if $\beta \in C$.

Strong expansive credibility states that if a sentence α is not credible, then any possible revision keeps a subset of A that implies $\neg\alpha$.[9] *Credibility lower bounding* states that if A is consistent, then its logical consequences are credible. *Uniform Credibility* states that if two sentences α and β are consistent with exactly the same subsets of A, then α and β are both credible or both non-credible.[10] We note that if a set satisfies either *single sentence closure* or *uniform credibility*, then it also satisfies *credibility of logical equivalents*.

Example 9 Let $A = \{p, p \vee \neg q, p \rightarrow \neg q\}$, $C = Cn(p)$ where Cn is purely truth-functional. Hence C satisfies *element consistency, single sentence closure and credibility of logical equivalents*. C does not satisfy *credibility lower bounding*, *expansive credibility*, *disjunctive completeness* (since $q \vee \neg q \in C$, but $q \notin C$ and $\neg q \notin C$) nor *uniform credibility* (since $A \cup \{p\} \nvdash \perp$) and $A \cup \{\neg q\} \nvdash \perp$ but $p \in C$ and $\neg q \notin C$).

Example 10 Let A be a consistent set and C be a set such that $\alpha \in C$ if and only if $A \nvdash \neg\alpha$. Hence C satisfies *element consistency, single sentence closure, disjunctive completeness, expansive credibility, credibility of logical equivalents, credibility lower bounding* and *uniform credibility*. Furthermore, we can ensure that C also satisfies the other properties mentioned for sets of credible sentences if we impose some conditions on the operator of CL base revision $\circledast$ on A that is induced by C. For example, if $\circledast$ satisfies *inclusion* and *vacuity*, then C also satisfies *revision credibility*.

[9]We note that more rigorously the expression "with respect to A and $\circledast$" should be added to the designation of *strong expansive credibility*, since it relates C with A and $\circledast$. This will be omitted since there is no risk of ambiguity whenever this property is mentioned along this paper. The same also applies to *revision credibility*.

[10]More rigorously the expression "with respect to A" should be added to the designation of the last two properties, since they relate C with A. This will be omitted since there is no risk of ambiguity whenever these properties are mentioned along this paper. The same also applies to *expansive credibility*.

3.2 Postulates for CL base revision

When considering a CL revision the *success* postulate is the one that we want to discard. In a realistic situation there must be beliefs that an agent should not incorporate even when revising by it. *Success* must be replaced by weaker postulates that are capable of expressing the properties that an operator of CL base revision should verify (Hansson et al. 2001; Fermé, Mikalef, and Taboada 2003).

Relative Success $\alpha \in A \circledast \alpha$ or $A \circledast \alpha = A$.
Disjunctive distribution If $\alpha \vee \beta \in A \circledast (\alpha \vee \beta)$, then $\alpha \in A \circledast \alpha$ or $\beta \in A \circledast \beta$.
Strict improvement If $\alpha \in A \circledast \alpha$ and $\vdash \alpha \rightarrow \beta$, then $\beta \in A \circledast \beta$.
Regularity If $A \circledast \alpha \vdash \beta$, then $\beta \in A \circledast \beta$.

Relative success states that either a sentence is incorporated in the revision of a set by it, or the original set is left unchanged. *Disjunctive distribution* states that if a disjunction belongs to the revision of a set by it, then the same thing happens regarding at least one of its disjuncts. *Strict improvement* states that if a certain sentence is incorporated when revising a set by it, then the same thing happens regarding every logical consequence of that sentence. *Regularity* says that if a sentence does not belong to the revision of a set by it, then that sentence is not implied by the revision of that set by any other sentence.

We propose also the following postulate:

Persistence If $A \cap A \circledast \beta \vdash \neg\beta$, then $A \cap A \circledast \alpha \vdash \neg\beta$.

Persistence states that if the formulae of A that are kept when revising it by β imply $\neg\beta$, then $\neg\beta$ is implied by the formulae of A that remain when revising it by any formula α.

The following observations relate some of the postulates mentioned above.

Observation 11 Let A be a belief base and $\circledast$ an operation on A.

(a) relevance and relative success imply disjunctive elimination.
(b) uniformity implies relative extensionality.
(c) relevance and relative success implies core-retainment.
(d) disjunctive elimination implies weak relative closure.
(e) relevance and success implies core-retainment.
(f) success and core-retainment imply vacuity.

Observation 12 Let A be a consistent belief base and $\circledast$ an operation on A. Consistency preservation, persistence, relative success and vacuity imply disjunctive distribution, strict improvement and regularity.

If α is credible, then it should be an element of the outcome of the revision of a set A by it. Therefore, a natural way to define a set of credible sentences C is by $C = \{\alpha : \alpha \in A \circledast \alpha\}$, where $\circledast$ is a CL base revision. The next theorem illustrates some properties that such a set satisfies whenever $\circledast$ satisfies some of the postulates mentioned in this section.

Observation 13 Let A be a consistent belief base, $\circledast$ be an operator on A and $C = \{\alpha : \alpha \in A \circledast \alpha\}$. Then:

(a) If $\circledast$ satisfies consistency preservation, then C satisfies element consistency.
(b) If $\circledast$ satisfies strict improvement, then C satisfies single sentence closure.
(c) If $\circledast$ satisfies disjunctive distribution, then C satisfies disjunctive completeness.
(d) If $\circledast$ satisfies vacuity, then C satisfies expansive credibility and credibility lower bounding.
(e) If $\circledast$ satisfies consistency preservation, persistence, relative success and vacuity, then C satisfies single sentence closure, disjunctive completeness, revision credibility, uniform credibility, credibility of logical equivalents and strong expansive credibility.

4 Axiomatic Characterizations

In this section we present axiomatic characterizations of CL base revision functions induced by different revision functions, namely by partial-meet revisions, by kernel revisions and by basic AGM-generated base revisions. In the representation theorems that we shall present in this section the postulates *relative success, consistency preservation, inclusion, vacuity* and *persistence* will be referred to as the ***core-postulates***.

4.1 CL base revision induced by a partial meet base revision

We start this section by presenting an axiomatic characterization of CL base revision operators induced by a partial meet revision operator and a set C (of credible sentences).

Theorem 14 Let A be a consistent belief base and $\circledast$ be an operation on A. Then the following conditions are equivalent:

1. $\circledast$ satisfies the core-postulates, relevance and uniformity.
2. $\circledast$ is an operator of CL base revision induced by a partial meet revision operator for A and a set C that satisfies element consistency and strong expansive credibility.

Proof Sketch:
(1 $\rightarrow$ 2) In the first step of this proof we show that $C_{\circledast} = \{\alpha : \alpha \in A \circledast \alpha\}$ satisfies the properties listed in condition 2. This follows from Observation 13. It can be shown that $A \circledast \alpha = A * \alpha$ if $\alpha \in C_{\circledast}$ and $A \circledast \alpha = A$ otherwise; where $*$ is a partial meet revision operator defined as in the proof of (Fermé, Mikalef, and Taboada 2003, 1 to 2 part of Theorem 4.3). **(2 $\rightarrow$ 1)** Let A be a belief base, $*$ be a partial meet revision operator on A and C be a set of sentences that satisfies *element consistency* and *strong expansive credibility*. Let $\circledast$ be a CL base revision induced by $*$ and C. Using Observation 1 it can be shown that $\circledast$ satisfies the postulates listed in condition 1. ■

It is worth to mention here that, since in the proof of the 1 to 2 part of Theorem 14 we used the set

$C = \{\alpha : \alpha \in A \circledast \alpha\}$, it follows from Observation 13, having in mind the postulates listed in condition 1, that this set also satisfies single sentence closure, disjunctive completeness, expansive credibility, revision credibility, uniform credibility, credibility of logical equivalents and credibility lower bounding. Hence these properties can also be added to the list of properties of C mentioned in 2. The same occurs regarding the other representation theorems that we shall present in this section.

At this point we must remark that in (Fermé, Mikalef, and Taboada 2003, Theorem 4.3) it was presented an axiomatic characterization of CL base revision operators induced by a partial meet revision and a set C satisfying *element consistency, single sentence closure, disjunctive completeness, expansive credibility, revision credibility, strong revision credibility* and *uniform credibility*.[11]

4.2 CL base revision induced by a kernel base revision

In this subsection we present an axiomatic characterization of CL base revision operators induced by a kernel revision operator and a set C.

Theorem 15 Let A be a consistent belief base and $\circledast$ be an operation on A. Then the following conditions are equivalent:

1. $\circledast$ satisfies the core-postulates, core-retainment and uniformity.
2. $\circledast$ is an operator of CL base revision induced by a kernel revision operator for A and a set C that satisfies element consistency and strong expansive credibility.

Proof Sketch:
(1 → 2) We define $C_\circledast = \{\alpha : \alpha \in A \circledast \alpha\}$. The proof that $C_\circledast$ satisfies the properties listed in condition 2 follows from Observation 13.
It holds that σ defined as follows:

$$\sigma(A \perp\!\!\!\perp \alpha) = \begin{cases} A \setminus (A \cap A \circledast \neg\alpha) & \text{if } \neg\alpha \in A \circledast \neg\alpha \\ \cup(A \perp\!\!\!\perp \alpha) & \text{otherwise} \end{cases}$$

is an incision function. Hence $A * \alpha = (A \setminus \sigma(A \perp\!\!\!\perp \neg\alpha)) \cup \{\alpha\}$ is a kernel revision function on A. Furthermore it holds that $A \circledast \alpha = A * \alpha$ if $\alpha \in C_\circledast$ and $A \circledast \alpha = A$ otherwise. **(2 → 1)** That $\circledast$ satisfies *core-retainment* follows from $\circledast$ definition and $*$ *core-retainment* (Observation 2). The proof that $\circledast$ satisfies the rest of the postulates listed in condition 1 uses Observation 2 and follows as in the (2 → 1) part of the proof of Theorem 14. ■

4.3 CL base revision induced by a smooth kernel base revision

In this subsection we present an axiomatic characterization of CL base revision operators induced by a smooth kernel revision operator and a set C.

[11] In fact, in the mentioned result, *uniform credibility* is not included among the properties that the set C is assumed to satisfy. However there is a small gap in the proof of that theorem which can be easily corrected if we add *uniform credibility* to the list of properties that the set C is required to satisfy.

Theorem 16 Let A be a consistent belief base and $\circledast$ be an operation on A. Then the following conditions are equivalent:

1. $\circledast$ satisfies the core-postulates, core-retainment, uniformity and weak relative closure.
2. $\circledast$ is an operator of CL base revision induced by a smooth kernel revision operator for A and a set C that satisfies element consistency and strong expansive credibility.

Proof Sketch:
(1 → 2) The incision function σ, defined as in the (1 → 2) part of the proof of Theorem 15, is smooth whenever $\circledast$ satisfies *weak relative closure*. The rest of the proof for this part follows as in the (1 → 2) part of the proof of Theorem 15. **(2 → 1)** That $\circledast$ satisfies *weak relative closure* follows from $\circledast$ definition and $*$ *weak relative closure* (Observation 3). That $\circledast$ satisfies the rest of the postulates listed in condition 1 follows as in the (2 → 1) part of the proof of Theorem 15. ■

4.4 CL base revision induced by a basic AGM-generated base revision

Now we present an axiomatic characterization of CL base revision operators induced by a basic AGM-generated base revision operator and a set C.

Theorem 17 Let A be a consistent belief base and $\circledast$ be an operation on A. Then the following conditions are equivalent:

1. $\circledast$ satisfies the core-postulates, disjunctive elimination and relative extensionality.
2. $\circledast$ is an operator of CL base revision induced by a basic AGM-generated base revision operator for A and a set C that satisfies element consistency and strong expansive credibility.

Proof Sketch:
(1 → 2) We define $C_\circledast = \{\alpha : \alpha \in A \circledast \alpha\}$. The proof that $C_\circledast$ satisfies the properties listed in condition 2 follows from Observation 13.
Let $*$ be an operator on A defined, for all $\alpha \in \mathcal{L}$, as follows:

$$A * \alpha = \begin{cases} A \circledast \alpha & \text{if } \alpha \in C_\circledast \\ (A \cap Cn(\alpha)) \cup \{\alpha\} & \text{otherwise} \end{cases}.$$

It can be shown that $*$ is a basic AGM-generated base revision by proving that $*$ satisfies: *consistency, success, inclusion, vacuity, relative extensionality* and *disjunctive elimination* (Observation 6). It also holds that $\circledast$ is a CL revision operator induced by $*$ and C. This follows from the definition of $*$ and $\circledast$ *relative success*. **(2 → 1)** Using Observation 6 it can be shown that $\circledast$ satisfies the postulates listed in condition 1 (*relative success, consistency preservation, inclusion* and *persistence* follow as in the (2 → 1) part of the proof of Theorem 14). ■

4.5 Maps between different CL base revision functions

In this subsection we establish the relation among the different kinds of CL base revision functions that we introduced in this section.

Theorem 18 Let A be a consistent belief base. Let CL-PMR, CL-KR, CL-SKR and CL-bAGMR represent the classes of CL operators of base revision induced by, respectively, partial meet, kernel, smooth kernel and basic AGM-generated base revision on A and a set C that satisfies *element consistency* and *strong expansive credibility*. Then:[12]

(a) CL-PMR⊂CL-SKR⊂CL-KR

(b) CL-bAGMR⊈CL-KR

(c) CL-SKR⊈CL-bAGMR

(d) CL-PMR⊂CL-bAGMR

That CL-PMR⊆CL-SKR⊆CL-KR and that CL-PMR⊆CL-bAGMR follows from the axiomatic characterizations presented in the Theorems 14, 15, 16 and 17 and the relation between postulates presented in Observation 11. To prove that CL-KR⊈CL-SKR, CL-SKR⊈CL-PMR, CL-bAGMR⊈CL-KR, CL-SKR⊈CL-bAGMR and that CL-bAGMR⊈CL-PMR it is enough to consider the counter-examples presented in the following example.

Example 19 Let $\circledast$ be a CL revision operator on a set A induced by a revision operator $*$ and a set C. Let Cn be purely truth-functional. Let $C = \{\alpha : \neg\alpha \notin Cn(\emptyset)\}$. It holds that C satisfies *element consistency*, and *strong expansive credibility*.

(a) Let $A = \{p, p \vee q, p \leftrightarrow q, r\}$. Hence $A \perp\!\!\!\perp (p \wedge q) = \{\{p, p \leftrightarrow q\}, \{p \vee q, p \leftrightarrow q\}\}$. Let $*$ be the smooth kernel revision based on a smooth incision function σ such that: $\sigma(A \perp\!\!\!\perp (p \wedge q)) = \{p, p \leftrightarrow q\}$. Hence $A * \neg(p \wedge q) = (A \setminus \sigma(A \perp\!\!\!\perp (p \wedge q))) \cup \{\neg(p \wedge q)\} = \{p \vee q, r, \neg(p \wedge q)\}$. Thus $\circledast$ is a CL-SKR. On the other hand $\neg(p \wedge q) \in C$. Thus $A \circledast \neg(p \wedge q) = A * \neg(p \wedge q) = \{p \vee q, r, \neg(p \wedge q)\}$. Hence $p \in A$, $p \notin A \circledast \neg(p \wedge q)$, $\{p \vee q, r, \neg(p \wedge q), p\} \nvdash \bot$, $\{p \vee q, r, \neg(p \wedge q), p \leftrightarrow q\} \vdash \bot$ and this violates *relevance*. Thus $\circledast$ is not a CL-PMR.

(b) Let $\mathcal{L}$ be the language that consists only of the two atomic sentences p and q and their truth-functional combinations. Let $A = \{p \wedge q\}$ and $\star$ be a basic AGM revision for $Cn(A)$. Hence $\star$ is a partial meet revision for $Cn(A)$ ((Alchourrón, Gärdenfors, and Makinson 1985)). It holds that $Cn(A) \perp p = \{Cn(p \leftrightarrow q), Cn(q)\}$. Let γ be a selection function for $Cn(A)$, such that $\gamma(Cn(A) \perp p) = \{Cn(p \leftrightarrow q)\}$. Hence $Cn(A) \star (\neg p) = Cn(Cn(p \leftrightarrow q) \cup \{\neg p\}) = Cn(\{p \leftrightarrow q, \neg p\}) = Cn(\neg q \wedge \neg p)$. Let $*$ be an operator defined for all α by $A * \alpha = (Cn(A) \star \alpha) \cap (A \cup \{\alpha\})$. Hence $*$ is a basic AGM-generated base revision and consequently $\circledast$ is a CL-bAGMR. It holds that $A * (\neg p) = (A \cup \{\neg p\}) \cap Cn(\neg q \wedge \neg p) = \{\neg p\}$. From the definition of C it holds that $\neg p \in C$. Thus $A \circledast (\neg p) = A * (\neg p) = \{\neg p\}$. Therefore $q \in A$ and $q \notin A \circledast (\neg p)$, from which it follows that $\circledast$ does not satisfy *core-retainment*. Hence $\circledast$ is not a CL-KR (nor a CL-SKR nor a CL-PMR).

(c) Let $A = \{p, p \vee q, p \to q\}$. Hence $A \perp\!\!\!\perp q = \{\{p, p \to q\}, \{p \vee q, p \to q\}\}$. Let $*$ be the kernel revision based on a smooth incision function σ such that: $\sigma(A \perp\!\!\!\perp q) = \{p, p \to q\}$. Hence $A * (\neg q) = (A \setminus \sigma(A \perp\!\!\!\perp q)) \cup \{\neg q\} = \{p \vee q, \neg q\}$. It holds that $\circledast$ is a CL-SKR. On the other hand, $\neg q \in C$, thus $A \circledast (\neg q) = A * (\neg q) = \{p \vee q, \neg q\}$. Hence $p \in A$, $p \notin A \circledast (\neg q)$ and $A \circledast (\neg q) \vdash q \vee p$. Therefore, $\circledast$ does not satisfy *disjunctive elimination*. Thus $\circledast$ is not a CL-bAGMR.

(d) Let $A = \{p, q, p \vee q, p \to q\}$. Hence $A \perp\!\!\!\perp q = \{\{p, p \to q\}, \{p \vee q, p \to q\}, \{q\}\}$. Let $*$ be the kernel revision based on a incision function σ such that: $\sigma(A \perp\!\!\!\perp q) = \{q, p \vee q, p \to q\}$. Hence $A * (\neg q) = \{p, \neg q\}$. It holds that $\circledast$ is a CL-KR. On the other hand, $\neg q \in C$, thus $A \circledast (\neg q) = A * (\neg q) = \{p, \neg q\}$. Hence $A \cap A \circledast (\neg q) = \{p\}$. Therefore $A \cap Cn(A \cap A \circledast (\neg q)) = A \cap Cn(p) = \{p, p \vee q\} \nsubseteq A \circledast (\neg q)$. Hence $\circledast$ does not satisfy *weak relative closure*. Hence $\circledast$ is not a CL-SKR

[12] In the statements of this theorem we will use $A \subset B$ to denote that A is a proper subset of B and $A \nsubseteq B$ to denote that A is not a subset of B.

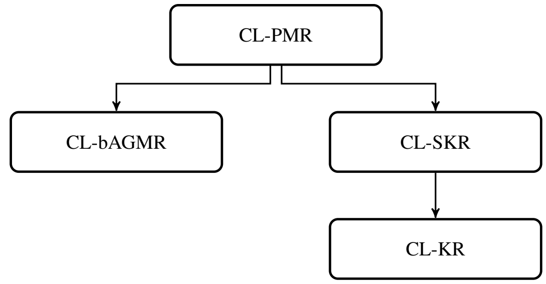

Figure 1: Map among different kinds of CL base revision functions.

In Figure 1 we present a diagram that summarizes the results presented in Theorem 18. In this diagram an arrow between two boxes symbolizes that the class of CL revision operators indicated at the origin of the arrow is a strict subclass of the class of CL revision operators that appears at the end of that arrow. The absence of an arrow between two kinds of CL revisions means that the corresponding classes are not related by means of inclusion.

5 Conclusion

This paper is an exploration of credibility-limited revision operators for belief bases. The main contributions of this paper are: (i) The definition and axiomatic characterization of two kinds of base revision functions; (ii) The proposal of new postulates for prioritized and non-prioritized revision and of new properties for the set of credibility sentences; (iii) The axiomatic characterization of four kinds of CL base revision functions; (iv) The study of the interrelations among these four kinds of functions. Our results are inspired by the CL revision operators defined in (Hansson et al. 2001) and the first adaptation of these operators to belief bases provided in (Fermé, Mikalef, and Taboada 2003). Regarding future works another perspective is to allow a less drastic behaviour for CL base revision operators. Operators defined here either accept a revision or completely reject it, if the new information is insufficiently credible. We are working on a definition of operators that allow partial acceptance of the new information, inspired in (Fermé and Hansson 1999).

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