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Penalty Logic-Based Representation of C-Revision

S. Laaziz,[1] **Y. Zeboudj,**[1] **S. Benferhat,**[2] **F. Haned Khellaf**[1]
[1]RIIMA, USTHB, Faculty of Electronique and Informatique, Algiers, Algeria.
[2] CRIL - CNRS UMR 8188, Artois University, Lens, France.

Abstract

Belief revision consists in modifying an epistemic state in the light of a new information. In this paper, we focus on the so-called multiple iterated belief revision process called C-revision. Epistemic states are represented in terms of Ordinal Conditional Functions OCF and penalty knowledge bases. The input is a set of consistent weighted formulas. We show that C-revision, defined at a semantic level using OCF, has a very natural counterpart in penalty logic.

Introduction

Belief revision (Alchourrón, Gärdenfors, and Makinson 1985; Williams 1995; Williams and Rott 2001), is an important field of research in artificial intelligence and knowledge representation areas. It consists in defining processes for modifying initial beliefs in the light of new information, considered as fully reliable.

Three main elements are necessary to define a revision process. The first one concerns the representation of current beliefs or, more generally, of epistemic states. In this paper, at the semantic level, we will use ordinal conditional functions OCF (Spohn 1988; 2012) to represent epistemic states. An ordinal conditional function, denoted by κ, is an uncertainty distribution where each element ω of the universe of discourse Ω (here a set of propositional logic interpretations) is associated with a positive integer number. $\kappa(\omega)$ is often interpreted as a degree of surprise that ω is the real world. Examples of belief revision methods that use OCF to represent epistemic states are C-revision (discussed in this paper) and the so-called transmutations (Williams 1994).

At the syntactic level, epistemic states are represented using weighted logics. They are sets of pairs of the form (ϕ_i, α_i) where ϕ_i is a propositional logic formula and α_i is the uncertainty degree associated with ϕ_i.

Weighted knowledge bases have been intensively used in the literature for handling uncertainty (such as in a possibilistic logic framework ((Lang 2001; Benferhat 2010; Dubois and Prade 2018)) or for handling inconsistency (Benferhat et al. 2002; Brewka 1989; Benferhat, Dubois, and Prade 1999; Williams 1995).

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Examples of weighted logics are (min-based and product) possibilistic logic (Dubois and Prade 2014; Lang 2001) and penalty logic (Pinkas 1991; Dupin De Saint-Cyr, Lang, and Schiex 1994; Darwiche and Marquis 2004). In this paper we use penalty logic where α_i's are positive integers.

The second element needed to define a revision process, concerns the representation of the new information. Typically, in belief revision the new information is encoded by a formula of propositional logic. Some works represent new information with uncertain observations which may represent a partition over a set of interpretations as in (Jeffrey 1965; Dubois and Prade 1997; Benferhat et al. 2009) . In some approaches, the input information is simply the whole epistemic as in (Benferhat et al. 2000). In this paper, the new information will be represented by a consistent set of weighted propositional logic formulas.

The last element concerns the definition of the revision operator itself, denoted by $\star$. In this paper, we will use the multiple revision operator, called c-revision, proposed in (Kern-Isberner and Huvermann 2015). This revision operator ($\star$) takes as input an ordinal conditional function κ, a consistent set of weighted formulas $\mathcal{S}$, and produces a new ordinal conditional function $\kappa \star \mathcal{S}$. The revision operation takes into account the fact that formulas of the input are issued from different and independent sources.

The main contribution of this paper consists in defining a syntactic representation of $\kappa \star \mathcal{S}$ using penalty logic which is a weighted logic where knowledge bases are sets of pairs (ϕ_i, α_i). ϕ_i's are propositional logic formulas and α_i's are positive integers. The weight α_i is often interpreted as a price (or cost) to pay if the propositional formula ϕ_i is not satisfied.

We show that C-revision, defined at the semantic level using ordinal conditional functions, has a very natural counterpart in penalty logic, defined at the syntactic level using weighted (penalty logic-based) knowledge bases.

The rest of this paper is organized as follows. Section 2 gives a refresher on ordinal conditional functions and penalty logic. While Sections 3 and 4 present belief revision and multiple iterated belief c-revision respectively. Section 5 gives the encoding of multiple iterated belief C-revision using penalty logic. Finally, Section 6 concludes the paper.

OCF and penalty logic

This section is divided into two subsections in which we first give a brief refresher on Ordinal Conditional Functions (OCFs) (for more details see (Spohn 1988; 2012)) and then we present penalty logic (for more details see (Dupin De Saint-Cyr, Lang, and Schiex 1994)).

Ordinal Conditional Functions

Let $\mathcal{L}$ be a propositional language based on a finite set of propositional variables. $\top$ et $\bot$ represent tautology and contradiction respectively. $\phi, \psi, ...etc$ represent formulas of $\mathcal{L}$. The set of interpretations is represented by Ω. An interpretation of Ω is denoted by ω.

An Ordinal Conditional Function distribution (Spohn 1988; 2012) can be simply viewed as a function that assigns to each interpretation ω of Ω an integer denoted by $\kappa(\omega)$. $\kappa(\omega)$ represents the degree of surprise of having ω as being the real world. $\kappa(\omega) = 0$ means that nothing prevents ω for being the real world. $\kappa(\omega) = 1$ means that ω is somewhat surprising to be the real world. $\kappa(\omega) = +\infty$ simply means that it is impossible for ω to be the real world.

For instance, suppose that we are interested in encoding our beliefs regarding the amenities and facilities offered by a hotel in Paris' downtown.

Let a be a propositional symbol to express the fact that a hotel has a kitchen in the room. Let s be a propositional symbol to express that a hotel has a swimming pool.

Assume that available beliefs are expressed by the following ordinal conditional function κ such that: $\kappa(\neg a \wedge \neg s) = 0$, $\kappa(a \wedge \neg s) = \kappa(\neg a \wedge s) = 1$ and $\kappa(a \wedge s) = +\infty$.

The ordinal conditional function, given above, first represents the fact that having none of the two amenities is the normal situation. And it is somewhat surprising to have one of the amenities. And lastly it is impossible that a hotel in Paris' downtown offers both amenities.

An OCF κ defined on Ω can be extended to formulas of the propositional language. The cost of a propositional logic formula ψ, denoted by $\kappa(\psi)$, is equal to the minimal cost of interpretations that satisfy γ :

$$\kappa(\psi) = min\{\kappa(\omega), \omega \models \psi\}$$

with by convention $\kappa(\bot) = +\infty$.

Penalty logic

Penalty logic is a weighted logic which has been introduced in (Pinkas 1991), and then developed in (Dupin De Saint-Cyr, Lang, and Schiex 1994; Pinkas and Loui 1992). It associates to each formula in the knowledge base a weight which represents a price to pay if the formula is not satisfied.

More precisely, a penalty knowledge base $\mathcal{PK}$ is a finite set of pairs (ϕ_i, α_i) such that ϕ_i's are formulas of the propositional language $\mathcal{L}$ and α_i's are strictly positive integers[1]. The integer α_i is the penalty associated with the formula ϕ_i. The higher the weight α_i is, the more important the formula ϕ_i is.

[1] As we will see later, we will allow a negative integer to be only associated with the contradiction formula $\bot$.

In particular, if for some formula ϕ_i we have $\alpha_i = +\infty$ then ϕ_i is considered as an integrity constraint that should absolutely be satisfied. In the following, a penalty base $\mathcal{PK}$ is denoted by:

$$\mathcal{PK} = \{(\phi_i, \alpha_i), i = 1, ..., n\}.$$

If the costs of formulas in a penalty base $\mathcal{PK}$ are all equal to $+\infty$, this means that no formula of $\mathcal{PK}$ should be violated. In this case, penalty logic is reduced to the propositional logic.

The following definition explains how to associate, to each weighted knowledge base, an ordinal conditional function over the set of interpretations.

Definition 1 *(Pinkas 1991)*
Each penalty (or weighted) base $\mathcal{PK} = \{(\phi_i, \alpha_i) : i = 1..n\}$ induces for each interpretation $\omega \in \Omega$ a cost, denoted $\kappa_{\mathcal{PK}}(\omega)$, which is equal to the sum of penalties of formulas of $\mathcal{PK}$ that are violated by ω. Namely $\forall \omega \in \Omega$:

$$\kappa_{\mathcal{PK}}(\omega) = \begin{cases} 0, \textit{if } \forall (\phi_i, \alpha_i) \in \mathcal{PK}, \omega \models \phi_i, \\ \sum \{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK}, \omega \not\models \phi_i\}, \textit{ otherwise} \end{cases}$$

In the following, $\kappa_{\mathcal{PK}}$ is simply called the ordinal conditional function OCF associated with $\mathcal{PK}$. The way the OCF distributions are produced from penalty logic bases is very close to the way possibility distributions are produced from product-based possibilistic knowledge bases.

One can easily check that if a formula ϕ appears several times in a penalty knowledge base $\mathcal{PK}$, we can replace all its occurrences by a single occurrence of the formula ϕ weighted by the sum of the weights associated with this formula.

Namely, if $(\phi, \alpha) \in \mathcal{PK}$ and $(\phi, \beta) \in \mathcal{PK}$ then $\mathcal{PK}$ is equivalent to $(\mathcal{PK} - \{(\phi, \alpha), (\phi, \beta)\} \cup \{(\phi, \alpha + \beta)\}.)$

Example 1 *Let $\mathcal{PK} = \{(a, 3), (b, 7), (\neg b, 2)\}$ be a penalty knowledge base. Its associated OCF distribution is illustrated in Table 1:*

Table 1: An example of OCF distribution $\kappa_{\mathcal{PK}}$ obtained from the knowledge base $\mathcal{PK}$ using Definition 1

ω	$\kappa_{\mathcal{PK}}(\omega)$
ab	2
$a\neg b$	7
$\neg ab$	5
$\neg a \neg b$	10

Note that in this example, there is no interpretation ω such that $\kappa_{\mathcal{PK}}(\omega) = 0$. This reflects that the penalty base $\mathcal{PK}$ is inconsistent.

Belief revision

Belief revision was originally introduced by Alchourron, Gärdenfors and Makinson (Alchourrón, Gärdenfors, and Makinson 1985) (Gärdenfors 2003). They proposed a set of axioms (called AGM postulates) to characterize the rationality of a revision process.

In (Darwiche and Pearl 1997), the authors have proposed an extension of the AGM model by adding postulates that manage the iterated revision. Jin and Thielscher (2007) have also proposed a new postulate for iterated revision called *Independance* postulate. This later was generalized by Delgrande and jin (2012) by introducing a new set of postulates that surpasses weakness of postulates proposed in (Darwiche and Pearl 1997).

In uncertainty theories, several works have been proposed for revising ordinal conditional functions (Häming and Peters 2010; Benferhat and Tabia 2010).

In possibility theory, the revision of possibilistic belief bases has been proposed, for example in (Benferhat et al. 2009; Dubois and Prade 1997), using the possibilistic counterpart of Jeffrey's rule (Jeffrey 1965).

Possibility theory has strong connections with ordinal conditional functions. In possibility theory, the uncertainty distribution; called a possibility distribution; is denoted by π. It assigns to each interpretation ω a positive real number in the unit interval $[0, 1]$.

The revision of a possibility distribution π by an input composed of a set of weighted formulas $\mu = \{(\phi_i, a_i), i = 1..n\}$ is as follows :

$$\forall(\phi_i, a_i) \in \mu, \forall\omega \vDash \phi_i, \pi(\omega|\mu) = a_i \otimes \pi(\omega|_{\otimes}\phi_i)$$

where $\otimes$ represents either the product operator or the min operator depending on the nature of the packaging used in the possibility theory. Note that, in Jeffrey's rule, as well as in its possibility theory counterpart, formulas of the input ϕ_i's $\in \mu$ represent a partition,

namely

$\bigvee\{\phi_i, \phi_i \in \mu\}$ is a tautology

and

$\forall i, \forall j, \phi_i \wedge \psi_j$ is a contradiction (with $i \neq j$)

The following section presents a revision process that operates on ordinal conditional functions. This revision operation is called c-revision.

C-revision and multiple iterated c-revision

This section recalls briefly the multiple C-revision introduced in (Kern-Isberner and Huvermann 2015; Kern-Isberner 2004). The multiple iterated C-revision considers that the input of the revision process is a set of weighted formulas (and not a single formula) $\mathcal{S} = \{(\xi_1, \beta_1), .., (\xi_n, \beta_n)\}$.

Note that ξ_i's are jointly consistent. So the way the input is viewed in C-revision departs from the one used in Jeffrey's rule where the input formulas induce a partition over Ω.

The C-revision is defined at a semantic level for ordinal conditional functions. It takes as input an OCF distribution κ, a consistent set of weighted formulas and produces a new OCF distribution denoted by $\kappa \star \mathcal{S}$. More precisely :

Definition 2 *Let κ be an OCF distribution, and let $\mathcal{S} = \{(\xi_1, \beta_1), .., (\xi_n, \beta_n)\}$ be a consistent set of weighted formulas where weights are positive integers ($\beta_i \in \mathcal{N}^+$).*
The new OCF distribution $\kappa \star \mathcal{S}$, obtained after the revision of κ by $\mathcal{S}$ using $\star$, is defined as follows :

$\forall\omega \in \Omega$,
$\kappa \star \mathcal{S}(\omega) = \kappa(\omega) - \kappa(\xi_1 \wedge ... \wedge \xi_n) + \sum_{i=1,\omega\not\models\xi_i}^{n} \beta_i$

where each β_i satisfies the following condition :
$\forall i \in \{1, .., n\}$,
$\beta_i > \kappa(\xi_1 \wedge ... \wedge \xi_n) - \min_{\omega\not\models\xi_i}\{\kappa(\omega) + \sum_{j\neq i,\omega\not\models\xi_j}^{n} \beta_j\}$.

One can easily check that if ω is a model of $\xi_1 \wedge ... \wedge \xi_n$ then $\kappa \star \mathcal{S}(\omega) = 0$.

Besides counter-models of ξ_i's are shifted up by their associated weights β_i's.

The presence of the sum operator ($\sum$) in the definition of $\kappa \star \mathcal{S}$ reflects the fact that the formulas ξ_i are independent.

Lastly, the inequality constraints on β_i's given in Definition 2 ensures that $\kappa \star \mathcal{S}(\neg\xi_i) > 0$.

It has been shown in (Kern-Isberner and Huvermann 2015) that multiple iterated c-revision satisfies the extended AGM postulates (as described in (Kern-Isberner and Huvermann 2015)), and the two postulates (PC3) and (PC4) of Delgrande and jin (2012).

It has also been argued in (Kern-Isberner and Huvermann 2015) that Jin's and Thielscher's (Ind) postulate (Jin and Thielscher 2007) is not necessary for multiple iterated c-revision.

Example 2 *Let us take again the OCF distribution given in Example 1. Table 2 is obtained from Table 1 by applying Definition 2 with : $\mathcal{S} = \{(a, 5), (\neg b, 7)\}$[2].*

Table 2: C-revision of the distribution given in Example 1 using $\mathcal{S} = \{(a, 5), (\neg b, 7)\}$.

ω	$\kappa_{\mathcal{PK}} \star \mathcal{S}(\omega)$
ab	2
$a\neg b$	0
$\neg ab$	10
$\neg a\neg b$	8

For instance:
$\kappa_{\mathcal{PK}} \star \mathcal{S}(a\neg b) = \kappa(a\neg b) - \kappa(a \wedge \neg b) + 0$ *(since $a\neg b \models a$ and $a\neg b \models \neg b$) = 7 - 7 = 0. and $\kappa_{\mathcal{PK}} \star \mathcal{S}(\neg ab) = \kappa(\neg ab) - \kappa(a \wedge \neg b) + 5$ (since $\neg ab \not\models a$) +7 (since $\neg ab \not\models \neg b$)= 5 - 7 + 12= 10.*

In (Benferhat and Ajroud 2016) the syntactic counterpart based on possibilistic weighted bases has been proposed. The computation of the function κ associated with $\mathcal{PK}$ is obtained by an equation similar to the one given in Definition 1 (the maximum operator is used instead of the sum operator).

The major disadvantage of the syntactic representation proposed in (Benferhat and Ajroud 2016) is that the obtained revised base is larger than the original knowledge base.

The aim of the following section is to propose a compact and direct encoding of multiple C-revision using penalty logic.

[2] For sake of simplicity, in this example we choose concrete values of β_i's. In the original definition of C-revision, β_i's are symbolic weights.

Syntactic revision of a penalty base

Let $\kappa_{\mathcal{PK}}$ be the ordinal conditional function associated with a knowledge base $\mathcal{PK}$ using Definition 1.

Recall that, from Definition 2, the revision operation C-revision is defined by :

$\forall \omega \in \Omega$,

$$\kappa_{\mathcal{PK}} \star \mathcal{S}(\omega) = \kappa_{\mathcal{PK}}(\omega) - \kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n) + \sum_{i=1, \omega \not\models u_i}^{n} \beta_i$$

Our aim is to provide a syntactic counterpart of $\kappa \star \mathcal{S}$. More precisely our aim is to compute a new weighted base $\mathcal{PK}1$ (from $\mathcal{PK}$ and $\mathcal{S}$) such that:

$\forall \omega$,

$$\kappa_{\mathcal{PK}1}(\omega) = \kappa_{\mathcal{PK}} \star \mathcal{S}(\omega)$$

Where $\kappa_{\mathcal{PK}1}$ (resp $\kappa_{\mathcal{PK}}$) is the ordinal conditional function associated with $\mathcal{PK}1$ (resp $\mathcal{PK}$) given by Definition 1, and $\kappa_{\mathcal{PK}} \star \mathcal{S}$ is the above C-revision of $\kappa_{\mathcal{PK}}$ by $\mathcal{S}$ using Definition 2.

Thus, given the above equation regarding the definition of C-revision, the syntactic C-revision requires the following steps:

- A syntactic computation, from the weighted base PK, of $\kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n)$;
- A syntactic computation of the weighted base associated with $\kappa_{\mathcal{PK}}(\omega) + \sum_{i=1, \omega \not\models \xi_i}^{n} \beta_i$;
- A syntactic computation of the weighted base associated with $\kappa \star \mathcal{S}$.

These three main steps are described in the following three subsections respectively.

Syntactic computation of the input weight

Let us start with the syntactic computation of $\kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n)$. We consider the following notations, given a penalty base $\mathcal{PK}$:

- $\mathcal{PK}^\star$: is the propositional base obtained by only considering propositional formulas of $\mathcal{PK}$ without taking into account their weights, namely $\mathcal{PK}^\star = \{\phi_i, (\phi_i, \alpha_i) \in \mathcal{PK}\}$. For instance, if $\mathcal{PK} = \{(\neg a, 2), (b, 7), (c \vee \neg b, 5), (a \vee c, +\infty)\}$ then $\mathcal{PK}^\star = \{\neg a, b, c \vee \neg b, a \vee c\}$
- $SW(K)$: is a function that sums the weights of a sub-base K of $\mathcal{PK}$: $SW(K) = \sum\{\alpha_i : (\phi_i, \alpha_i) \in K\}$. For instance, if $\mathcal{PK} = \{(\neg a, 2), (b, 7), (c \vee \neg b, 5), (a \vee c, +\infty)\}$ and $\mathcal{K} = \{(b, 7), (c \vee \neg b, 5), (a \vee c, +\infty)\}$ then $SW(K) = 7$.

Then the syntactic computation of $\kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n)$ is obtained using the following proposition:

Proposition 1 *Let :*

- *$\mathcal{PK}$: be a penalty base ;*
- *ψ : be a consistent formula (here $\psi = \xi_1 \wedge ... \wedge \xi_n$) ;*
- *A: be a sub-set of $\mathcal{PK}$ such that : $A^\star \wedge \psi$ consistent and $\nexists B \subseteq \mathcal{PK}, B^\star \wedge \psi$ consistent, $SW(B) > SW(A)$.*

We have :

$$\begin{aligned} \kappa_{\mathcal{PK}}(\psi) &= min\{\kappa_{\mathcal{PK}}(\omega), \omega \models \psi\} \\ &= SW(\mathcal{PK}) - SW(A) \end{aligned} \quad (1)$$

Proof 1 *From Definition 1 we have :*

$$\kappa_{\mathcal{PK}}(\psi) = min\{\kappa_{\mathcal{PK}}(\omega), \omega \models \psi\}$$

$$= min_\omega\{\sum\{\alpha_i, \omega \models \psi \wedge \neg\phi_i\}\}$$

$$= min_\omega\{SK(\mathcal{PK}) - \sum\{\alpha_i, \omega \models \phi_i \wedge \psi\}\}$$

$$= SW(\mathcal{PK}) - max_\omega\{\sum\{\alpha_i, \omega \models \phi_i \wedge \psi\}\}$$

$$= SW(\mathcal{PK}) - max_\omega\{SW(C), C \subseteq \mathcal{PK}, \omega \models C^\star \wedge \psi\}\}$$

$$= SW(\mathcal{PK}) - max\{SW(C), C \subseteq \mathcal{PK}, C^\star \wedge \psi \text{ consistent}\}\}$$

$$= SW(\mathcal{PK}) - SW(A)$$

(for sake of the proof, $\sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK}\}$ is replaced by $\sum\{\alpha_i\}$)

The computation of the sub-base A (and $SW(A)$) can be obtained using a polynomial number of calls to a Partial Max-SAT test. This step is not developed in this paper.

Example 3 *Let us continue our example with:*

$$\mathcal{PK} = \{(a, 3), (b, 7), (\neg b, 2)\}$$

and:

$$\mathcal{S} = \{(a, 5), (\neg b, 7)\}.$$

Using Proposition 1 :

$$\kappa_{\mathcal{PK}}(a \wedge \neg b) = SW(\mathcal{PK}) - SW(A) = 7.$$

An example of the sub-set A such that :

- *$(A^\star \wedge a \wedge \neg b)$ is consistent, and*
- *$(\nexists B \subseteq \mathcal{PK}, B$ consistent with $(a \wedge \neg b), SW(B) > SW(A))$*

is $A = \{(a, 3), (\neg b, 2)\}$.
One can also easily check that :

$$\kappa_{\mathcal{PK}}(a \wedge \neg b) = 12 - 5 = 7.$$

Next section is devoted to the integration of the input in the penalty knowledge base.

The integration of the input $\mathcal{S}$ in the weighted base

This subsection shows that integration of the new information into the penalty knowledge base is immediate as indicated in Proposition 2:

Proposition 2 *Let $\mathcal{PK} = \{(\phi_i, \alpha_i) : i = 1..n\}$ be a penalty base, and $\kappa_{\mathcal{PK}}$ be its associated distribution using Definition 1. Let $\mathcal{S} = \{(\xi_1, \beta_1), ..., (\xi_n, \beta_n)\}$ be the input. The syntactic counterpart of :*

$$\kappa_{\mathcal{PK}'}(\omega) = \kappa_{\mathcal{PK}}(\omega) + \sum_{i=m, \omega \not\models \xi_i}^{n} \beta_i$$

is :

$$\mathcal{PK}' = \mathcal{PK} \cup \{(\xi_i, \beta_i), i = 1, .., n\}.$$

The proof is immediate. It is enough to notice that $\forall i \in \{1, .., n\}$ and $\forall \omega \in \Omega$,

$$\kappa_{\mathcal{PK} \cup \{(\xi_i, \beta_i)\}}(\omega) = \begin{cases} \kappa_{\mathcal{PK}}(\omega) + \beta_i, & \text{if} \omega \not\models \xi_i. \\ \kappa_{\mathcal{PK}}(\omega), & \text{otherwise} \end{cases} \quad (2)$$

Example 4 *Let us continue the previous example. Applying proposition 2 gives us :*
$\mathcal{PK}'' = \mathcal{PK} \cup \{(a, 5)\} \cup \{(\neg b, 7)\}$
$= \{(a, 3), (b, 7), (\neg b, 2)\} \cup \{(a, 5)\} \cup \{(\neg b, 7)\}$
$= \{(a, 3), (b, 7), (\neg b, 2), (a, 5), (\neg b, 7)\}$

The associated distribution of $\mathcal{PK}^{''}$ is given in Table 3 :

Table 3: The distribution associated with $\mathcal{PK}^{''}$ after integrating the input

ω	$\kappa_{\mathcal{PK}}(\omega)$
ab	9
$a\neg b$	7
$\neg ab$	17
$\neg a \neg b$	15

Shifting down an OCF by $\kappa(\xi_1 \wedge ... \wedge \xi_n)$

The last step provides the syntactic characterization of shifting down the weight of each interpretation $\kappa(\omega)$ by $\kappa(\xi_1 \wedge ... \wedge \xi_n)$. This shifting operation ensures that the resulted OCF $k^{'} = \kappa(\omega) - \kappa(\xi_1 \wedge ... \wedge \xi_n)$ is normalized (namely there will exist at least an interpretation with a weight equals to 0).

In this section, we consider a very slight extension of the syntactic revision based on penalty logic, with negative weights only associated to the formula ($\perp$). These negative weights express a guaranteed reward. However weights associated with non-contradictory formulas $\phi_i \neq \perp$ remain positive. The other definitions, in particular Definition 1, also remain unchanged.

Proposition 3 *Let $\mathcal{PK} = \{(\phi_i, \alpha_i) : i = 1..n, \alpha_i \in \mathbb{R}^\star\}$ be the penalty base obtained in the previous step (Proposition 2), and $\kappa_{\mathcal{PK}}$ be its associated distribution. The syntactic counterpart of :*

$$\kappa_{\mathcal{PK}'}(\omega) = \kappa_{\mathcal{PK}}(\omega) - \kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n) \quad (3)$$

is :

$$\mathcal{PK}' = \mathcal{PK} \cup \{\perp, -\kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n)\}$$

The proof is immediate by applying Definition 1. Indeed, $\forall \omega, \kappa_{\mathcal{PK}'} =$

$$= \begin{cases} 0 & \text{if } \omega \models \mathcal{PK}^{'} \\ \sum_{\omega \not\models \phi_i} \{\alpha_i \ : (\phi_i, \alpha_i) \in \mathcal{PK}^{'}\} & \text{otherwise} \end{cases}$$

$$+ \begin{cases} 0 & \text{if } \omega \models \perp \\ -\kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n) & \text{otherwise} \end{cases}$$

$$= \kappa^{'}_{\mathcal{PK}}(\omega) - \kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n)$$

Example 5 *Let us illustrate the last step using Proposition 3. We have:*

$$\mathcal{PK}''' = \mathcal{PK}'' \cup \{\perp, -\kappa_{\mathcal{PK}}(\xi_1 \wedge ... \wedge \xi_n)\}$$

The final penalty base is :

$$\mathcal{PK}''' = \{(a, 3), (b, 7), (\neg b, 2), (a, 5), (\neg b, 7), (\perp, -7)\}$$

and its associated distribution is given in Table 4. We can clearly check that Table 4 is the same as the one of Table 2.

Table 4: Distribution of the resulted base which represents the C-revision of $\kappa_{\mathcal{PK}}$ with $\mathcal{S} = \{(\xi_1, \beta_1), .., (\xi_n, \beta_n)\}$

ω	$\kappa_{\mathcal{PK}}(\omega)$
ab	2
$a\neg b$	0
$\neg ab$	10
$\neg a \neg b$	8

Propositions 1-3 provide the characterization of the C-revision using weighted logic bases.

Note that the space complexity is linear with respect to the initial knowledge base for the three steps proposed in Propositions 1, 2 and 3.

The computational time complexity is also linear for steps given in Proposition 2 and Proposition 3. However computing $\kappa(\xi_1 \wedge ... \wedge \xi_n)$ (the step given in Proposition 1) needs a polynomial number of calls to a Max-SAT prover (an NP-Complete problem) with respect to the size of the knowledge base.

Conclusion

Multiple iterated c-revision is a revision process that modifies an OCF by taking into account a set of independent formulas.

In this paper, multiple iterated c-revision has been encoded using penalty logic. This is done in three steps: i) computing $\kappa(\xi_1 \wedge ... \wedge \xi_n)$, ii) computing $\kappa(\omega) + \sum_{i=m,\omega \not\models \xi_i}^{n} \beta_i$ and iii) shifting down an OCF κ by a constant number.

A future work is to provide an experimental study by comparing the syntactic computation of c-revision, given in this paper, with the one given in (Benferhat and Ajroud 2016).

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