

# Overlapping Coalition Formation in Multi-Sensor Networks

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## Abstract

We investigate *overlapping coalition formation* in multi-sensor networks where sensor agents can track multiple targets within their overlapping fields of view simultaneously. We employ a game-theoretic approach to model the problem of sensors coordination as that of an overlapping coalition formation problem. Our model seeks to find the minimum-sized overlapping coalition structure that maximizes the overall social welfare of sensor networks, i.e., ensuring targets coverage maximization while also minimizing the number of sensors needed. We then show that finding the optimal overlapping coalition structure of a set of agents in this type of environment is *NP*-complete.

## Introduction

### Background

Cooperation among autonomous agents in multi-agent environments is fundamental for agents to successfully achieve goals for which they lack enough resources. Agents' resources and skills vary, hence a motivation for agents to cooperate on tasks that are otherwise difficult for individual agents to complete or for which better results can be achieved by working in a group. One way of modeling such cooperation is via *coalition formation*. A coalition is a formal agreement among self-interested agents to complete mutually beneficial tasks jointly.

Examples of coalition formation can be found in *business* (e.g., organizations forming coalitions to make more sales and hence more profits), in *academia* (e.g., professors forming coalitions to publish articles), in *search and rescue* (e.g., robotic agents forming coalitions in large natural disaster environments to save life and properties), and in *voting* (e.g., voters forming coalitions to win elections). Our mundane day to day activities are not exempted from coalition formation influence. Thus, we cooperate with others to solve problems that may be difficult to accomplish individually. This difficulty may be due to a number of factors, such as time criticality of tasks, distribution of individual skills and/or resources, and the need for our physical presence in multiple work places simultaneously.

Much research on coalition formation has been restricted to *non-overlapping* coalition where agents are explicitly assumed to participate or belong to exactly one coalition at a time. The optimal *coalition structure* that emerges to execute tasks is a *partition* (i.e., disjoint coalitions which are subsets) of the original set of agents. There are environments, however, where it is appropriate for agents to belong to more than one coalition at a time. For example, in multi-sensor networks, agents (i.e., sensors) can track multiple targets within their overlapping fields of view simultaneously.

We refer to coalition formation where agents may belong to multiple coalitions simultaneously as being *overlapping*. We investigate overlapping coalition formation model in multi-sensor networks. Multi-sensor networks are usually composed of several thousands sensor nodes (Vinyals, Rodriguez-Aguilar, and Cerquides 2008). Our choice of overlapping coalition formation in multi-sensor networks is informed by the emergence of small and inexpensive sensors that have found applications in these large environments. Their appropriateness for modeling autonomous self-aware sensors in a flexible way is another important reason (Vinyals, Rodriguez-Aguilar, and Cerquides 2008).

### Motivation

Figure 1 depicts representation for a simple sensor node and its *field of view*. The field of view is the area bounded by the sector within which targets are observable. The area of the sector determines the *range of observation* that is available to the sensor. Thus, sensor nodes have varying range of fields of view. Sensor nodes orient their fields of view to indicate active sensing states when tracking targets. This is achieved by rotating in a circular mode within their fields of view. When a node senses the presence of targets within its

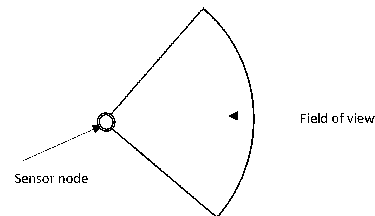


Figure 1: A simple sensor node and its field of view

field of view, the sensor reorients to track the targets.

A simple motivation for this problem considers a wireless sensor network of four sensor agents,  $a_1, a_2, a_3$ , and  $a_4$  depicted in figures 2 and 3. Figure 2 shows the initial deployment of the sensors in an area with three targets,  $t_1, t_2$ , and  $t_3$  within the vicinity.

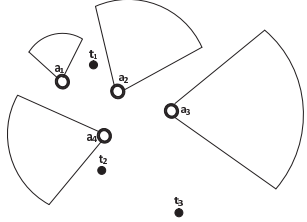


Figure 2: A wireless sensor network of four agents,  $a_1, a_2, a_3$ , and  $a_4$  with initial deployment in an area with three targets,  $t_1, t_2$ , and  $t_3$  in the vicinity.

The four sensors (see Figure 2) *distributedly* reorient from their initial active sensing states forming coalitions to track the three targets as shown in Figure 3. There are three coalitions,  $C_1 = \{a_1, a_2\}$ ,  $C_2 = \{a_3, a_4\}$ , and  $C_3 = \{a_3\}$  of the sensor nodes that are formed to track the three targets,  $t_1, t_2$ , and  $t_3$  respectively. Coalitions  $C_2$  and  $C_3$  are however *overlapping* as  $C_2 \cap C_3 \neq \emptyset$ . The coalitions remain for as long as the targets remain in the vicinity of the sensors or until more *important* targets appear; at which time the sensors review their coalitions on how best to track the new targets without losing focus of the current targets. We infer the importance of a target in a multi-sensor network environment by the degree of risk such a target may pose compared to other targets currently within the vicinity of the network.

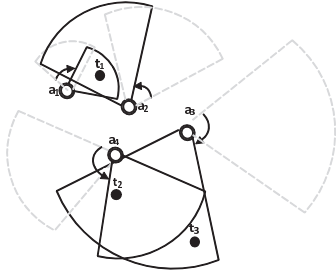


Figure 3: Sensors *distributedly* reorient forming coalitions to track targets. There are three coalitions,  $C_1 = \{a_1, a_2\}$ ,  $C_2 = \{a_3, a_4\}$ , and  $C_3 = \{a_3\}$  that are formed to track the three targets,  $t_1, t_2$ , and  $t_3$ , respectively. Coalitions  $C_2$  and  $C_3$  are *overlapping* as  $C_2 \cap C_3 \neq \emptyset$ .

There are several possible ways sensor nodes can coordinate to form coalitions. We refer to the three coalitions above together (i.e.,  $\{\{a_1, a_2\}, \{a_3, a_4\}, \{a_3\}\}$ ) as an *overlapping coalition structure*. This is just one of the several overlapping coalition structures from coordination among the sensor agents. Agents' decisions to participate in particular coalitions are influenced by important factors such as the *coalition value* (i.e., the largest value a coalition can

achieve by cooperating) and also by the *allocation* of the coalition value (i.e., the distribution of the coalition value among the members of the coalition). We consider this problem to be one of *coalition structure generation*. A *coalition structure* in non-overlapping domain is a partition of the set of agents. In our setting, an overlapping coalition structure is not necessarily a partition of the set of agents since coalitions may overlap. Our main results are as follows:

- We use a game-theoretic approach to model the problem of sensors coordination as that of an overlapping coalition formation problem. Our model seeks to find the minimum-sized overlapping coalition structure that maximizes the overall social welfare of multi-sensor networks.
- We then show that finding the optimal overlapping coalition structure of a set of agents in this type of environment is *NP*-complete.

## Previous Work

Coalition formation is widely studied and has been applied to many problems including task allocations (Shehory and Kraus 1995), multilateral trades (Yeung, Poon, and Wu 1999), multi-sensor networks (Dang et al. 2006), mobile networks (Lee and Chen 2006), and human coalition formation (Khandaker and Sohs 2009). Much research on coalition formation has been restricted to *non-overlapping* coalition where it is assumed that agents may only participate in a single coalition at a time, so the optimal coalition structure that emerges to execute tasks is a partition of the original set of agents. We are concerned in this work with *overlapping coalition formation* in multi-sensor networks.

The maiden study of overlapping coalition formation is the work of Shehory and Kraus (Shehory and Kraus 1996). Their approach is appropriate for agents in a Distributed Problem Solving (DPS) system where agents are concerned with the global performance of the system rather than individual benefits. The paper provides insights into overlapping coalition formation in a restricted blocks-world environment where goals or tasks have precedence order. Agents can complete tasks in sequence by belonging to several coalitions required for the ordered tasks using appropriate capabilities or skills as tasks demand. The work assumes a sub-additive environment where addition of new agents to coalitions may lead to overhead from coordination and communication costs that grows with the size of coalitions. Thus, they have restrictions on the size of coalitions that may form, and, in particular, the grand coalition may not form. Their overlapping coalition strategy is restricted to precedence blocks-world environment and similar domains which is not appropriate for the multi-sensor network environments that we consider in this work.

Dang et al. (Dang et al. 2006) propose two efficient *centralized* algorithms suitable for overlapping coalition formation in multi-sensor network environments. Similar to Shehory and Kraus work, the paper is concerned with the overall welfare of the entire system in a *cooperative* environment. They seek to find a coalition structure of agents that maximizes the system's welfare. Hence, their algorithms' performance is judged by the entire welfare of the

network. The idea of their overlapping coalition formation ensues when sensors belong to multiple coalitions tracking multiple targets simultaneously. Since multi-sensor networks are usually composed of several thousands of sensor nodes (Vinyals, Rodriguez-Aguilar, and Cerquides 2008), it is clear that centralizing the network may be infeasible for many reasons - due majorly to resources. A single agent may not be powerful enough to have all the required resources (e.g., hardware) to globally solve the overlapping coalition formation problem in the multi-sensor environment using Dang et al.'s algorithms exclusively.

In contrast to Dang et al.'s, our model assumes that agents operate in distributed multi-sensor network environments.

## Definitions and Notation

Our treatment of overlapping coalition formation in multi-sensor networks uses ideas from *coalitional game theory*. We give the following definitions and notation that are used in the paper. Let  $I = \{1, \dots, n\}$  be a set of  $n \in \mathbb{N}$  agents. The non-empty subsets,  $C \subseteq I$  are called *coalitions*.

**Definition 1.** Let  $(I, v)$  define a transferable utility coalitional game.  $v : 2^I \rightarrow \mathbb{R}$  is called the characteristic function of the game, and associates with each coalition  $C$ , a real-valued payoff  $v(C)$ .  $v(C)$  is the largest value that  $C$  can attain by cooperating. The term transferable means that  $v(C)$  can be shared in any manner that the members of  $C$  choose.

**Definition 2.** Cost, reward, and value of a coalition. Let  $C$  be a coalition of sensor agents that is tracking a target  $t$ . Let the sensing cost  $SC_{a_i}$  and the communication cost  $CC_{a_i}$  be the costs incurred by each sensor  $a_i \in C$ . We define

$$Q(C) = \sum_{a_i \in C} SC_{a_i} + CC_{a_i}$$

as the overall cost of forming the coalition  $C$ . Furthermore, let  $R(C)$  be the reward achieved by coalition  $C$  from tracking target  $t$ . We define  $v(C)$ , the coalitional value achieved by  $C$  as the net income gained from tracking  $t$  as

$$v(C) = R(C) - Q(C).$$

**Definition 3.** An overlapping coalition structure over  $I$  is a collection of non-empty subsets  $OCS_I = \{C_1, \dots, C_k\}$  such that

$$\bigcup_{i=1}^k C_i = I \text{ and } C_i \cap C_j \neq \emptyset$$

for some  $i, j$ ,  $1 \leq i, j \leq k$ , and such that  $i \neq j$ .

This definition implies that the distinct coalitions in  $OCS_I$  are not necessarily disjoint. For example, suppose we have three sensor agents,  $I = \{a_1, a_2, a_3\}$ . The following are some possible overlapping coalition structures for  $I$ :

$$\{\{a_1\}, \{a_1, a_2\}, \{a_3\}\}, \{\{a_2, a_3\}, \{a_3\}\}, \{\{a_1, a_2\}, \{a_2, a_3\}\}$$

**Definition 4.** Let  $OCS_I$  be an overlapping coalition structure over  $I$ . The value of  $OCS_I$  denoted by  $V(OCS_I)$  is

$$V(OCS_I) = \sum_{C \in OCS_I} v(C).$$

**Definition 5.** Overlapping Coalition Structure Generation (OCSG) Problem. Let  $\Gamma(I)$  be the set of all coalition structures for  $I$ . The optimal overlapping coalition structure  $OCS_I^*$ , is given as:

$$OCS_I^* = \arg \min_{OCS_I \in \Gamma(I)} |OCS_I| \left( \arg \max_{OCS_I \in \Gamma(I)} V(OCS_I) \right)$$

We seek to find the minimum-sized overlapping coalition structure that maximizes the overall social welfare of the system. We want to maximize targets coverage while minimizing the number of sensors. We prefer coalition structures which are small in size because they are easier to form and manage. Also, intra and inter coalition coordination, communications, and other overheads increase with coalition structure size.

**Example 1.** Consider a wireless sensor network of three sensor agents,  $a_1, a_2$ , and  $a_3$ , with initial deployment in an area with two targets,  $t_1$  and  $t_2$ , in their vicinity. We required that all targets be covered. Let the values for tracking targets  $t_1$  and  $t_2$  by coalitions of the sensor agents in this environment be define as follows.

$t_1$ :

$$\begin{aligned} v(\{a_1\}) &= v(\{a_2\}) = v(\{a_3\}) = 3 \\ v(\{a_1, a_2\}) &= v(\{a_1, a_3\}) = v(\{a_2, a_3\}) = 6 \end{aligned}$$

$t_2$ :

$$\begin{aligned} v(\{a_1\}) &= v(\{a_2\}) = v(\{a_3\}) = 2 \\ v(\{a_1, a_2\}) &= v(\{a_1, a_3\}) = v(\{a_2, a_3\}) = 4 \end{aligned}$$

In the non-overlapping coalition domain, the optimal coalition structure consists of any two-agent coalition tracking  $t_1$  and a single coalition tracking  $t_2$  for a total value of 8. For example,  $\{\{a_1, a_2\}, \{a_3\}\}$ . However, in the overlapping coalition domain, the optimal coalition structure consists of any two-agent coalitions tracking  $t_1$  and  $t_2$  for a total value of 10. For example,  $\{\{a_1, a_2\}, \{a_2, a_3\}\}$ . Note that since a coalition can track only one target at a time, then the grand coalition consisting of all the agents should not form in both the overlapping and non-overlapping domains.

## Complexity Analysis

We show that the OCSG problem is NP-complete. The well known Set Cover Problem (SCP) is defined as follows. Given a collection of  $n$  sets  $S_1, S_2, \dots, S_n$  and an integer  $k$ . Is there a subcollection of  $k$  sets  $S_{i1}, S_{i2}, \dots, S_{ik}$  such that

$$\bigcup_{i=1}^n S_i = \bigcup_{j=1}^k S_{ij}.$$

The subcollection of the  $k$  sets includes possibly overlapping sets. The SCP problem is known to be NP-complete (Garey and Johnson 1979). Observe that the problem of coalition structure generation in the non-overlapping domain, that is also known to be NP-complete (Sandholm et

al. 1999), is a special case of the *OCSG* problem. We can reduce *SCP* to *OCSG*. We provide a formal prove of the following result:

**Theorem 1.** *Finding the optimal overlapping coalition structure of a set of agents is NP-complete.*

**Lemma 1.** *OCSG is in NP.*

*Proof.* We consider the following decision version of the *OCSG* problem: Let  $I$  be a set of  $n \in \mathbb{N}$  agents and  $\Gamma(I)$  be the set of all coalition structures for  $I$ . Given coalition values  $v(C)$  for some coalitions  $C \subseteq I$  and positive real numbers  $p$  and  $q$ , does there exists in  $\Gamma(I)$  an  $OCS_I$  of size  $|OCS_I| \leq p$ , with value  $V(OCS_I) \geq q$ ?

Let us define a nondeterministic algorithm  $\hat{A}$  that takes as input an instance of the *OCSG* problem. We define  $\hat{A}$  to first guess some structure  $OCS_I$ . Then, we have  $\hat{A}$  verify that some coalitions in  $OCS_I$  are overlapping and that  $OCS_I$  defines an overlapping coalition structure for  $I$ . Furthermore,  $\hat{A}$  can efficiently check that  $OCS_I$  indeed contains at most  $p$  coalitions, and then computes its value. Clearly, all of these verifications can be done in polynomial time. In particular, computing  $V(OCS_I)$  involves summing the values of each coalition  $C$  in  $OCS_I$ , and there are at most  $p$  such coalitions. Thus, *OCSG* is in NP.  $\square$

**Lemma 2.** *OCSG is NP-Hard.*

*Proof.* We show that the *OCSG* problem is NP-hard by reducing the *SCP* problem to it. We define a polynomial transformation from *SCP* to *OCSG* as follows. Given a collection of  $n$  sets  $S_1, S_2, \dots, S_n$  and an integer  $k$  with  $|\bigcup_{i=1}^n S_i| = q$  as an instance of *SCP*. The corresponding instance of *OCSG* has a set  $I = \bigcup_{i=1}^n S_i$  of agents and a collection of coalitions  $C_1, C_2, \dots, C_n$  that are respectively identical to  $S_1, S_2, \dots, S_n$ . Note also that  $|I| = q$ . For any coalition  $C_i$ , the value  $v(C_i)$  of the coalition is set to  $|C_i|$  if  $S_i$  belongs to a set cover in the *SCP* instance and 0 otherwise. Finally, we set  $p = k$ . This construction can be done in time that is polynomial in the size of the *SCP* instance.

Next, we need to show that the original instance of the *SCP* is a yes instance if and only if the *OCSG* instance is also a yes instance. Suppose there is a set cover  $S = \{S_1, S_2, \dots, S_k\}$  of size at most  $k$  in the *SCP* instance, then coalitions  $OCS_I = \{C_1, C_2, \dots, C_k\}$  constitute an overlapping coalition structure of size  $|OCS_I| \leq p = k$  with value  $V(OCS_I) \geq q$ . This is because each  $C_i$  in *OCSG* corresponds to a set  $S_i$  in *SCP* and hence has value  $v(C_i) = |C_i|$  for a total value of  $\sum_{i=1}^k v(C_i) = q$ .

Now suppose  $OCS_I = \{C_1, C_2, \dots, C_p\}$  is an overlapping coalition structure for  $I$ , having size  $|OCS_I| \leq p$  and with value  $V(OCS_I) \geq q$ . Since each set  $C_i$  in the *OCSG* instance is associated with each set  $S_i$  in the *SCP* instance and there are exactly  $p$  such coalitions implies that we have a set cover  $S = \{S_1, S_2, \dots, S_k\}$  of size at most  $k = p$  in the *SCP* instance. Furthermore, the fact that  $V(OCS_I) \geq q$  implies that  $|\bigcup_{i=1}^k S_i| = q$  since each coalition  $C_i$  has a value  $v(C_i) = |C_i|$  if it is in the  $OCS_I$  or 0 otherwise. Thus, *OCSG* is NP-Hard.  $\square$

## Conclusions

We consider overlapping coalition formation in multi-sensor networks where it is appropriate for sensor agents to belong to more than one coalition at a time while tracking targets within their vicinity. The purpose of the overlapping coalition formation ideas is to maximize targets coverage while also minimizing the number of sensors that are needed. We use a game-theoretic approach to model this sensors coordination problem. Our model is formulated to find the minimum-sized overlapping coalition structure that maximizes the social welfare of sensor networks. We also show that finding the optimal overlapping coalition structure of a set of agents in this type of environment is NP-complete.

Having established the computational complexity of the overlapping coalition formation problem in this environment, a natural future work is to develop efficient approximation algorithms that sensors can distributedly use to coordinate. Theoretical and empirical evaluations of such algorithms to establish performance will also be considered.

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