Negated Min-Based Possibilistic Networks

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Abstract

Possibilistic networks are important tools for reasoning under uncertainty. They are compact representations of joint possibility distributions that encode available expert knowledge. The first part of the paper defines the concept of negated possibilistic network which will be used to encode the reverse of a joint possibility distribution. The second part of the paper proposes a propagation algorithm to compute a possibility degree of each event in the negated possibilistic network. Our algorithm is based on the use of a junction tree associated to the initial graphical structure.

Introduction

Graphical models (Darwiche 2009; Garcia and Sabbadin 2008) provide efficient tools to deal with uncertain pieces of information. In this paper, we are interested in possibilistic networks (Gebhardt and Kruse 1997; Benferhat 2010) which are important methods to efficiently represent and analyse uncertain information. They allow a flexible representation and a handling of independence relationships which are primordial to efficiently represent uncertain pieces of information.

As in a probabilistic Bayesian network (Darwiche 2009), a possibilistic network is a Directed Acyclic Graph (DAG), where nodes correspond to variables and edges represent causal or influence relationships between variables. Uncertainty is expressed by means of conditional possibility distributions for each node in the context of its parents.

There are two major definitions of possibility theory: min-based (or qualitative) possibility theory and product-based (or quantitative) possibility theory. At the semantic level, these two theories share the same definitions, including the concepts of possibility distributions, necessity measures, possibility measures and the definition of normalization conditions. However, they differ in the way they define possibilistic conditioning. Indeed, in a possibility theory, there are two main definitions of possibilistic conditioning. The first one is called min-based conditioning (or qualitative-based conditioning) which is appropriate in situations where only the ordering between events is important. In this case, the unit interval [0,1] is viewed as an ordinal

scale where only the minimum and the maximum operations are used for propagating uncertainty degrees. The second definition of conditioning is called product-based conditioning (or quantitative-based conditioning) where the unit interval is used in a general sense.

The two definitions of conditioning in a possibility theory framework lead to two types of possibilistic networks: product-based possibilistic networks (or quantitative-based networks) and min-based possibilistic networks (or qualitative-based networks) (Gebhardt and Kruse 1997; Benferhat 2010). In both possibilistic networks, several efficient inference algorithms have been proposed (BenAmor, Benferhat, and Mellouli 2003; Ayachi, Ben-Amor, and Benferhat 2013). As in a probabilistic network, the inference algorithms depend on the structure of the initial graph: if the graph is simply connected, then a polynomial algorithm based on message passing mechanism can be used. In the case of multiply connected networks, a graphical transformation from the initial graph is required to a secondary structure, such as *junction trees*, is needed.

In this paper, we only deal with min-based possibilistic networks. We present a definition of a negated min-based possibilistic network. In fact, the negation of a possibility distribution may be needed in different contexts. This can be encountered in decision making problems under uncertainty where the pessimistic utility function is expressed by the negated possibility distribution (Benferhat et al. 2009). Therefore, it is important to define a possibilistic network that encodes the negation of some possibility distribution. One natural solution is to keep the same graphical representation as the original graphical model and adapt the inference process in order to calculate the possibility degree of each event given some observed variables.

The rest of this paper is organized as follows: next section briefly recalls basic concepts of the possibility theory framework. Section 3 provides an overview of possibilistic networks which allow a graphical representation of uncertain information. In section 4, we detail the representation and the reasoning process for negated possibility distributions. Section 5 concludes the paper.

Basics Concepts of Possibility Theory

In order to deal with uncertain and imprecise data, several theories of uncertainty have been proposed. This paper fo-

cuses on possibility theory (Dubois and Prade 2012).

Let $\mathcal{V} = \{A_1, ..., A_N\}$ be a set of variables. We denote by $\mathbb{D}_{Ai} = \{a_{i1}, ..., a_{in}\}$ the domain associated with the variable A_i. a_{ij} denotes any instance j of A_i. The universe of discourse is denoted by $\Omega = \times_{A_i \in V} \mathbb{D}_{A_i}$, which is the Cartesian product of all variables' domains in V. Each element $\omega \in \Omega$ is called an interpretation which represents a possible state of world (or universe of discourse). It is denoted by $\omega = (a_{1i}, ..., a_{nm})$. $\phi, \psi...$ represent subsets of Ω (events).

Possibility Distribution

The basic element in a possibility theory is the notion of a possibility distribution π which corresponds to a mapping from the set of interpretations Ω to the uncertainty scale $[0, 1]$. This distribution allows the encoding of our knowledge on real world. $\pi(\omega) = 1$ means that ω is fully possible and $\pi(\omega) = 0$ means that it is impossible to ω to be the real world. A possibilistic scale can be interpreted in two ways:

- in a qualitative way if the possibility degrees only reflect an ordering between different states of the world,

- in a quantitative way if the affected values make sense in numerical scale.

This paper focuses on ordinal interpretation of uncertainty scales.

A possibility distribution π is said to be $\alpha-$normalized, if its normalization degree $h(\pi)$ is equal to α, namely:

$$h(\pi) = \max_{\omega} \pi(\omega) = \alpha. \qquad (1)$$

If $\alpha = 1$, then π is said to be normalized.

Given a possibility distribution π on the universe discourse Ω, two dual measures are defined for each event $\phi \subseteq \Omega$:

- **Possibility measure:** this measure evaluates to what extent ϕ is consistent with our knowledge encoded by π:

$$\Pi(\phi) = \max_{\omega \in \Omega} \{\pi(\omega) : \omega \in \Omega \text{ and } \omega \models \phi\}. \qquad (2)$$

- **Necessity measure:** it is the dual of possibility measure. The necessity measure evaluates to which level ϕ is certainly implied by our knowledge, represented by π:

$$N(\phi) = 1 - \Pi(\neg\phi). \qquad (3)$$

Possibilistic conditioning (Coletti, Petturiti, and Vantaggi 2013) consists in revising the initial knowledge, encoded by a possibility distribution π, by the arrival of a new certain information $\phi \subseteq \Omega$. The initial distribution π is then replaced by another one, denoted $\pi' = \pi(. \mid \phi)$. The two interpretations of the possibilistic scale (qualitative and quantitative) induce two definitions of possibilistic conditioning: product-based conditioning and min-based conditioning. In this paper, we only focus on min-based conditioning defined by:

$$\pi(\omega \mid \phi) = \begin{cases} 1 & \text{If } \pi(\omega) = \Pi(\phi) \text{ and } \omega \in \phi \\ \pi(\omega) & \text{If } \pi(\omega) < \Pi(\phi) \text{ and } \omega \in \phi \\ 0 & \text{otherwise.} \end{cases} \qquad (4)$$

We also use a so-called min-based independence relation, also known as a non-interactivity relation. This relation is obtained by using the min-based conditioning (Equation 4) and it is defined by:

$$\forall x, y, z \; \Pi(x \wedge y \mid z) = \min(\Pi(x \mid z), \Pi(y \mid z)). \qquad (5)$$

Possibilistic Networks

There are two ways to define possibilistic networks in a possibility theory framework depending on the use of possibilistic conditioning (Coletti, Petturiti, and Vantaggi 2013). In this paper, we only focus on min-based possibilistic networks. A min-based possibilistic network (Fonck 1992; Benferhat 2010) over a set of variables $\mathcal{V}$, denoted by $\Pi G_{min} = (G, \pi)$, is characterized by:

1. **A graphical component:** which is represented by a Directed Acyclic Graph (DAG) where nodes correspond to variables and arcs represent dependence relations between variables.

2. **Numerical components:** these components quantify different links in the DAG by using local possibility distributions for each node A in the context of its parents, denoted by $Par(A)$. More precisely:

   - For every root node A ($Par(A) = \emptyset$), uncertainty is represented by a priori possibility degree $\pi(a)$ for each instance $a \in \mathbb{D}_A$, such that $\max_{a \in \mathbb{D}_A} \pi(a) = 1$.

   - For the rest of the nodes ($Par(A) \neq \emptyset$), uncertainty is represented by the conditional possibility degree $\pi(a \mid u_A)$ for each instance $a \in \mathbb{D}_A$ and for any instance $u_A \in \mathbb{D}_{Par(A)}$ (where $\mathbb{D}_{Par(A)}$ represents the Cartesian product of all variable's domains in $Par(A)$), such that $\max_{a \in \mathbb{D}_A} \pi(a \mid u_A) = 1$, for any u_A.

The set of a priori and conditional possibility degrees induce a unique joint possibility distribution π_{min} defined by:

$$\pi_{min}(A_1, ..., A_N) = \min_{i=1..N} \pi(A_i \mid U_i). \qquad (6)$$

One of the most common tasks performed on possibilistic networks is the *possibilistic inference* which consists in determining how the realization of some specific values of some variables, called observations or evidence, affects the remaining variables (Huang and Darwiche 1996). The problem of computing posteriori marginal distributions on nodes in arbitrary possibilistic networks is known to be a hard problem (Borgelt, Gebhardt, and Kruse 1998) except for singly connected graphs which ensure the propagation in polynomial time (Fonck 1992). One of well-known propagation algorithm is the so-called junction tree algorithm (Darwiche 2009). The idea is to transform the initial graph into a tree on which the propagation algorithm can be achieved in an efficient way. More recent works have been proposed and based on compilation process (Darwiche 2009) (Ayachi, BenAmor, and Benferhat 2013) of parameters instead of graphs.

Negated Possibilistic Network

This section contains the main contributions of this paper which consists in defining the syntactic counterpart of negated possibility distributions. In many situations, one may need to compute negated possibility distributions. For instance, in decision making under uncertainty, computing pessimistic decision comes down to choosing a decision $d \in D$ which maximises the qualitative utility given by:

$$u_*(d) = min_{\omega \in \Omega} max(1 - \pi(\omega), \mu(\omega))$$

where D is a set of decisions, π and μ are two possibility distributions representing agent's beliefs and preferences respectively. The natural question in this case is: if π and μ are compactly represented by two min-based possibilistic networks, how to represent $(1 - \mu)$ by a min-based possibilistic networks? Our aim consists then in defining a new possibilistic network $\Pi G_{neg} = (G', \pi_N)$ encoding the joint possibility distribution $\pi_{neg} = 1 - \pi_{min}$ which corresponds to the negation of the first network ΠG_{min}. Once the negated possibilistic network is defined, we show how to perform queries over ΠG_{neg}.

Construction of Negated Possibilistic Network

Recall that a min-based possibilistic network $\Pi G_{min} = (G, \pi)$ is defined by its graphical component G and a set of conditional possibility distributions $\pi(A_i \mid U_{A_i}), \forall A_i \in V$. It encodes a unique joint possibility distribution π_{min} described by equation 6.

The negated min-based possibilistic network $\Pi G_{neg} = (G', \pi_N)$ is defined as follows:

Definition 1. *Let $\Pi G_{min} = (G, \pi)$ be a min-based possibilistic network.*

- *It has the same graphical component as ΠG_{min} namely: $G' = G$,*
- *The negated possibility distributions relative to each variable $A \in v$ is given by:*
  - *For each root node, $\forall a \in \mathcal{D}_A \; \pi_N(a) = 1 - \pi(a)$,*
  - *For the rest of the node, $\forall a \in \mathcal{D}_A$ and $\forall u_A \in \mathcal{D}_{Par(A)}$ $\pi_N(a \mid u_A) = 1 - \pi(a \mid u_A)$.*

The set of a priori and conditional possibility degrees induces a unique joint possibility distribution defined by the following max-based chain rule:

Definition 2. *Given a negated min-based possibilistic network $\Pi G_{neg} = (G, \pi_N)$, we define its associated joint possibility distribution, denoted by π_{neg}, using the following max-based chain rule:*

$$\pi_{neg}(A_1, ..., A_N) = \max_{i=1..N} \pi_N(A_i \mid U_i) \qquad (7)$$

The following proposition guarantees that the negated possibilistic network encodes the joint possibility $\pi_{neg} = 1 - \pi_{min}$. Note that in the Definition 2, the joint distribution is defined using the maximum operation instead of minimum operation. This leads to a definition similar to the notion of guaranteed possibility distributions defined in (Dubois, Hajek, and Prade 2000). Hence, to some extent, a negated min-based network can be viewed as a guaranteed-based network (Ajroud et al. 2009) and guaranteed-based possibilistic logic can be used to represent preferences (Benferhat et al. 2002).

Proposition 1. *Let $\Pi G_{min} = (G, \pi)$ be a min-based possibilistic network encoding the possibility distribution π_{min}. Let ΠG_{neg} be the negated min-based possibilistic network associated with ΠG_{min} using the above definitions (Definition 1 and Definition 2). Then, we have:*

$$\pi_{neg}(A_1, ..., A_N) = 1 - \pi_{min}(A_1, ..., A_N) \qquad (8)$$

Proof. The proof is immediate. By definition, we have :

$$\begin{aligned}
\pi_{neg}(A_1, ..., A_N) &= \max_{i=1..N} \pi_N(A_i \mid U_i) \\
&= \max_{i=1..N} [1 - \pi(A_i \mid U_i)] \\
&= 1 - \min_{i=1..N} \pi(A_i \mid U_i) \\
&= 1 - \pi_{min}(A_1, ..., A_N).
\end{aligned}$$

$\square$

Example 1. *Let us consider the min-based network $\Pi G_{min} = (G, \pi)$, composed of the DAG of Figure 1 and the initial possibility distributions associated with variables X, Y, Z and V are given in tables 2 and 1. We assume that the variables are binary variables.*
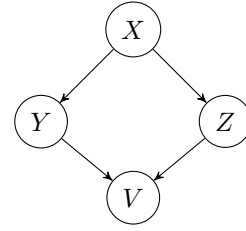

Figure 1: Example of DAG

Table 1: Initial possibility distributions and their negation.

X	$\pi(X)$	$\pi_N(X)$
x	.1	**.9**
$\neg x$	1	**0**

Table 2: Initial possibility distributions and their negation on Y given X.

Y	X	$\pi(Y \mid X)$	$\pi_N(Y \mid X)$
y	x	.9	**.1**
y	$\neg x$	1	**0**
$\neg y$	x	1	**0**
$\neg y$	$\neg x$	.6	**.4**

Table 3: Initial possibility distributions and their negation on Z given X.

Z	X	$\pi(Z \mid X)$	$\pi_N(Z \mid X)$
z	x	.2	**.8**
z	$\neg x$	1	**0**
$\neg z$	x	1	**0**
$\neg z$	$\neg x$	1	**0**

Table 4: Initial possibility distributions and their negation on V given Y and Z.

V	Y	Z	$\pi(V\mid Y,Z)$	$\pi_N(V\mid Y,Z)$	V	Y	Z	$\pi(V\mid Y,Z)$	$\pi_N(V\mid Y,Z)$
v	y	z	.8	**.2**	$\neg v$	y	z	1	**0**
v	y	$\neg z$	1	**0**	$\neg v$	y	$\neg z$	.7	**.3**
v	$\neg y$	z	.2	**.8**	$\neg v$	$\neg y$	z	1	**0**
v	$\neg y$	$\neg z$	.3	**.7**	$\neg v$	$\neg y$	$\neg z$	1	**0**

The negated possibilistic network $\Pi G_{neg} = (G, \pi_N)$ is such that its graphical component is the same as the one given by the DAG of Figure 1. Its numerical component is the negation of possibility distributions given in Tables 1, 2, 3 and 4.

Using the min-based chain rule (Equation 6) we obtain the joint possibility distribution given in table 5. The joint

Table 5: Joint possibility distribution π_{min} and its negation π_{neg}.

X	Y	Z	V	π_{min}	$\boldsymbol{\pi_{neg}}$	X	Y	Z	V	π_{min}	$\boldsymbol{\pi_{neg}}$
x	y	z	v	.1	**.9**	$\neg x$	y	z	v	.8	**.2**
x	y	z	$\neg v$	.1	**.9**	$\neg x$	y	z	$\neg v$	1	**0**
x	y	$\neg z$	v	.1	**.9**	$\neg x$	y	$\neg z$	v	1	**0**
x	y	$\neg z$	$\neg v$	.1	**.9**	$\neg x$	y	$\neg z$	$\neg v$	.7	**.3**
x	$\neg y$	z	v	.1	**.9**	$\neg x$	$\neg y$	z	v	.2	**.8**
x	$\neg y$	z	$\neg v$	.1	**.9**	$\neg x$	$\neg y$	z	$\neg v$	.6	**.4**
x	$\neg y$	$\neg z$	v	.1	**.9**	$\neg x$	$\neg y$	$\neg z$	v	.3	**.7**
x	$\neg y$	$\neg z$	$\neg v$	.1	**.9**	$\neg x$	$\neg y$	$\neg z$	$\neg v$	.6	**.4**

possibility distribution associated with ΠG_{neg} obtained using the max-based chain rule (Equation 7) is given by the same table (in bold). We can for instance check that:

$$\pi_{neg}(\neg x, y, z, v) = 1 - \pi_{min}(\neg x, y, z, v)$$

Indeed,
$\pi_{neg}(\neg x, y, z, v) =$
$\max(\pi_N(\neg x), \pi_N(y \mid \neg x), \pi_N(z \mid \neg x), \pi_N(v \mid y, z)).$
Then,
$\pi_{neg}(\neg x, y, z, v) = \max(0, 0, 0, .2) = .2$
$1 - \pi_{min}(\neg x, y, z, v) = 1 - .8 = .2$

Next section shows how to propagate uncertainty in negated possibilistic networks.

Uncertainty Propagation in Negated Possibilistic Network

The negated min-based possibilistic network defined just above will be used for computing a marginal distribution for each variable. More precisely, we propose a propagation algorithm in the spirit of the one based on junction tree structure. The principle of our algorithm is to transform the initial graph into a secondary structure known as a junction tree noted $\mathcal{JT}$ (Darwiche 2009). The procedure of building the junction tree is the same as the one used in a probabilistic Bayesian network and in a min-based possibilistic network. However, the propagation step in negated networks is different from the one used in probabilistic Bayesian networks and the one used in min-based possibilistic networks. Indeed, the propagation process in negated possibilistic networks is reduced in two steps, instead of the three stages identified in the propagation process in possibilistic networks (BenAmor, Benferhat, and Mellouli 2003). The first step consists in *initialization* of the junction tree allowing its quantification using the initial distributions. The second one, associated with *handling queries*, allows to provide the marginal distribution for each variable.

More precisely, the different steps can be depicted as follows:

A) **Building junction tree:**
The construction of the junction tree $\mathcal{JT}$ from the initial DAG G needs three steps (Darwiche 2009):

1. **Moralization of the initial graph $G_{\oplus}$:** It consists in creating an undirected graph from the initial one by adding links between the parents of each variable, and replacing directed arcs by undirected ones.

2. **Triangulation of the moral graph:** It allows to identify sets of variables that can be gathered as clusters or cliques denoted by C_i. Several heuristics have been proposed in order to find the best triangular graph which minimizes the size of clusters.

3. **Construction of the optimal junction tree $\mathcal{JT}$:** A junction tree is built, as in the case of Bayesian networks, by connecting the clusters, representing cliques of the triangulated graph, identified in the previous step. Once adjacent clusters have been identified, between each pair of clusters C_i and C_j, a separator S_{ij} containing their common variables, is inserted.

B) **Propagation process:**
The propagation process involves two steps:

1. **Initialization:** Once the junction tree is achieved, we proceed to quantify this new structure by transforming initial conditional possibility distributions into local joint distributions attached to each cluster and separator.

   For each cluster C_i (respectively separator S_{ij}), we assign a potential π_{C_i} (respectively $\pi_{S_{ij}}$).

   Definition 3. *Let $\mathcal{JT}$ be a junction tree associated with the initial graph G. The unique joint distribution, noted $\pi_{\mathcal{JT}}$ associated to the junction tree $\mathcal{JT}$ is defined by:*

   $$\pi_{\mathcal{JT}}(A_1, ..., A_N) = \max_{i=1..m} \pi_{C_i}, \qquad (9)$$

   where m is the number of clusters in $\mathcal{JT}$.

   The quantification of the junction tree $\mathcal{JT}$ is done using the initial possibility distributions as follows:

   - For each cluster C_i, $\pi_{C_i} \leftarrow 0$,
   - For each separator S_{ij}, $\pi_{S_{ij}} \leftarrow 0$,
   - For each variable A_i, choose a cluster C_i containing $\{A_i\} \cup \{Par(A_i)\}$: $\pi_{C_i} = max(\pi_{C_i}, \pi_N(A_i \mid U_i))$.

   The following proposition shows that the initialized junction tree encodes the same possibility distribution as the one of initial negated possibilistic network.

   Proposition 2. *Let $\Pi G_{neg} = (G, \pi_N)$ be the negated min-based possibilistic network associated with ΠG_{min}. Let $\mathcal{JT}$ be the junction tree corresponding to ΠG_{neg} generated using the above initialization procedure. Let π_{neg} be the joint distribution encoded by ΠG_{neg} using Equation 7 and $\pi_{\mathcal{JT}}$ be the joint possibility distribution encoded by $\mathcal{JT}$ using Equation 9. Then, $\forall i, i = 1, ..., n$*

   $$\pi_{neg}(A_1, ..., A_N) = \pi_{\mathcal{JT}}(A_1, ..., A_N). \qquad (10)$$

2. **Handling queries:**
   The computation of the marginal possibility distributions relative to each variable $A_i \in \mathcal{V}$ can be accomplished by marginalizing the potential of each cluster and choosing the maximum one as follows:

Proposition 3. *Let $\Pi G_{neg} = (G, \pi_N)$ be the negated min-based possibilistic network associated to ΠG_{min}. Let $\mathcal{JT}$ be the junction tree corresponding to ΠG_{neg} generated using the above initialization procedure. The possibility distribution $\pi_N(A_i)$ relative to each variable $A_i \in \mathcal{V}$ is done by:*

$$\forall A_i \in \mathcal{V}, \; \pi_N(A_i = a_i) = \max_{i=1..m} \max_{A_i = a_i} \pi_{C_i} \quad (11)$$

Proof. The proof is immediate. Indeed,

$$
\begin{aligned}
\pi_N(A_i = a_i) \;& = \pi_{\mathcal{JT}}(A_i = a_i) \\
& = \max_{V \backslash A_i} \pi_{\mathcal{JT}}(A_1, .., a_i, .., A_N) \\
& = \max_{V \backslash A_i} [\max_{i=1..m} \pi_{C_i}] \\
& \text{by using Equation 9} \\
& = \max_{i=1..m} [\max_{V \backslash A_i} \pi_{C_i}] \\
& = \max_{i=1..m} [\max_{A_i = a_i} \pi_{C_i}].
\end{aligned}
$$

$\square$

Example 2 (Example continued).

- *Applying the procedure described above, the junction tree associated with the DAG of Figure 1, is given by Figure 2. it contains two clusters*
  $C_1 = \{X, Y, Z\}$ *and*
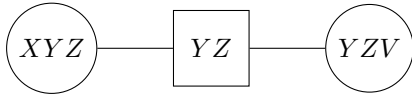 $C_2 = \{Y, Z, V\}$ *and their separator*
  $S_{12} = \{Y, Z\}$.

Figure 2: Junction tree

- *For the quantification step of the obtained junction tree, the potential assigned to each cluster is given in Table 6. As the variable x has no parent, the variable Y is with its parent in C_1 and similarly for the variable Z. The initialization procedure affects the potential π_{C_1} for the first cluster as follows:*

$$\pi_{C_1} = \max(0, \pi_N(X), \pi_N(Y \mid X), \pi_N(Z \mid X)).$$

However, the variable V is with their parents (Y and Z) in the second cluster C_2. From the initialization procedure, the potential π_{C_2} is given by:

$$\pi_{C_2} = \max(0, \pi_N(V \mid Y, Z)).$$

For example, the potential $\pi_{C_2}(yzv)$ is computed as fol-

Table 6: Potentials affected to clusters.

X	Y	Z	π_{C_1}	Y	Z	V	π_{C_2}
x	y	z	.9	y	z	v	.2
x	y	$\neg z$	.9	y	z	$\neg v$	0
x	$\neg y$	z	.9	y	$\neg z$	v	0
x	$\neg y$	$\neg z$	.9	y	$\neg z$	$\neg v$	.3
$\neg x$	y	z	0	$\neg y$	z	v	.8
$\neg x$	y	$\neg z$	0	$\neg y$	z	$\neg v$	0
$\neg x$	$\neg y$	z	.4	$\neg y$	$\neg z$	v	.7
$\neg x$	$\neg y$	$\neg z$	.4	$\neg y$	$\neg z$	$\neg v$	0

Table 7: Joint possibility distribution associated to $\mathcal{JT}$.

X	Y	Z	V	$\pi_{\mathcal{JT}}(X,Y,Z,V)$	X	Y	Z	V	$\pi_{\mathcal{JT}}(X,Y,Z,V)$
x	y	z	v	.9	$\neg x$	y	z	v	.2
x	y	z	$\neg v$	.9	$\neg x$	y	z	$\neg v$	0
x	y	$\neg z$	v	.9	$\neg x$	y	$\neg z$	v	0
x	y	$\neg z$	$\neg v$	.9	$\neg x$	y	$\neg z$	$\neg v$	.3
x	$\neg y$	z	v	.9	$\neg x$	$\neg y$	z	v	.8
x	$\neg y$	z	$\neg v$	.9	$\neg x$	$\neg y$	z	$\neg v$	.4
x	$\neg y$	$\neg z$	v	.9	$\neg x$	$\neg y$	$\neg z$	v	.7
x	$\neg y$	$\neg z$	$\neg v$	.9	$\neg x$	$\neg y$	$\neg z$	$\neg v$	.4

lows:

$$
\begin{aligned}
\pi_{C_2}(yzv) \;& = \max(0, \pi_N(v \mid yz)) \\
& = \max(0, .2) \\
& = .2.
\end{aligned}
$$

The joint possibility distribution induces by junction tree is computed using Equation 9. The corresponding results are reported in Table 7.

- *For the propagation, we need to compute the marginal possibility distribution $\pi_N(\neg x)$.*
  Using Equation 11, we get:

$$
\begin{aligned}
\pi_N(\neg x) \;& = \max(\max_{YZ} \pi_{C_1}, \max_{YZV} \pi_{C_2}) \\
& = \max(.4, .8) \\
& = .8.
\end{aligned}
$$

We get the same result using the negated min-based possibilistic network. Indeed,

$$\pi_N(\neg x) = \max_{YZV} \pi_{neg} = .8.$$

Conclusion

This paper proposed a compact representation of the negation of a possibility distribution. The obtained negated min-based possibilistic network preserves the same graphical component as the initial min-based possibilistic network. It also induces a unique joint possibility distribution obtained

by a max-based chain rule. The propagation process allows the computation of marginal distributions using junction tree. This result is important since it shows that the computational complexity of querying a negated possibilistic network is the same as the one of querying a standard min-based possibilistic network. As future works, we plan to apply the results of this paper for computing pessimistic optimal decisions (BenAmor, Fargier, and Guezguez 2014) (BenAmor, Essghaier, and Fargier 2014). Indeed, pessimistic utility is expressed by the negation of possibility distribution representing agent's knowledge. Thus, by analogy to the work developed in (Benferhat, Khellaf, and Zeddigha 2015) for optimistic criteria, the proposed framework will leads to offer a new graphical model for computing optimal decision using pessimistic criteria.

Acknowledgements

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