Multiple Iterated Belief Revision Without Independence

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Abstract

Multiple iterated revision needs advanced belief revision techniques beyond the classical AGM theory that are able to integrate several (propositional) pieces of new information into epistemic states. A crucial feature of this kind of revision is that the multiple pieces of information should be dealt with separately, which has usually been understood as requiring some kind of independence among the different propositions under revision. Therefore, previous works have proposed several independence postulates which should ensure this. In this paper, we present an approach to multiple iterated revision that can do without those independence postulates. More precisely, we propose a method to revise ordinal conditional functions (so-called Spohn's ranking functions) by a set of propositional beliefs that satisfies the epistemic AGM postulates and the Darwiche and Pearl postulates for iterated revision, as well as some other postulates for multiple iterated revision, but none of the independence postulates that have been proposed so far. This shows that those independence postulates are not necessary for ensuring the adequate handling of multiple pieces of information under revision.

1 Introduction

AGM revision (Alchourrón, Gärdenfors, and Makinson 1985) is the classical theory for performing belief revision in propositional frameworks. Given a set of beliefs K and a new information A, AGM theory sets up a frame of postulates to ensure that the revised belief set $K * A$ follows a paradigm of minimal (reasonable) change. Since AGM theory considers only deductively closed belief sets, it has long been assumed that the revision by multiple pieces of information A, B can be handled adequately by taking their conjunction $A \wedge B$ as input for the revision process and so boils down to classical AGM revision. However, when the AGM framework was extended by considering iterated revision (Darwiche and Pearl 1997), it became apparent that identifying several pieces of information with their conjunction leads to counterintuitive results, as in the following adder-and-multiplier example of which we recall the version described in (Delgrande and Jin 2012):

Example 1 ((Delgrande and Jin 2012)) *Suppose an electric circuit contains an adder and a multiplier. The atomic propositions a and m denote respectively that the adder and the multiplier are working. Initially we have no information about this circuit; then we learn that the adder and the multiplier are working: $A = a \wedge m$. Thereafter, someone tells us that the adder is actually not working: $B = \neg a$. At this point, the postulates for iterated revision of Darwiche and Pearl ((DP) postulates) (Darwiche and Pearl 1997) imply that we have to 'forget' that the multiplier is working because of $B \models \neg A$.*

What has long been conceived as a problem with the (DP) postulates is rather a problem caused by identifying multiple pieces of information with their conjunction – from the set $\{a, m\}$, we can eliminate a (basically) without problems whereas the elimination of a from the formula $a \wedge m$ needs more involved logical considerations. Being able to make a difference between considering the set vs. the formula under revision requires to express a kind of independence between the elements of the set. A postulate that aims at overcoming this problem for iterated revision of epistemic states is the *Independence postulate* (Jin and Thielscher 2007) that has received some attention. One of the most recent publications for multiple iterated revision in the spirit of AGM is (Delgrande and Jin 2012) where belief revision of epistemic states by sets of sentences is studied. In particular, the latter authors generalize the *Independence postulate*, discuss several other postulates and the relationships among them, and coin the term *parallel revision* for revision operators that satisfy a specific subset of the postulates.

In this paper, we make a proposal for multiple iterated belief revision that is closer to AGM and the DP postulates (Darwiche and Pearl 1997) but nevertheless ensures that multiple revision does not collapse to usual AGM revision by single sentences. In particular, we show that the independence postulates of (Delgrande and Jin 2012; Jin and Thielscher 2007) are not necessary for multiple revision but are rather aiming at enforcing or strengthening the acceptance of beliefs in the posterior epistemic state. By adapting conditional c-revisions (Kern-Isberner 2004) to the propositional case, we present a versatile and constructive schema for multiple iterated belief revision in the context of Spohn's ranking functions (Spohn 1988) which has become a particularly basic and popular framework for studying belief change. Therefore, our approach has two advantages over other approaches having been proposed so far:

First, the schema for our multiple iterated revisions called c-revisions is quite concise and clear without forcing the user to consider involved recursive definitions. Second, it satisfies all major postulates from the literature except for the independence postulates.

This paper is organized as follows: In section 2, we recall details on propositional logic and ordinal conditional functions (OCF). Afterwards, related works on multiple iterated revision are briefly summarized in section 3; in particular, we include a collection of postulates that are relevant for the results in this paper. We elaborate on propositional c-revisions in section 4 and conclude by highlighting the main contributions of this paper in section 5. Some technical details or proofs have been omitted due to lack of space.

2 Preliminaries on logic and OCF

Let $\mathcal{L}$ be a finitely generated propositional language, with atoms $a, b, c, \ldots$, and with formulas $A, B, C, \ldots$. For conciseness of notation, we will omit the logical *and*-connector, writing AB instead of $A \wedge B$, and overlining formulas will indicate negation, i.e. $\overline{A}$ means $\neg A$. Let Ω denote the set of possible worlds over $\mathcal{L}$; Ω will be taken here simply as the set of all propositional interpretations over $\mathcal{L}$. $\omega \models A$ means that the propositional formula $A \in \mathcal{L}$ holds in the possible world $\omega \in \Omega$; then ω is a *model* of A. As usual, let $\models$ also denote the classical entailment relation between propositions. By slight abuse of notation, we will use ω both for the model and the corresponding conjunction of all positive or negated atoms. For a finite set $\mathcal{S}$ of formulas in $\mathcal{L}$, let $\bigwedge \mathcal{S}$ denote the conjunction of all formulas in $\mathcal{S}$; $\bigwedge \mathcal{S}$ is also called the *propositional content* of $\mathcal{S}$. $Mod(\mathcal{S})$ denotes the set of all models of $\mathcal{S}$ and conversely, given a set of possible worlds $\Omega' \subseteq \Omega$, $\mathcal{T}(\Omega')$ denotes the set of formulas which are true in all elements of Ω'. The classical consequences of $\mathcal{S}$ are given by $Cn(\mathcal{S}) = \{B \in \mathcal{L} \mid \bigwedge \mathcal{S} \models B\}$. Furthermore, $\overline{\mathcal{S}}$ stands for the set of all negated sentences from $\mathcal{S}$, i.e., $\overline{\mathcal{S}} = \{\overline{S} \mid S \in \mathcal{S}\}$. For a subset $\mathcal{S}_1 \subseteq \mathcal{S}$, the *completion of* $\mathcal{S}_1$ *in* $\mathcal{S}$ is defined by $C_{\mathcal{S}}(\mathcal{S}_1) = \mathcal{S}_1 \cup \overline{(\mathcal{S} \backslash \mathcal{S}_1)}$. For two sets $\mathcal{S}_1, \mathcal{S}_2$ of formulas of $\mathcal{L}$, let $\mathcal{S}_1 \parallel \mathcal{S}_2 = \{\mathcal{S}'_1 \subseteq \mathcal{S}_1 \mid \mathcal{S}'_1 \cup \mathcal{S}_2 \text{ is consistent}\}$ be the set of all subsets of $\mathcal{S}_1$ that are consistent with $\mathcal{S}_2$. Furthermore, for a set of formulas $\mathcal{S}$ and a possible world ω, let $\mathcal{S}|\omega = \{A \in \mathcal{S} \mid \omega \models A\}$ be the subset of all formulas in $\mathcal{S}$ which are satisfied by ω.

Conditionals $(B|A)$ over $\mathcal{L}$, i.e., $A, B \in \mathcal{L}$, are meant to express uncertain, defeasible rules "If A then plausibly B".

Ordinal conditional functions (OCFs), (also called *ranking functions*) $\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$ with $\kappa^{-1}(0) \neq \emptyset$, were introduced first by Spohn (Spohn 1988). They express degrees of plausibility of propositional formulas A by specifying degrees of disbeliefs of their negations $\overline{A}$. More formally, we have $\kappa(A) := \min\{\kappa(\omega) \mid \omega \models A\}$, so that $\kappa(A \vee B) = \min\{\kappa(A), \kappa(B)\}$. Hence, due to $\kappa^{-1}(0) \neq \emptyset$, at least one of $\kappa(A), \kappa(\overline{A})$ must be 0. A proposition A is believed, $\kappa \models A$, if $\kappa(\overline{A}) > 0$ (which implies particularly $\kappa(A) = 0$). Degrees of plausibility can also be assigned to conditionals by setting $\kappa(B|A) = \kappa(AB) - \kappa(A)$. A conditional $(B|A)$ is accepted in the epistemic state represented by κ, written as $\kappa \models (B|A)$, iff $\kappa(AB) < \kappa(A\overline{B})$, i.e. iff AB is more plausible than $A\overline{B}$.

3 Postulates and related work

In this section, we recall briefly the relevant parts of belief revision theory, with a special focus on results for multiple belief revision.

Alchourron, Gärdenfors, and Makinson were the first to consider logic-based quality criteria for changing a belief set (i.e., a deductively closed set of formulas) K by a new proposition A, resulting in a posterior belief set $K * A$ with a revision operator $*$ (Alchourrón, Gärdenfors, and Makinson 1985). These so-called *AGM postulates* have been providing a base for most work on belief revision since then. A particular useful extension to the AGM theory was provided by Darwiche and Pearl (Darwiche and Pearl 1997) when dealing with iterated change, i.e., the multiple usage of the revision operator to process multiple pieces of information successively, as in $(K * A) * B$. This put the focus on revision strategies and epistemic states Ψ as entities that represent the knowledge resp. plausible beliefs of agents, with $Bel(\Psi)$ denoting the set of most plausible (propositional) beliefs. In (Darwiche and Pearl 1997), the original AGM postulates were modified to handle revision of epistemic states Psi instead of belief sets K. Later on in (Delgrande and Jin 2012), these epistemic AGM postulates were adapted to deal with multiple revision, i.e., with revision by sets of propositions $\mathcal{S}$. We recall the epistemic AGM postulates as phrased by (Delgrande and Jin 2012):

Let Ψ be an epistemic state with associated belief set $Bel(\Psi)$, and let $\mathcal{S} \subset \mathcal{L}$ be a set of propositional formulas.

($\Psi 1$) $Cn(Bel(\Psi * \mathcal{S})) = Bel(\Psi * \mathcal{S})$

($\Psi 2$) $\mathcal{S} \subseteq Bel(\Psi * \mathcal{S})$ (Success)

($\Psi 3$) $Bel(\Psi * \mathcal{S}) \subseteq Cn(Bel(\Psi) \cup \mathcal{S})$

($\Psi 4$) If $Bel(\Psi) \cup \mathcal{S}$ is consistent, then $Cn(Bel(\Psi) \cup \mathcal{S}) \subseteq Bel(\Psi * \mathcal{S})$

($\Psi 5$) $Bel(\Psi * \mathcal{S})$ is consistent iff $\mathcal{S}$ is consistent.

($\Psi 6$) If $\mathcal{S}_1 \equiv \mathcal{S}_2$, then $Bel(\Psi * \mathcal{S}_1) = Bel(\Psi * \mathcal{S}_2)$

($\Psi 7$) $Bel(\Psi * (\mathcal{S}_1 \cup \mathcal{S}_2)) \subseteq Cn(Bel(\Psi * \mathcal{S}_1) \cup \mathcal{S}_2)$

($\Psi 8$) If $Bel(\Psi * \mathcal{S}_1) \cup \mathcal{S}_2$ is consistent, then $Cn(Bel(\Psi * \mathcal{S}_1) \cup \mathcal{S}_2) \subseteq Bel(\Psi * (\mathcal{S}_1 \cup \mathcal{S}_2))$

Following (Katsuno and Mendelzon 1991) and (Darwiche and Pearl 1997), AGM revisions of epistemic states can be realized via *faithful assignments* that map each epistemic state Ψ to a total preorder (called *faithful ranking*) $\preceq_\Psi$ on possible worlds Ω in such a way that $\omega_1 \approx_\Psi \omega_2$ if $\omega_1, \omega_2 \models Bel(\Psi)$, and $\omega_1 \prec_\Psi \omega_2$ if $\omega_1 \models Bel(\Psi), \omega_2 \not\models Bel(\Psi)$, where $\approx_\Psi$ and $\prec_\Psi$ are defined in the ususal way from $\preceq_\Psi$ by $\omega_1 \approx_\Psi \omega_2$ if both $\omega_1 \preceq_\Psi \omega_2$ and $\omega_2 \preceq_\Psi \omega_1$ hold, and $\omega_1 \prec_\Psi \omega_2$ if $\omega_1 \preceq_\Psi \omega_2$ holds, but not $\omega_2 \preceq_\Psi \omega_1$. This representation theorem of (Darwiche and Pearl 1997) has also been adapted by (Delgrande and Jin 2012) to the case of revising by multiple propositions:

Proposition 1 ((Delgrande and Jin 2012)) *A revision operator* $*$ *satisfies* $(\Psi 1) - (\Psi 8)$ *iff there exists a faithful ranking* $\preceq_\Psi$ *for* Ψ*, such that for any set of sentences* $\mathcal{S}$ *it holds that* $Bel(\Psi * \mathcal{S}) = \mathcal{T}(\min(Mod(\mathcal{S}), \preceq_\Psi))$.

This proposition allows us to study epistemic AGM-style revisions by focussing on total preorders that turn out to be a suitable representation of epistemic states for the purpose of revision. By assigning ordinal resp. natural numbers to the strata of total preorders, we obtain ordinal conditional functions (OCFs) which have been found to be a particularly typical and convenient formal frame for studying belief revision. OCFs κ can be taken as representations of faithful rankings via $\omega_1 \leqslant_\kappa \omega_2$ iff $\kappa(\omega_1) \leqslant \kappa(\omega_2)$, and therefore as semi-quantitative representations of epistemic states. Here, we have $\mathit{Bel}(\kappa) = \mathcal{T}(\{\omega \in \Omega \mid \kappa(\omega) = 0\})$, i.e., the agent believes exactly the propositions that are valid in all most plausible models.

More postulates have been proposed to validate the suitability of revision operators. For iterated revision, Darwiche and Pearl (Darwiche and Pearl 1997) suggested the following four postulates:

(DP1) If $B \vdash A$, then $\mathit{Bel}((\Psi * A) * B) = \mathit{Bel}(\Psi * B)$

(DP2) If $B \vdash \neg A$, then $\mathit{Bel}((\Psi * A) * B) = \mathit{Bel}(\Psi * B)$

(DP3) If $A \in \mathit{Bel}(\Psi * B)$, then $A \in \mathit{Bel}((\Psi * A) * B)$

(DP4) If $\neg A \notin \mathit{Bel}(\Psi * B)$, then $\neg A \notin \mathit{Bel}((\Psi * A) * B)$

Except for (DP2), these postulates are widely accepted. For (DP2), there exist several examples which aim at illustrating seeming flaws of (DP2). One of these examples is the adder-and-multiplier example from the introduction. However, as we pointed out in the introduction, the problems in this example are caused rather by identifying the information a, b with $a \wedge b$, not by (DP2) in itself. This raised the necessity of considering multiple revisions more seriously. In (Delgrande and Jin 2012), several postulates for multiple iterated revision (called *parallel revision* there) were put forward. The main idea was to realize a kind of independence between the new pieces of information under revision and thus to generalize the *independence postulate* proposed in (Jin and Thielscher 2007):

(Ind) If $\neg A \notin \mathit{Bel}(\Psi * B)$, then $A \in \mathit{Bel}((\Psi * A) * B)$.

As an adaptation of (Ind) for the case of multiple revision, (Delgrande and Jin 2012) propose the postulate of *evidence retainment (Ret)*:

(Ret) If $A \in \mathcal{S}_1$, and for all $\mathcal{S}_c \subseteq \mathcal{S}_1$ which are consistent with $\mathcal{S}_2 (\neq \emptyset)$ we have $\neg A \notin \mathit{Bel}(\Psi * (\mathcal{S}_c \cup \mathcal{S}_2))$, then $A \in \mathit{Bel}((\Psi * \mathcal{S}_1) * \mathcal{S}_2)$.

On the semantical side, (Ret) is characterized by the following condition:

(Ret^{sem}) If $\mathcal{S}|\omega_2 \subset \mathcal{S}|\omega_1$, then $\omega_1 \preceq_\Psi \omega_2$ implies $\omega_1 \prec_{\Psi * \mathcal{S}} \omega_2$.

(Ret) (and (Ind) likewise) enforces beliefs A of the first revision set $\mathcal{S}_1$ in the iterated revision $(\Psi * \mathcal{S}_1) * \mathcal{S}_2$ if there are (basically) no conflicts with Ψ revised by the second revision set $\mathcal{S}_2$. To see this, consider the case $\mathit{Bel}(\Psi) \models \mathcal{S}_1$ and $\mathcal{S}_1 \cup \mathcal{S}_2$ being consistent, and assume additionally that for some $A \in \mathcal{S}_1$, we have $\neg A \notin \mathit{Bel}(\Psi * (\mathcal{S}_c \cup \mathcal{S}_2))$ for all $\mathcal{S}_c \subseteq \mathcal{S}_1$. Then from (Ret) we can conclude that $A \in \mathit{Bel}((\Psi * \mathcal{S}_1) * \mathcal{S}_2)$ which is strange because it either implies $\Psi * \mathcal{S}_1 \neq \Psi$ violating the minimal change paradigm (note that $\mathit{Bel}(\Psi) \models \mathcal{S}_1$), or (quite magically) makes A a belief in $\Psi * \mathcal{S}_2$. However, a weaker, more cautious form of retainment makes perfect sense as a general guideline for multiple iterated revisions without enforcement, this postulate is termed (PC4) in (Delgrande and Jin 2012) and is understood to generalize (DP4) for the multiple case:

(PC4) If for all $\mathcal{S}_c \subseteq \mathcal{S}_1$ that are consistent with $\mathcal{S}_2 (\neq \emptyset)$ we have $\neg A \notin \mathit{Bel}(\Psi * (\mathcal{S}_c \cup \mathcal{S}_2))$, then $\neg A \notin \mathit{Bel}((\Psi * \mathcal{S}_1) * \mathcal{S}_2)$.

According to (Delgrande and Jin 2012), (PC4) is semantically characterized by the following condition on worlds:

(PC4^{sem}) If $\mathcal{S}|\omega_2 \subseteq \mathcal{S}|\omega_1$, then $\omega_1 \preceq_\Psi \omega_2$ implies $\omega_1 \preceq_{\Psi * \mathcal{S}} \omega_2$.

A similar postulate that generalizes (DP3) to the multiple case is listed in (Delgrande and Jin 2012) as (PC3) which we recall together with its semantic characterization:

(PC3) If for every $\mathcal{S}_c \in \mathcal{S}_1 || \mathcal{S}_2$ where $\mathcal{S}_2 \neq \emptyset$ we have that $A \in \mathit{Bel}(\Psi * (\mathcal{S}_c \cup \mathcal{S}_2))$, then $A \in \mathit{Bel}((\Psi * \mathcal{S}_1) * \mathcal{S}_2)$

(PC3^{sem}) If $\mathcal{S}|\omega_2 \subseteq \mathcal{S}|\omega_1$, then $\omega_1 \prec_\Psi \omega_2$ implies $\omega_1 \prec_{\Psi * \mathcal{S}} \omega_2$

Finally, beside (Ret), (PC3), and (PC4), another postulate, (P) which is to implement *success preservation*, has been recommended in (Delgrande and Jin 2012) for multiple iterated revision operators:

(P) If $\mathcal{S}_1 \subset \mathcal{S}$ and $\mathcal{S}_1 \cup (\overline{\mathcal{S} \backslash \mathcal{S}_1}) \nvdash \bot$, then
$\mathcal{S}_1 \subseteq \mathit{Bel}((\Psi * \mathcal{S}) * (\overline{\mathcal{S} \backslash \mathcal{S}_1}))$.

In (Delgrande and Jin 2012), the authors apply their general approach to parallel belief revision to OCFs, proposing an OCF-based parallel belief revision. For the purpose of comparison with our approach, we recall their definition here.

Definition 1 (parallel OCF-revision) *For an OCF κ and a finite set $\mathcal{S}$ of satisfiable sentences, the* parallel OCF-revision of κ by $\mathcal{S}$*, denoted by $\kappa \otimes \mathcal{S}$, is defined inductively in the following way:*

1. *For $\omega \in \min_\kappa(\mathcal{S})$, we set $(\kappa \otimes \mathcal{S})(\omega) = 0$.*
2. *Assume that for $i > 0$, for all subsets $\mathcal{S}_1 \subseteq \mathcal{S}$ with $|\mathcal{S}| - |\mathcal{S}_1| < i$, and for all $\omega_1 \in \min_\kappa(C_{\mathcal{S}}(\mathcal{S}_1))$, $(\kappa \otimes \mathcal{S})(\omega_1)$ has been already set. Then, let $\mathcal{S}_2 \subset \mathcal{S}$ with $|\mathcal{S}| - |\mathcal{S}_2| = i$. For $\omega_2 \in \min_\kappa(C_{\mathcal{S}}(\mathcal{S}_2))$, we set*
   $(\kappa \otimes \mathcal{S})(\omega_2) = 1 + \max\{(\kappa \otimes \mathcal{S})(C_{\mathcal{S}}(\mathcal{S}'_1)),$
   $(\kappa \otimes \mathcal{S})(C_{\mathcal{S}}(\mathcal{S}'_1)) + \kappa(C_{\mathcal{S}}(\mathcal{S}_2)) - \kappa(C_{\mathcal{S}}(\mathcal{S}'_1))$
   $\mid \mathcal{S}_2 \subset \mathcal{S}'_1 \subseteq \mathcal{S}$ *and* $|\mathcal{S}_2| + 1 = |\mathcal{S}'_1|\}$
3. *For $\omega \notin \min_\kappa(C_{\mathcal{S}}(\mathcal{S}|\omega))$, we set $(\kappa \otimes \mathcal{S})(\omega) =$*
   $= (\kappa \otimes \mathcal{S})(C_{\mathcal{S}}(\mathcal{S}|\omega)) + \kappa(\omega) - \kappa(C_{\mathcal{S}}(\mathcal{S}|\omega))$.

In (Delgrande and Jin 2012) it was shown that parallel OCF-revision satisfies (P), (Ret), (PC3) and (PC4).

In this paper, we are going to show that (Ret) (and therefore also (Ind)) is not needed for handling multiple (iterated) revisions appropriately. Actually, these postulates should be understood as optional postulates that can be used if such an enforcement is intended, but not as crucial ingredients for (multiple) iterated revision. Nevertheless, we show that our approach is compatible with (PC3) and (PC4) while we also question (P) as being too strong.

4 C-revisions for OCFs by sets of sentences

In (Kern-Isberner 2001), revision operators for multiple iterated revision by sets of conditionals have been proposed. There, the principle of conditional preservation that had first been proposed by (Darwiche and Pearl 1997) as a crucial aspect for iterated revision was elaborated in full detail as an invariance property (see also (Kern-Isberner 2004)) and played a major role for the revision of OCFs. This principle brings forth a clear formal schema for multiple conditional revision of OCFs (called c-revisions) which can be adapted for the case of multiple propositional revision by identifying propositions with conditionals having a tautological antecedent: $A \equiv (A|\top)$ expresses that A is a plausible belief (as a part of the set $\mathcal{S}$) that is used to revise an OCF κ. For further details on the background of this theory, we refer to (Kern-Isberner 2001). For this paper, we present the adapted schema as our novel approach for multiple propositional revision of OCFs:

Let κ be an OCF, and $\mathcal{S} = \{A_1, \ldots, A_n\}$ be a finite, consistent set of propositional formulas. A *(propositional) c-revision $\kappa * \mathcal{S}$ of κ by $\mathcal{S}$* has the form

$$\kappa * \mathcal{S}(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\substack{i=1 \\ \omega \models \overline{A_i}}}^{n} \kappa_i^- \quad (1)$$

with natural numbers $\kappa_0, \kappa_1^-, \ldots \kappa_n^-$, where the κ_i's are *impact factors* associated with the pieces A_i of new information and κ_0 is a normalizing constant ensuring κ^* to be an OCF. The (Success) postulate $(\Psi 2)$ requires that the κ_i^- must be determined so as to satisfy

$$\kappa_i^- > \min_{\omega \models A_i}\{\kappa(\omega) + \sum_{\substack{j \neq i \\ \omega \models \overline{A_j}}} \kappa_j^-\} - \min_{\omega \models \overline{A_i}}\{\kappa(\omega) + \sum_{\substack{j \neq i \\ \omega \models \overline{A_j}}} \kappa_j^-\}. \quad (2)$$

Note that each κ_i implements an impact of A_i which is independent of the specific ω (i.e., the outcomes of the other A_j) while logical interdependencies among the A_i can be taken into account via (2). The following lemma allows us to simplify the schema (1) for propositional c-revisions further. First, it is possible to determine the normalizing constant κ_0 for c-revisions in general. Second, we might alleviate the handling of c-revisions by observing that the first minimum in (2) is constant for all $A_i \in \mathcal{S}$, depending only on the prior κ.

Lemma 1 *Let $\kappa^* = \kappa * \mathcal{S}$ be a propositional c-revision of κ by $\mathcal{S} = \{A_1, \ldots, A_n\}$ of the form (1) with parameters κ_i^- satisfying (2). Then $\kappa_0 = -\kappa(S) = -\kappa(A_1 \ldots A_n)$, and for any $A_i \in \mathcal{S}$, it holds that*

$$\min_{\omega \models A_i}\{\kappa(\omega) + \sum_{\substack{j \neq i \\ \omega \models \overline{A_j}}} \kappa_j^-\} = \kappa(A_1 \ldots A_n).$$

We summarize these results for a handy definition of c-revisions for multiple propositional belief change.

Definition 2 ((Propositional) C-revisions for OCFs) *Let κ be an OCF specifying a prior epistemic state, and let $\mathcal{S} = \{A_1, \ldots, A_n\}$ represent new information. Then a* (propositional) c-revision of κ by $\mathcal{S}$ *is given by*

$$\kappa * \mathcal{S}(\omega) = \kappa^*(\omega) = -\kappa(A_1 \ldots A_n) + \kappa(\omega) + \sum_{\substack{i=1 \\ \omega \models \overline{A_i}}}^{n} \kappa_i^- \quad (3)$$

with non-negative integers κ_i^- satisfying

$$\kappa_i^- > \kappa(A_1 \ldots A_n) - \min_{\omega \models \overline{A_i}}\{\kappa(\omega) + \sum_{\substack{j \neq i \\ \omega \models \overline{A_j}}} \kappa_j^-\}. \quad (4)$$

Each c-revision κ^* of κ by $\mathcal{S}$ is characterized by the vector $(\kappa_1^-, \ldots, \kappa_n^-)$ of non-negative integers satisfying (4), we indicate this briefly by writing $\kappa^* \sim (\kappa_1^-, \ldots, \kappa_n^-)$, which implies (3) and (4) to hold.

We will omit the attribute "propositional" in the following and speak of c-revisions of OCFs by sets of sentences. Note that the attribute "multiple" has been left out right from the beginning, since c-revisions can handle multiple pieces of information in general.

Since (3) and (4) provide a general schema for revision operators, many c-revisions are possible. However, one might impose further constraints on the parameters κ_i^-. One option is to take the minimal κ_i^- satisfiying (4) ensuring that the resulting OCF values are as least implausible as possible.

Definition 3 (Minimal c-revisions) *A* minimal c-revision of κ by $\mathcal{S}$ *is a c-revision $\kappa^* \sim (\kappa_1^-, \ldots, \kappa_n^-)$ such that the vector $(\kappa_1^-, \ldots, \kappa_n^-)$ is pareto-minimal, i.e., no other c-revision $\kappa' \sim (\kappa'^-_1, \ldots, \kappa'^-_n)$ of κ by $\mathcal{S}$ exists with $\kappa'^-_i \leqslant \kappa_i^-$ for all i, and $\kappa'^-_i < \kappa_i^-$ for at least one i, $1 \leqslant i \leqslant n$.*

For the case $n = 1$, i.e., $\mathcal{S} = \{A\}$, it is straightforward to compute that the minimal c-revision is uniquely determined by $\kappa * A = \kappa$ if $\kappa \models A$, and by

$$\kappa * A(\omega) = \begin{cases} \kappa(\omega) - \kappa(A) & \text{if } \omega \models A, \\ \kappa(\omega) + 1 & \text{if } \omega \models \overline{A}. \end{cases} \quad (5)$$

if $\kappa \not\models A$. So, in the case $n = 1$, the minimal c-revision nearly coincides with the $\bullet$-revision operator of (Darwiche and Pearl 1997) except for leaving κ unchanged if already $\kappa \models A$. In particular, any c-revision $\kappa * A$ fulfills all (DP) postulates from (Darwiche and Pearl 1997).

Now, we turn to the genuine multiple case and illustrate the approach of propositional multiple c-revisions by a simple, but typical example. In particular, we show how it differs from AGM revision by the propositional content.

Example 2 *We consider a vocabulary $\Sigma = \{A_1 = a, A_2 = b\}$ and a prior OCF κ, as given in the second column of Table 1. In particular, $\kappa \models \overline{a}\overline{b}$. Now, we revise κ by $\mathcal{S} = \{a, b\}$ according to (3) and (4). The propositional content of $\mathcal{S}$ is $S = ab$. We have $\kappa \models \overline{a}\overline{b}$ and $\kappa(ab) = 4$, so we obtain $\kappa * \{a, b\}(\omega) = -4 + \kappa(\omega) + \sum_{\substack{i=1 \\ \omega \models \overline{A_i}}}^{2} \kappa_i^-$ with $\kappa_1^- > 4 - \min\{1, \kappa_2^-\}$ and $\kappa_2^- > 4 - \min\{1, \kappa_1^-\}$, see the third column in Table 1. The pareto-minimal parameters κ_1^-, κ_2^- satisfying this system of inequalities are $\kappa_1^- = 4 = \kappa_2^-$, so*

ω	$\kappa(\omega)$	$\kappa * \mathcal{S}(\omega)$	$(\kappa * \mathcal{S})_{min}(\omega)$	$\kappa * ab(\omega)$	$(\kappa * ab)_{min}(\omega)$
ab	4	$-4+\kappa(\omega)$	0	$-4+\kappa(\omega)$	0
$a\overline{b}$	1	$-4+\kappa(\omega)+\kappa_2^-$	1	$-4+\kappa(\omega)+\kappa^-$	2
$\overline{a}b$	1	$-4+\kappa(\omega)+\kappa_1^-$	1	$-4+\kappa(\omega)+\kappa^-$	2
$\overline{a}\overline{b}$	0	$-4+\kappa(\omega)+\kappa_1^-+\kappa_2^-$	4	$-4+\kappa(\omega)+\kappa^-$	1

Table 1: Prior κ, both generic $\kappa * \mathcal{S}$ and $\kappa * ab$, and both minimal $\kappa * \mathcal{S}$ and minimal $\kappa * ab$ for Example 2

we obtain the minimal c-revision $(\kappa * \{a, b\})_{min}$ as given in the fourth column of Table 1. For the c-revision of κ by ab (with corresponding κ^-), we obtain the generic and minimal c-revision (with minimal $\kappa^- = 4 + 1 = 5$) as given in the fifth and sixth columns of Table 1.

Not only the outcomes of the minimal solutions to these revision problems are substantially different (fourth and sixth columns of Table 1), but also the employed strategies of change are structurally different (third and fifth columns of Table 1). Nevertheless, the revision strategies share the common idea to shift falsifying worlds uniformly, but multiple c-revisions consider each propositional input in $\mathcal{S}$ as structurally independent from the other propositions in $\mathcal{S}$. It is important to emphasize that this independence is realized on a structural level, i.e., each κ_i^- is a uniform and independent component of the sum making up the revised OCF, but logical dependencies between the input propositions in $\mathcal{S}$ result in numerical influences among the κ_i^-. Note that in this example, the system of inequalities determining the κ_i^- is completely symmetric in κ_1^- and κ_2^-, reflecting the symmetry both of the revision task and the prior information with respect to a and b.

Even though c-revisions clearly differ from propositional AGM revisions by the propositional content as example 2 shows, they comply with the postulates for multiple epistemic AGM revision:

Proposition 2 *C-revision operators satisfy the epistemic AGM-Postulates $(\Psi 1) - (\Psi 8)$.*

It is straightforward to show this proposition by making use of (3) and (4), and Lemma 1. Similarly, it can be proved easily that c-revisions comply with generalizations of the ideas of (Darwiche and Pearl 1997) in the form of the postulates (PC3) and (PC4) from (Delgrande and Jin 2012):

Proposition 3 *C-revisions satisfy (PC3) and (PC4).*

Summarizing so far, c-revisions satisfy major postulates that have been proposed for multiple iterated belief revision. Now, we investigate the relationships between c-revisions and the independence postulates (Ind) and (Ret) that have been proposed as being crucial for multiple revision. Since (Ret) generalizes (Ind), it is enough to consider (Ret). The following proposition shows that c-revisions (even minimal c-revisions) *do not satisfy (Ret).*

Proposition 4 *C-revisions do not satisfy (Ret).*

The proof of this proposition is given by the following counterexample:

Example 3 *We consider a vocabulary $\Sigma = \{a, b\}$, the sets $S_1 = \{A_1^1 = a, A_2^1 = ab\}$ and $S_2 = \{a\overline{b} \vee \overline{a}b\}$ plus the uniform OCF κ_u with $\kappa_u(\omega) = 0$ for every possible world ω. Hence, we get $S_1||S_2 = \{\{a\}, \emptyset\}$ and therefore $S_c^1 = \{a\}$ and $S_c^2 = \emptyset$ as the only elements of $S_1||S_2$. Now, we revise κ_u by $S_c^1 \cup S_2 = \{A_1 = a, A_2 = a\overline{b} \vee \overline{a}b\}$ and by $S_c^2 \cup S_2 = \{A_2 = a\overline{b} \vee \overline{a}b\}$ according to (3) and (4). Since $\kappa_u(\omega) = 0$ for every possible ω, we obtain $\kappa_u * \{a, a\overline{b} \vee \overline{a}b\}(\omega) = \sum_{i=1,\, \omega \models \overline{A_i}}^{2} \kappa_i'^-$ with (non-negative) parameters $\kappa_1'^- > 0$ and $\kappa_2'^- > 0$, and $\kappa_u * \{a\overline{b} \vee \overline{a}b\}(\omega) = \sum_{i=2,\, \omega \models \overline{A_i}}^{2} \kappa_i''^-$ with a (non-negative) parameter $\kappa_2''^- > 0$. Choosing pareto-minimal parameters $\kappa_1'^- = \kappa_2'^- = \kappa_2''^- = 1$ yields $Bel((\kappa_u * (S_c^1 \cup S_2))_{\min}) = Cn(\{a\overline{b}\})$ and $Bel((\kappa_u * (S_c^2 \cup S_2))_{\min}) = Cn(\{a\overline{b} \vee \overline{a}b\})$. Thereby, we obtain $\neg a \notin Bel((\kappa_u * (S_c \cup S_2))_{\min})$ for every $S_c \in S_1||S_2$. Accordingly, the premise of (Ret) is satisfied for $\alpha = a$. However, contrary to the assertion of (Ret), $a \notin Bel(((\kappa_u * S_1)_{\min} * S_2)_{\min})$: analogously to the previous revision, we obtain $\kappa_u * S_1(\omega) = \sum_{i=1,\, \omega \models \overline{A_i^1}}^{2} \kappa_i^-$ with parameters $\kappa_1^- > -\kappa_2^-$ and $\kappa_2^- > 0$. The pareto-minimal parameters κ_1^-, κ_2^- satisfying this system of inequalities are $\kappa_1^- = 0$ and $\kappa_2^- = 1$. Since we have $(\kappa_u * S_1)_{\min}(a\overline{b} \vee \overline{a}b) = 1$, we obtain $(\kappa_u * S_1) * S_2(\omega) = -1 + \kappa_u * S_1(\omega) + \sum_{\omega \models ab \vee \overline{a}\overline{b}} \kappa^-$ with a parameter $\kappa^- > 1 - \min_{\omega \models ab \vee \overline{a}\overline{b}} \{\kappa_u * S_1(\omega)\} = 1$. The pareto-minimal parameter κ^- satisfying this inequality is $\kappa^- = 2$. We find that $Bel(((\kappa_u * S_1)_{\min} * S_2)_{\min}) = Cn(\{a\overline{b} \vee \overline{a}b\})$. Contrary to (Ret), $\alpha = a \notin Bel(((\kappa_u * S_1)_{\min} * S_2)_{\min})$.*

This shows that c-revisions realize an independent processing of multiple pieces of information under revision without having to obey (Ret).

Also postulate (P) which is recommended for multiple revision in (Delgrande and Jin 2012), too, is not satisfied by c-revisions. We will illustrate this by a meaningful example to show that (P) should not be adopted blindly.

Example 4 *Suppose we consider a murder case. Pieces of evidence on the murderer are provided by three (more or less independent, but in any case not completely reliable) witnesses claiming that (s)he is tall (t), wears a Rolex (r), and wears very expensive shoes (s). In the prior epistemic state κ of the police inspector, as shown in the table below, also the atom w (being wealthy) is considered. Note that $\dot{t}$ stands for any one of t or $\overline{t}$, so κ is quite indifferent with respect to t, i.e., possible worlds are assigned the same rank* *not regarding whether t is true or false. This models a reasonable kind of semantic independence between t and any combination or w, r, or s, whereas a plausible relationship between the variables w, r and s can be found in κ expressing that wealthy people tend to wear Rolex watches and very expensive shoes, in fact, we have $\kappa \models (rs|w)$.*

ω	$\kappa(\omega)$	ω	$\kappa(\omega)$	ω	$\kappa(\omega)$	ω	$\kappa(\omega)$
$\dot{t}rsw$	1	$\dot{t}\overline{r}sw$	2	$\dot{t}r\overline{s}w$	2	$\dot{t}\overline{r}\,\overline{s}w$	4
$\dot{t}rs\overline{w}$	5	$\dot{t}\overline{r}s\overline{w}$	3	$\dot{t}r\overline{s}\,\overline{w}$	3	$\dot{t}\overline{r}\,\overline{s}\,\overline{w}$	0

A minimal c-revision $\kappa * \mathcal{S}$ with $\mathcal{S} = \{t, r, s\}$ can be computed from (3) and (4) with $\kappa_1^- = \kappa_2^- = \kappa_3^- = 1$ (we omit the details here). Then, observation s turns out to be false, the third witness realized that the expensively looking brand label on the shoes was just a fake, and so we c-revise $\kappa * \mathcal{S}$ by $\{\overline{s}\}$, obtaining $\kappa_1^* = (\kappa * \mathcal{S}) * \{\overline{s}\}$ according to (5). In this situation, postulate (P) could be applied with $\mathcal{S}_1 = \{t, r\}, \overline{\mathcal{S}\backslash\mathcal{S}_1} = \{\overline{s}\}$, claiming that $(\kappa * \mathcal{S}) * \overline{\mathcal{S}\backslash\mathcal{S}_1} \models \mathcal{S}_1$ should hold; in particular, according to (P), one should still believe that the murderer wore a Rolex. However, using iterated c-revision in the indicated way, we find that $(\kappa * \mathcal{S}) * \{\overline{s}\} \models t$ ($\kappa_1^*(\overline{t}) = 1$), but $(\kappa * \mathcal{S}) * \{\overline{s}\} \not\models r$ – just to the contrary, we obtain $\kappa_1^*(\overline{r}) = 0$ and $\kappa_1^*(r) = 1$, exactly as for the prior κ. We argue that this is reasonable since coming to know that s is false casts doubt on w due to the background knowledge $\kappa \models (rs|w)$ which leads to giving up r also in $(\kappa * \mathcal{S}) * \{\overline{s}\}$ – given that the shoes were a fake, the watch might also be a fake. (P) does not allow to take such background knowledge from the prior epistemic state into account.

So, c-revisions do not generally satisfy (P). Again, we raise the point that (P) might be too strong for multiple iterated revision since it ignores semantical relationships that are encoded in the prior epistemic state. Finally, we compare c-revisions with the parallel revision approach for OCF from (Delgrande and Jin 2012). Already the simple example 2 allows an informative comparison here.

Example 5 *We continue Example 2 and apply parallel OCF-revision (cf. Definition 1) to this belief revision problem. In the following table, the result is compared to minimal c-revision.*

ω	$\kappa(\omega)$	$(\kappa * \mathcal{S})_{min}(\omega)$	$(\kappa \otimes \mathcal{S})(\omega)$
ab	4	0	0
$a\overline{b}$	1	1	1
$\overline{a}b$	1	1	1
$\overline{a}\overline{b}$	0	4	2

Whereas the inductive definition of parallel belief revision only ensures that the difference between $\overline{a}\overline{b}$ and $a\overline{b}$ resp. $\overline{a}b$ is noticeable, the c-revision approach demands this difference to be uniquely determined by the impact factors of the formulas in $\mathcal{S}$, not by the specific change context which is given by the particular ω.

This example shows that although the parallel revision operator of (Delgrande and Jin 2012) was designed to satisfy the strong (Ret) postulate, the structures that it impose on the revised epistemic state are weaker than the structures resulting from c-revisions.

5 Conclusion and future work

Multiple iterated belief revision addresses the problem of revising expistemic states by sets of sentences and extends the classical AGM theory by two dimensions: First, several pieces of new information are considered jointly but also each piece on its own, and second, epistemic structures (typically, total preorders) are revised instead of just dealing with belief sets. This paper presents an approach to multiple iterated belief revision (called c-revisions) that make use of Spohn's ranking functions as a prototypical environment for investigating belief change and is based on a concise and elegant schema for setting up the posterior ranking function (following the ideas of (Kern-Isberner 2004)). We showed that our approach satisfies all major postulates from the literature without having to obey the independence postulates (Ind) (Jin and Thielscher 2007) and (Ret) (Delgrande and Jin 2012). In this way, we clarified the view on the field of multiple iterated revision by distinguishing between the major line, following (Alchourrón, Gärdenfors, and Makinson 1985) and (Darwiche and Pearl 1997)), and optional extensions, like (Ind) and (Ret). As part of our ongoing and future work, we elaborate in more depth on the relationships between the principle of conditional preservation (Darwiche and Pearl 1997; Kern-Isberner 2001) and the principles having been proposed for multiple iterated revision.

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