Proceedings of the Twenty-Eighth International Florida Artificial Intelligence Research Society Conference

# Stability in Role Based Hedonic Games

**Matthew Spradling** and **Judy Goldsmith**
University of Kentucky
mjspra2@uky.edu and goldsmit@cs.uky.edu

### Abstract

In the hedonic coalition formation game model *Roles and Teams Hedonic Games (RTHG)* (Spradling et al. 2013), agents view teams as compositions of available roles. An agent's utility for a partition is based upon which role she fulfills within the coalition and which additional roles are being fulfilled within the coalition. Goals for matchmaking in this setting include forming partitions which optimize some function of the utility. Optimization problems related to finding Perfect, MaxSum and MaxMin partitions in RTHG are all known to be NP-hard. In this paper, we introduce a *Role Based Hedonic Game* model (RBHG) which has no fixed team size and a more relaxed set of compositions. We consider the related problem of stability in RBHG. Given a set of available movements for agents, a partition is stable iff no agent would choose to move from the partition to another partition. We show NP-completeness for several RBHG stability problems and coNP-completeness for two verification problems.

## Team Formation in MMOs

In massively multiplayer online games (MMOs), social interaction and team formation is one of the top areas of modern research (Achterbosch, Pierce, and Simmons 2008). Major hits of this genre includes massively multiplayer online roleplaying games (MMORPGs) such as *World of Warcraft* and multiplayer online battle arenas (MOBAs) such as *League of Legends* and *DoTA 2*. Recent work has studied the impact of perceived gender in MMO social interactions (Stabile 2014), the relation between players' motivating passions and how they interact in MMO communities (Fuster et al. 2014), and decision support systems for detection of disruptive players (Shim, Kim, and Kim 2014), among others.

When forming teams in a MOBA or MMORPG, players are interested in finding team assignments in which their skills and goals are complimentary. This would allow the players the best opportunity to achieve their goals together, enhancing enjoyment of the game and profitability for the developers.

In this work, we ask only "which partition does an agent prefer?" and not "why does an agent prefer a partition?" This preference information is represented by integer utilities.



When a central authority is tasked with assigning players to competitive teams, three important goals arise. First, the teams need to be formed quickly. Second, the teams should be of a good overall quality to ensure some optimal social welfare function is satisfied. Third, the players should individually accept their team assignments.

Because a player's utility only depends on their own team assignment, MOBA and MMORPG team formation are examples of hedonic coalition formation games. Due to the large population size in a popular MOBA, and the fact that skill sets are generally represented by the "roles" which players fulfill on a team, this team formation problem can be modeled as a *Roles and Teams Hedonic Game (RTHG)* (Spradling et al. 2013). In this model, agents represent their utilities for partitions based only upon the roles they fulfill on their own teams and the roles fulfilled by their teammates.

Research on coalition formation games has looked at stability and optimization on a variety of criteria, such as achieving MaxSum or MaxMin utility. A MaxSum partition is one in which the sum total utility is maximized. A MaxMin partition is one in which the utility of lowest-utility coalition is maximized. Finding an optimal partition under either criterion is NP-hard in RTHG (Spradling et al. 2013).

In this work, we define *Role Based Hedonic Games* (RBHG). This model relaxes the RTHG requirements for a fixed team size and consideration of all possible compositions. We consider the problem of finding and verifying *stable* assignments in RBHG. In the context of RBHG, we define *individually rational*, *Nash stable*, *individually stable*, *core stable*, *strict core stable*, *contractual strict core stable*, *envy-free* and *Pareto optimal* partitions. We compare these stability problems to the related stability problems in *Additively Separable Hedonic Games (ASHG)*. We prove that verification of *Pareto optimality* and *contractual strict core stability* are both coNP-complete, and the remaining stability problems are all NP-complete.

## Stable Teams in MMOs

Because computing a MaxSum partition in RTHG is NP-hard, heuristics approximation algorithms have been proposed (Spradling et al. 2013). Though a central authority might wish to maximize total utility for a partition, this goal relies upon the the agents' acceptance of the assignment. When agents have autonomy and hedonic utilities, they can and will choose to make local movements for improvement. A partition from the central authority should make such changes unnecessary.

Finding an optimal partition is not a sufficient or worthwhile goal if agents don't stick to the plan. In an MMO, players are unlikely to know or care about the global utility of a partition. They will be more interested in changing the assignment to improve their own utilities. The players are not a captive audience. Should players not find partitions to their liking, they may switch to other games or even *read books* in the worst case. The MMO industry is highly competitive and the players can be quite fickle.

In order to improve acceptance of partitions by the population, finding *stable* partitions needs to be the focus. While we expect agents to make movements which gravitate towards a stable partition, it is worthwhile to partition them such that these changes are easy or unnecessary. If a player is consistently matched to a team which they find unacceptable, the player may quit the game altogether rather than have to improve every assignment offered.

## Related Work: Stability in Hedonic Games

Hedonic games are used to model situations in which agents need to be matched to teams and only care about the utility of their own teams, not others. The original motivation for this model stems from an economic setting where agents join teams to produce goods for in-team consumption (Drèze and Greenberg 1980). The goal for an agent in this setting is to be matched to the most self-fulfilling team.

A hedonic game consists of a set of agents and a preference profile of the agents and the set of possible partitions, where a partition is a set of teams (subsets of agents). The game is hedonic in that an agent's preference for a partition is determined only by the coalition to which the agent is assigned (Sung and Dimitrov 2010). It has been shown that, for general hedonic games, finding a core stable coalition structure is NP-complete (Ballester 2004). This does not exclude the possibility of finding such solutions quickly in some special cases of hedonic games.

Ballester showed that, for hedonic games which can be represented by an individually rational list of coalitions, checking the existence of Nash stable, individually stable or core stable partitions is NP-complete (Ballester 2004). These problems, in addition to the problem of finding a strict core stable partition, remain NP-complete for hedonic games with additively separable preferences (Sung and Dimitrov 2010). Additional hedonic game variants include anonymous hedonic games (Banerjee, Konishi, and Sönmez 2001) and the Group Activity Selection Problem (GASP) (Darmann et al. 2012), among others (Hajduková 2006).

Additively Separable Hedonic Games (ASHG) (Bogomolnaia and Jackson 2002; Aziz, Brandt, and Seedig 2011) model situations where agents have additively separable utilities for each other that may or may not be symmetric. Many stability problems in ASHG have been shown to be NP-complete or coNP-complete. We outline several such problems later in the paper.

The ASHG model is useful if it can be assumed that all agents have had an opportunity to evaluate one another individually. When we match such agents into pairs, the problem of finding a stable assignment reduces to the stable roommates problem (Irving 1985; Irving and Manlove 2002), for which fast algorithms exist.

In an MMO, communications network (Saad et al. 2010), or vehicular network (Saad et al. 2011b), the population may be relatively large and anonymous. It may be too much to assume that agents will all have had an opportunity to evaluate one another. Furthermore, in these settings we often need to form teams of varying sizes.

In an MMO where players select avatars having different skills and abilities, success of a team can depend on which roles are fulfilled by the subset of avatars selected. While players are largely anonymous, preferences in terms of roles performed by teammates to achieve some objective can be described. This adds to what it means to be an agent. Rather than having agents who are immutable objects, a single agent can be "worth" different amounts depending upon what she is doing for her team. This task-dependent agent valuation also has application with distributed task allocation in wireless agents (Saad et al. 2011a).

When partitioning and RBHG instance, each agent is assigned to a role and has preferences over which role to select for itself given a team composition.

## ASHG and RBHG Definitions

In this section we define two hedonic game models, ASHG and RBHG. While we consider RBHG to be a better model for MMO matchmaking, little was known about the problem of finding stable assignments in RBHG. We leverage a relationship with ASHG, for which the complexities of many stability problems were already known.

### Additively Separable Hedonic Games

An *Additively Separable Hedonic Game* (ASHG) (Bogomolnaia and Jackson 2002; Aziz, Brandt, and Seedig 2011) instance consists of:

- $N$: A population of players
- $V$: A utility function vector $V$ over all pairs of agents in $N$, where $v_i(n_j) \in V$ is an integer representing the utility player $n_i$ has for having player $n_j$ on its team. We assume that $v_i(n_i) = 0$ for all $n_i \in N$.

A solution to an ASHG instance is a partition $\pi$ composed of a set of teams $T$, where each team $t \in T$ is a set of agents $t \subseteq N$ and $\pi(n_i) = t^i$ is the single team to which an agent $n_i$ is matched. Because ASHG is a hedonic game, utility of a player for a partition $\pi$ is equal to that player's utility for its team $\pi(n_i) = t^i$. Let $v_i(t) = \sum_{j \in t} v_i(j)$ be the *utility* of player $n_i$ for team $t$. A player $n_i$'s utility for $\pi$ is $v_i(t^i)$.

Because some researchers (Bogomolnaia and Jackson 2002; Ballester 2004; Gairing and Savani 2010), have dealt with special cases of ASHGs, much of the recent research refers to the definition just given as *general ASHGs*.

### Role Based Hedonic Games

A *Role Based Hedonic Game* (RBHG) instance consists of:

- $P$: A population of players
- $R$: A set of roles
- $C$: A set of compositions, where a composition $c \in C$ is a multiset (bag) of roles from $R$.
- $U : P \times R \times C \to \mathbb{Z}$ defines the utility function $u_i(r, c)$ for each player $p_i$. We assume that for all $p_i \in P$ and for all $r \in R$, $u_i(r, \{r\}) = 0$.

The *Roles and Teams Hedonic Game* (RTHG) model assumes a fixed team size $m$ which all teams share and that the set of compositions $C$ includes all possible multisets of $R$ (Spradling et al. 2013). In RBHG, there is no fixed team size and the set $C$ need not include all compositions.

We reason that these generalizations are appropriate. Observe that certain compositions may be considered universally unacceptable either by the population (a MOBA team of all healers which has little chance of winning) or a central authority (a military commander forming a sniper team with no snipers). Team size may not have reason to be fixed in real-world scenarios. Even in a MOBA, where team size is usually fixed for each team, some players may join a queue as a preformed "buddy group" needing only a few additional players. A central authority could leverage the RBHG model to find smaller sub-teams to complete these groups.

A solution to a RBHG instance is a partition $\pi$. For each player $p_i \in P$, $\pi(p_i) = (r^i, c^i, t^i)$, where $p_i$ is assigned to both a role $r^i$ and team of players $t^i$ and $c^i$ is the bag of roles to which players in $t^i$ are assigned.

Because RBHG is a hedonic game, utility of a player for a partition $\pi$ is equal to that player's utility for $\pi(p_i) = (r^i, c^i, t^i)$. For simplicity, we will refer to a player's $p_i$ utility for $\pi$ as $u_i(r^i, c^i)$.

## Stability Definitions

Different notions of stability depend on different constraints on agent movements. In this section, we define what is meant by "stability" in hedonic games, what it means for an agent to move, and provide definitions for several stability problems in RBHG. We compare these to the related NP-complete and coNP-complete ASHG stability problems.

### Forms of Movement

A partition for an RBHG instance is stable when agents having no incentive, or perceived improvement of utility, for changing their assigned role and team. Changing involves some sort of *movement* by the agents. In this subsection, we discuss the general notion of agents' movements.

Any movement begins from a partition $\pi$ and results in a new partition $\pi'$. This change may be made by individual agents or as joint movements by a group of agents made in unison. When we say that a partition $\pi$ of a hedonic coalition formation game is *stable* we mean that, given a set of possible movements, no agents would improve utility by taking such movements.

When we say that a player $i$ *moves from* a team $t$ to a team $t'$, we mean that the partition $\pi$ containing $t$ and $t'$ is modified such that $t := t - \{i\}$ and $t' := t' \cup \{i\}$. This creates a new partition $\pi'$ in which $i$ is a member of $t'$ and not a member of $t$. When we say that a team *breaks off* from a partition we mean that these agents *move from* their current teams in the partition and form a new team $t'$ together. When the group of agents breaks off, this creates a new partition $\pi'$ which includes the team $t'$.

**Individually Rational** A partition $\pi$ is *individually rational* (IR) iff no player can benefit by moving from its team $t^i$ to a team by itself.

In ASHG, $\pi$ is individually rational iff all players $n_i \in N$ have utility $v_i(t^i) \geq 0$. In RBHG, $\pi$ is individually rational iff all players $p_i \in P$ have utility $u_i(r^i, c^i) \geq 0$.

**Nash Stable** A partition $\pi$ is *Nash stable* (NS) iff no player $p_i \in P$ can benefit by moving from its team $t^i$ to another (possibly empty) team $t'$.

In ASHG, $\pi$ is Nash stable iff $\pi$ is individually rational and it holds that for all $n_i \in N$, for all $t' \in \pi$, $v_i(t^i) \geq v_i(t' \cup \{n_i\})$. In RBHG, $\pi$ is Nash stable iff $\pi$ is individually rational and it holds that for all $p_i \in P$, for each $t' \in \pi$ having a composition $c'$, for all $r' \in R$, $u_i(r^i, c^i) \geq u_i(r', c' \cup \{r'\})$.

**Observation 1** *(Sung and Dimitrov 2010) Checking whether an* NS *partition exists in an ASHG is NP-hard.*

**Individually Stable** A partition $\pi$ is *individually stable* (IS) iff no player $p_i \in P$ can benefit by moving from its team $t^i$ to another (possibly empty) team $t'$ while not making members of $t'$ worse off.

In ASHG, $\pi$ is individually stable iff $\pi$ is individually rational and it holds that for all $n_i \in N$, for all $t^j \in \pi$, if $v_i(t^i) < v_i(t^j \cup \{n_i\})$ then $v_j(t^j) > v_j(t^j \cup \{n_i\})$ for some $j \in t^j$. In RBHG, $\pi$ is individually stable iff $\pi$ is individually rational and it holds that for all $p_i \in P$, for all $t^j \in \pi$, for all $r' \in R$, if $u_i(r^i, c^i) < u_i(r', c^j \cup \{r'\})$ then $u_j(r^j, c^j) > u_j(r^j, c^j \cup \{r'\})$ for some $j \in t^j$.

**Observation 2** *(Sung and Dimitrov 2010) Checking whether an* IS *partition exists in an ASHG is NP-hard.*

**Core Stable** A team $t'$ *blocks* a partition $\pi$ if each player $i \in t'$ has greater utility for $t'$ than its current team $t^i \in \pi$. A partition $\pi$ which admits no blocking coalition is said to be *in the core* or *core stable* (CS). If the core is *empty*, this means that there are no *core stable* partitions.

In ASHG, a team $t'$ blocks a partition $\pi$ iff there is a set $N' \subseteq N$ where $t'$ is a team consisting of all agents in $N'$ and $v_i(t') > v_i(t^i)$ for all $n_i \in N'$. In RBHG, a team $t'$ having a composition $c'$ blocks a partition $\pi$ iff there is a set $P' \subseteq P$ and an assignment of agents in $P'$ to the bag of roles $c$ such that $u_i(r^i, c) > u_i(r^i, c^i)$ for all $p_i \in P'$.

**Observation 3** *(Sung and Dimitrov 2010; Aziz, Brandt, and Seedig 2011) Checking whether a non-empty* CS *partition exists in an ASHG is NP-hard.*

**Strict Core Stable** A team $t'$ *weakly blocks* a partition $\pi$ if each player $i \in t'$ has greater or equal utility for $t'$ compared to its current team $t^i \in \pi$ and at least one player $j \in t'$ has greater utility for $t'$ than its current team $t^j \in \pi$. A partition $\pi$ which admits no weakly blocking coalition is said to be *in the strict core* or *strict core stable* (SCS). If the strict core is *empty*, this means that there are no *strict core stable* partitions.

In ASHG, a team $t'$ weakly blocks a partition $\pi$ iff there is a set $N' \subseteq N$ where $t'$ is a team consisting of all agents in $N'$, $v_i(t') \geq v_i(t^i)$ for all $n_i \in N'$, and $v_j(t') > v_j(t^j)$ for at least one $n_j \in N'$. In RBHG, a team $t'$ having a composition $c'$ weakly blocks a partition $\pi$ iff there is a set $P' \subseteq P$ and an assignment of agents in $P'$ to the bag of roles $c$ such that $u_i(r^i, c) \geq u_i(r^i, c^i)$ for all $p_i \in P'$ and $u_j(r^j, c) > u_j(r^j, c^j)$ for at least one $p_j \in P'$

**Observation 4** *(Sung and Dimitrov 2010; Aziz, Brandt, and Seedig 2011) Checking whether a non-empty strictly core stable partition exists in an ASHG is NP-hard.*

**Contractual Strict Core Stable** A partition $\pi$ is said to be *in the contractual strict core* or *contractual strict core stable* (CSCS) iff any *weakly blocking* team $t'$ makes at least one player $n_j \in N \backslash t'$ worse off when breaking off.

In ASHG, a player $n_j \in N$ is worse off when a *weakly blocking* team $t'$ breaks off if some agent $n_i \in t'$ was formerly on $t^j \in \pi$ and $v_j(t^j - \{n_i\}) < v_j(t^j)$. In RBHG, a player $n_j \in N$ is worse off when a *weakly blocking* team $t'$ breaks off if some agent $p_i \in t'$ was formerly on $t^j \in \pi$ in a role $r^i$ and $u_j(r^j, c^j - \{r^i\}) < u_j(r^j, c^j)$.

**Observation 5** *(Aziz, Brandt, and Seedig 2011) Verifying whether the partition is* contractual strict core stable *in ASHGs is coNP-complete.*

**Pareto Optimal** A partition $\pi$ is Pareto optimal (PO) iff there is no partition $\pi'$ such that each agent has utility greater than or equal to their utility for $\pi$ and at least one agent has greater utility for $\pi'$ than for $\pi$ (Aziz, Brandt, and Seedig 2011).

In ASHG, a partition $\pi$ is Pareto optimal iff there is no partition $\pi'$ such that $v_i(t'^i) \geq v_i(t^i)$ for all $n_i \in N$ and $v_j(t'^j) > v_j(t^j)$ for at least one $n_j \in N$. In RBHG, a partition $\pi$ is Pareto optimal iff there is no partition $\pi'$ such that $u_i(r'^i, c'^i) \geq u_i(r^i, c^i)$ for all $p_i \in P$ and $u_j(r'^j, c'^j) > u_j(r^j, c^j)$ for at least one $p_j \in P$.

**Observation 6** *(Aziz, Brandt, and Seedig 2010; 2011) Verifying whether a given partition $\pi$ is* Pareto optimal *in ASHGs is coNP-complete.*

**Envy Free** A partition $\pi$ is *envy free* (EF) iff no player has utility for her team that is less than her utility for another agent's team.

In ASHG, $\pi$ is EF iff no player $n_i \in N$ has utility $v_i(t^i) < \Sigma_{k \in t^j} v_i(k)$, for some player $j \in N$ on team $t^j \in \pi$. In RBHG, $\pi$ is EF iff no player $p_i \in P$ has utility $u_i(r^i, c^i) < u_i(r^j, c^j)$, for some player $p_j \in P$ on team $t^j \in \pi$ in role $r^j$.

**Observation 7** *(Aziz, Brandt, and Seedig 2010) Checking whether there exists a partition which is both envy free and Nash stable in ASHGs is NP-complete even if preferences are symmetric.*

## Stability Complexity in RBHG

**Definition 1** *The language* NS RBHG *consists of instances of RBHG for which there exists a Nash stable solution.*

**Theorem 1** NS RBHG *is NP-complete.*

**Proof 1** To see that NS RBHG is in NP, we observe that Nash stability for a single RBHG agent can be verified in time $O(|P| \cdot |R|)$. Verifying the property for all agents requires time $O(|P|^2 \cdot |R|)$.

Next, we construct a reduction, $f$, from NS ASHG to NS RBHG. Let $A$ be an instance of NS ASHG with a population $N$ and utility function vector $V$.

We define $f(A) = (P, R, C, U)$ to be an instance of RBHG. We set $|P| = |N|$, $R = \{r_1, ..., r_{|N|}\}$, and $C = \mathcal{P}(R)$. We set $U$ for all agents as follows:

$$u_i(r, c) = \begin{cases} \Sigma_{j \in c} v_i(j) & \text{if } r = r^i \\ & \text{and } r^i \in c \\ -\text{MaxAbsValue}(A) \cdot |P| - 1 & \text{o.w.} \end{cases}$$

**Observation 8** *The only partitions with positive values consist of coalitions where, for each $p_i \in P$, $p_i$ is assigned to role $r^i$. Therefore we limit $C$ to such coalitions.*

Let $f(A)$ be in NS RBHG and let $\pi$ be a Nash stable partition of $f(A)$. From $\pi$ we construct a partition $\pi'$ of $A$ using $f^{-1}$. Since $\pi$ maps each $p_i \in P$ to the role $r^i$ representing $f^{-1}(p_i) = n_i \in N$, we have that $f^{-1}(\pi) = \pi'$ is a well-defined partition of $A$. We claim that $\pi'$ is Nash stable.

Suppose there were an agent $n_i \in N$ and a team $t' \in \pi'$ such that $v_i(t' \cup \{n_i\}) > v_i(t^i)$. Then by the construction, there must be an agent $p_i \in P$ and a team $t' \in \pi$ having composition $c'$ such that $u_i(r^i, c' \cup \{r^i\}) > u_i(r^i, c^i)$. This contradicts the premise that $\pi$ is Nash stable. Therefore if $f(A)$ is in NS RBHG then $A$ is in NS ASHG.

Now let $\pi$ be a Nash stable partition of $A$. Let $\pi' = f(\pi)$ be the corresponding partition in $f(A)$. For each agent $p_i \in P$, $u_i(r^i, c^i) = v_i(t^i)$ where $t^i \in \pi$ is composed of the agents represented by the roles in $c^i$. By the same argument as in the previous case, we get that $\pi'$ is also Nash stable. Therefore if $A$ is in NS ASHG then $f(A)$ is in NS RBHG.

Therefore $f(A)$ is in NS RBHG iff $A$ is in NS ASHG. Thus, we have shown that $f$ is a reduction from NS ASHG to NS RBHG. □

**Definition 2** *The language* EF NS RBHG *consists of those instances of RBHG for which there exists an envy free Nash stable solution.*

**Theorem 2** EF NS RBHG *is NP-complete.*

**Proof Sketch 2** We use the same reduction as in Theorem 1, and show that the reduction preserves envy freeness and Nash stability of the partitions. □

**Definition 3** *The language* IS RBHG *consists of those instances of RBHG for which there exists an individually stable solution.*

**Theorem 3** IS RBHG *is NP-complete.*

**Proof Sketch 3** The reduction from Theorem 1 also preserves individual stability of partitions.
□

**Definition 4** *The language* CS RBHG *consists of those instances of RBHG for which there exists a non-empty core stable solution.*

**Theorem 4** CS RBHG *is NP-complete.*

**Proof 4** We will show that the reduction given in the proof of Theorem 1 is also a reduction from CS ASHG to CS RBHG.

Let $f(A)$ be in CS RBHG and let $\pi$ be a core stable partition of $f(A)$. From $\pi$ we construct a partition $\pi'$ of $A$ using $f^{-1}$. Since $\pi$ maps each $p_i \in P$ to the role $r^i$ representing $f^{-1}(p_i) = n_i \in N$, we have that $f^{-1}(\pi) = \pi'$ is a well-defined partition of $A$. We claim that $\pi'$ is core stable.

Suppose there were a subset of agents $N' \subset N$ such that, for each $n_i \in N'$, $v_i(N') > v_i(t^i)$. Then by the construction, there must be a subset of agents $P' \subset P$ such that, for each $p_i \in P'$, $u_i(r^i, c') > u_i(r^i, c^i)$ where $c' \subset R$ is composed of the roles represented by the agents in $P'$. This contradicts the premise that $\pi$ is core stable. Therefore if $f(A)$ is in CS RBHG then $A$ is in CS ASHG.

Now let $\pi$ be a core stable partition of $A$. Let $\pi' = f(\pi)$ be the corresponding partition of $f(A)$. For each agent $p_i \in P$, $u_i(r^i, c^i) = v_i(t^i)$ where $t^i \in \pi$ is composed of the agents represented by the roles in $c^i$. By the same argument as in the previous case, we get that $\pi'$ is also core stable. Therefore if $A$ is in CS ASHG then $f(A)$ is in CS RBHG.

Therefore $f(A)$ is in CS RBHG iff $A$ is in CS ASHG. Thus, we have shown that $f$ is a reduction from CS ASHG to CS RBHG. □

**Definition 5** *The language* SCS RBHG *consists of those instances of RBHG for which there exists a non-empty strict core stable solution.*

**Theorem 5** SCS RBHG *is NP-complete.*

**Proof Sketch 5** We use the same reduction as in Theorem 1, and show that the reduction preserves strict core stability of the partitions. □

**Definition 6** *The* grand coalition *for RBHG is an partition $\pi$ of all agents to a single team $t$. In RBHG, there are several possible grand coalitions with different distributions of roles.*

**Definition 7** *The language* GRAND PO RBHG *consists of those instances of RBHG for which there exists a partition consisting of the* grand coalition *and some assignment of agents to roles that is Pareto optimal.*

**Theorem 6** GRAND PO RBHG *is coNP-complete.*

**Proof 6** First we show that GRAND PO RBHG is in coNP. Given two partitions $\pi$ and $\pi'$ for an instance of RBHG, we can check in polynomial time if $\pi'$ is a partition such that $u_i(r'^i, c'^i) \geq u_i(r^i, c^i)$ for all $p_i \in P$ and $u_j(r'^j, c'^j) > u_j(r^j, c^j)$ for at least one $p_j \in P$. Thus, given an instance $f(A)$ of RBHG and a grand coalition $\pi$, it is NP to decide that $\pi$ is *not* Pareto optimal.

We will show that the reduction given in the proof of Theorem 1 is also a reduction from GRAND PO ASHG to GRAND PO RBHG.

Let $f(A)$ be in GRAND PO RBHG and let $\pi$ be a Pareto optimal partition of $f(A)$ consisting of the *grand coalition*. Observe that each agent in $p_i \in \pi$ must be assigned to $r^i \in R$, or else $\pi$ could not be Pareto optimal.

**Observation 9** *By the construction, each agent $p_i \in P$ has $u_i(r, c) = -MaxAbsValue(A) \cdot |P| - 1$ when $r \neq r^i$. Since $\pi$ is a Pareto optimal partition, each agent $p_i \in \pi$ must be assigned to $r^i \in R$. Otherwise the partition could be improved by assigning each agent $p_i$ to its role $r^i$.*

*Consider for example an instance $f(A)$ of RBHG where $P = \{p_1, p_2, p_3, p_4\}$. Denote the composition $c_g = \{r^1, r^2, r^3, r^4\}$. Let $u_1(r^1, c_g) = 1$, $u_2(r^2, c^g) = -2$, $u_3(r^3, c_g) = 0$ and $u_4(r^4, c^g) = 2$. Let $-MaxAbsValue(A) = -2$. Let $\pi_g$ be a partition consisting of the* grand coalition *with the composition $c_g$. Should any agent change roles to form a new partition $\pi'_g$, this will decrease the utility of each agent to $-MaxAbsValue(A) \cdot |P| - 1 = -9$.*

*Therefore if a partition $\pi$ consisting of the* grand coalition *is Pareto optimal, it can only be one in which each agent $p_i \in P$ is assigned to $r^i \in R$. This holds for any instance generated by the construction. We can guarantee that there is no $\pi'_g$ with a composition $c'_g \neq c_g$ which is Pareto optimal. However, $\pi_g$ is not guaranteed to be Pareto optimal.*

From $\pi$, a Pareto optimal partition of $f(A)$, we construct a partition $\pi'$ of $A$ using $f^{-1}$. Since $\pi$ maps each $p_i \in P$ to the role $r^i$ representing $f^{-1}(p_i) = n_i \in N$, we have that $f^{-1}(\pi) = \pi'$ is a well-defined partition of $A$. We claim that $\pi'$ is Pareto optimal.

Suppose there were a partition $\pi'_\delta$ of $A$ such that, for each $n_i \in N$ assigned to its team $t^i_\delta \in \pi'_\delta$, $v_i(t^i_\delta) \geq v_i(N)$ and for at least one $n_j \in N$ assigned to its team $t^j_\delta \in \pi_delta'$, $v_j(t^j_\delta) > v_i(N)$. Then by the construction, there must be a partition $f(\pi'_\delta)$ of $f(A)$ such that each agent $p_i$ is assigned to $r^i$ and, for each $p_i \in P$, $u_i(r^i, c^i_\delta) \geq u_i(r^i, c^i)$ and, for at least one $p_j \in P$, $u_j(r^j, c^j_\delta) > u_i(r^j, c^j)$. This contradicts the premise that $\pi$ is Pareto optimal. Therefore if $f(A)$ is in GRAND PO RBHG then $A$ is in GRAND PO ASHG.

Now let $\pi$ be a Pareto optimal partition of $A$. Let $\pi' = f(\pi)$ be the corresponding partition of $f(A)$. For each agent $p_i \in P$, $u_i(r^i, c^i) = v_i(t^i)$ where $t^i \in \pi$ is composed of agents represented by roles in $c^i$. By the same argument as in the previous case, we get that $\pi'$ is also Pareto optimal. Therefore if $A$ is in GRAND PO ASHG then $f(A)$ is in GRAND PO RTHG.

Therefore $f(A)$ is in GRAND PO RBHG iff $A$ is in GRAND PO ASHG. Thus, we have shown that $f$ is a reduction from GRAND PO ASHG to GRAND PO RBHG. □

**Definition 8** *The language* GRAND CSCS RBHG *consists of those instances of RBHG for which there exists a partition consisting of the* grand coalition *and some assignment of agents to roles which is contractual strict core stable.*

**Theorem 7** GRAND CSCS RBHG *is coNP-complete.*

**Proof Sketch 7** We use the same reduction as in Theorem 1, and show that the reduction preserves contractual strict core stability of the partitions. □

## Conclusions

In this paper, we defined the RBHG model and investigated the complexity of finding stable partitions of RBHG instances. We considered different notions of stability in coalition formation games in terms of allowable movements from a given partition.

We defined several stability problems in RBHG. We showed that finding Nash stable, envy free and Nash stable, individually stable, core stable, and strict core stable solutions are all generally NP-complete. We showed that finding contractual strict core stable and Pareto optimal solutions are both coNP-complete.

We plan to develop heuristic approximations for these stability problems in general RBHG. Additionally, new restrictions on utility functions, the sets of available roles and compositions, and the sets of movement available to agents may be considered. Leveraging restrictions on these variables may allow for fast algorithms on special instances of RBHG which have not yet been investigated.

When agents can make decisions about their partitions, the primary goal should be to make those decisions easy. We propose that, in any setting where agent utilities are hedonic and the agents are afforded some ability to defect from a partition, stable partition should be considered the primary goal for a central authority. Optimization of a utility function will not matter if the agents can simply defect from the partition we worked so hard to achieve. Available methods of movement should be considered for each real world setting. Stability against such movements must be ensured before any optimization can be considered worthwhile.

## Acknowledgments

This work is partially supported by the Duncan E. Clarke Memorial Innovation Award and by the National Science Foundation, under grant CCF-1215985. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. We also thank Dr. De Liu for his helpful comments on the importance of stability in MMO team formation.

## References

Achterbosch, L.; Pierce, R.; and Simmons, G. 2008. Massively multiplayer online role-playing games: the past, present, and future. *Computers in Entertainment (CIE)* 5(4):9.

Aziz, H.; Brandt, F.; and Seedig, H. G. 2010. Optimal partitions in additively separable hedonic games. *arXiv preprint arXiv:1005.4540*.

Aziz, H.; Brandt, F.; and Seedig, H. G. 2011. Stable partitions in additively separable hedonic games. In *Proc. AAMAS '11*, 183–190.

Ballester, C. 2004. NP-completeness in hedonic games. *Games and Economic Behavior* 49(1):1–30.

Banerjee, S.; Konishi, H.; and Sönmez, T. 2001. Core in a simple coalition formation game. *Social Choice and Welfare* 18(1):135–153.

Bogomolnaia, A., and Jackson, M. 2002. The stability of hedonic coalition structures. *Games and Economic Behavior* 38(2):201–230.

Darmann, A.; Elkind, E.; Kurz, S.; Lang, J.; Schauer, J.; and Woeginger, G. 2012. Group activity selection problem. In *Internet and Network Economics*. Springer. 156–169.

Drèze, J., and Greenberg, J. 1980. Hedonic coalitions: Optimality and stability. *Econometrica* 48(4):987–1003.

Fuster, H.; Chamarro, A.; Carbonell, X.; and Vallerand, R. J. 2014. Relationship between passion and motivation for gaming in players of massively multiplayer online role-playing games. *Cyberpsychology, Behavior, and Social Networking* 17(5):292–297.

Gairing, M., and Savani, R. 2010. Computing stable outcomes in hedonic games. In *Algorithmic Game Theory*. Springer. 174–185.

Hajduková, J. 2006. Coalition formation games: A survey. *International Game Theory Review* 8(04):613–641.

Irving, R. W., and Manlove, D. F. 2002. The stable roommates problem with ties. *Journal of Algorithms* 43(1):85–105.

Irving, R. W. 1985. An efficient algorithm for the stable roommates problem. *Journal of Algorithms* 6(4):577–595.

Saad, W.; Han, Z.; Basar, T.; Hjorungnes, A.; and Song, J. B. 2010. Hedonic coalition formation games for secondary base station cooperation in cognitive radio networks. In *Wireless Communications and Networking Conference (WCNC)*, 1–6.

Saad, W.; Han, Z.; Basar, T.; Debbah, M.; and Hjorungnes, A. 2011a. Hedonic coalition formation for distributed task allocation among wireless agents. *Mobile Computing, IEEE Transactions on* 10(9):1327–1344.

Saad, W.; Han, Z.; Hjorungnes, A.; Niyato, D.; and Hossain, E. 2011b. Coalition formation games for distributed cooperation among roadside units in vehicular networks. *Selected Areas in Communications, IEEE Journal on* 29(1):48–60.

Shim, J. Y.; Kim, T. H.; and Kim, S. W. 2014. Decision support of bad player identification in MOBA games using PageRank based evidence accumulation and normal distribution based confidence interval. *International Journal of Multimedia & Ubiquitous Engineering* 9(8).

Spradling, M.; Goldsmith, J.; Liu, X.; Dadi, C.; and Li, Z. 2013. Roles and teams hedonic game. In *Proc. Algorithmic Decision Theory*. Springer. 351–362.

Stabile, C. 2014. I will own you: Accountability in massively multiplayer online games. *Television & New Media* 15(1):43–57.

Sung, S.-C., and Dimitrov, D. 2010. Computational complexity in additive hedonic games. *European Journal of Operational Research* 203(3):635–639.