

Comparing Air Mission Routes from a Combat Survival Perspective

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Abstract

An aircraft flying inside hostile territory is exposed to the risk of getting detected and tracked by the enemy's sensors, and subsequently hit by its weapons. This paper describes a combat survivability model that can be used for assessing the risks associated with a mission route. In contrast to previous work, the model describes both the risk of getting tracked and the risk of getting hit, as well as the dependency between these risks. Three different ways of using the model for comparing routes from a combat survival perspective are suggested. The survivability for the end point, i.e., the probability of flying the entire route without getting hit, is a compact way of summarizing the risks. Visualizing how the risks vary along the route can be used for identifying critical parts of the mission. Finally, assigning weights to different risks allow the opportunity to take preferences regarding risk exposure into account.

Keywords: Survivability, Air mission, Markov model, Route planning, Fighter aircraft, Unmanned aerial vehicle

Introduction

An aircraft flying in hostile territory is exposed to the risk of getting detected, tracked and hit by the enemy's ground-based air defense system. The acceptable risk level for a route depends on, for instance, the importance of the mission and whether the aircraft is manned or unmanned. Schulte (2001) has described three sometimes conflicting goals for air missions; flight safety, mission accomplishment and combat survival. These goals need to be considered when planning air mission routes for manned fighter aircraft as well as unmanned aerial vehicles (UAVs). It is often not possible to accomplish the mission without exposing the aircraft to any risk, since the enemy positions the weapons in order to obstruct the mission. It would therefore be useful to compare different possible routes from the combat survival perspective, in order to determine where to fly. This requires a model that can describe the risk with flying the separate routes and a method for comparing the risks for different routes.

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Risk calculation for route have earlier been discussed in the literature, for instance in the context of decision support for fighter pilots to optimize the flight from a survival perspective (Randleff 2007) and in the context of route planning for UAVs in hostile environments (Zheng et al. 2005; Ruz et al. 2007). These models described the momentary risk at each point of the route and thereafter summed risks over the entire route. However, Ögren and Winstrand (2005) argued that actual risk is not a sum, but a product reflecting the combined probability of surviving all path segments. This is reasonable based on the observation that the risk of getting hit during one part of the route depends on where the aircraft has already flown. For instance, the aircraft can only get hit at some point, if it has survived the earlier parts of the mission. An attempt to model this dependency without sampling the route is the survivability model presented in (Erlandsson et al. 2011), which was based on a continuous Markov model with two states. However, none of the models discussed above explicitly described the dependency between the risk of getting tracked and the risk of getting hit. Discussions with domain experts therefore resulted in the suggestions of extending the two state model to include more states, see (Helldin and Erlandsson 2012). This extension would result in a better resolution in the evaluation of the routes. Furthermore, it would also describe the behavior of the enemy in a more realistic way. For example, the enemy must detect and track the aircraft with enough accuracy before firing a weapon.

The aim of the model presented here is to enable comparison of routes from a combat survival perspective. In the previous models it was natural to calculate the *survivability* for the routes. Survivability for a point of interest is here defined as the probability that the aircraft has not been hit up to that point of the route. Hence, the survivability for the end point of the route describes the probability that the aircraft can fly the entire route without getting hit. However, the extended survivability model raises new questions regarding the comparison of different routes, since the evaluation may include not only the survivability but also the risk that the aircraft gets detected and tracked.

This paper first presents the extended model and analyzes its behavior in a scenario. Thereafter, different ways of using the model for comparing air mission routes are suggested and discussed.

Model

The purpose of the survivability model is to evaluate a route from a combat survival perspective. However, uncertainties regarding, for instance, the enemy's capabilities and intentions make the evaluation of the route uncertain. Furthermore, the route is extended in time and the survival for one part of the route depends on what has happened earlier in the route. It is reasonable to consider the survival for the route as a stochastic process. A common model for stochastic processes is the Markov model, which due to its simplicity is used in many applications, e.g., reliability theory (Ben-Daya et al. 2009), life and sickness insurance (Alm et al. 2006) and medical decision making (Sonnenberg and Beck 1993; Briggs and Sculpher 1998). The survivability model is therefore based on a Markov model.

Markov Models

A continuous Markov model is used to model a stochastic process $X(t)$ that at every time t can be in one of a discrete number of states and where:

$$P[X(t + \Delta) = j | X(t) = i] = \lambda_{ij} \Delta,$$

$$P[X(t + \Delta) = i | X(t) = i] = 1 - \sum_{k \neq i} \lambda_{ik} \Delta,$$

for an infinitesimal time step of size Δ . Hence, if the present state of the process is known, knowledge regarding the previous states will not affect the probability of the future states. This property is known as the Markov property. λ_{ij} is known as the transition intensities and describes the rate of transitions from state i to state j . For Markov models with finite state space, the transition intensities can be used to define the rate matrix Λ , with the i, j th entry

$$\Lambda_{ij} = \begin{cases} \lambda_{ij}, & i \neq j \\ -\sum_{k \neq i} \lambda_{ik}, & i = j. \end{cases}$$

Let $\mathbf{p}(t)$ be the vector describing the state probabilities at time t , i.e., $p_j = P[X(t) = j]$. If all intensities are constant, then

$$\mathbf{p}(t)^T = \mathbf{p}(0)^T e^{\Lambda t}, \quad (1)$$

where $e^{\Lambda t}$ denotes the matrix exponential. Further information about Markov models can be found for instance in (Yates and Goodman 2005).

Survivability Model

The survivability model presented in this paper is a continuous Markov model with five states: Undetected, Detected, Tracked, Engaged and Hit, see Figure 1. The enemy's ground-based air defense system is described with sensor and weapon areas. The aircraft is usually faster and more mobile than the systems on the ground. This motivates that the sensor and weapon areas are modeled as stationary. The transition intensities in the model depend on the relative geometry between the aircraft and these areas. For instance, when the aircraft is outside the enemy's sensor and weapon

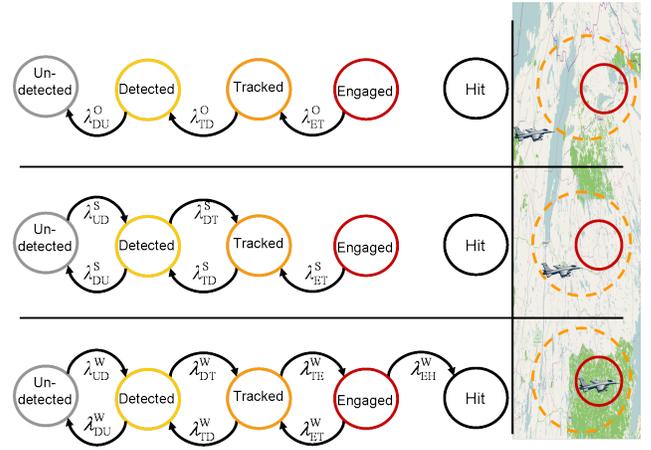


Figure 1: The survivability model is a Markov model with five states. The structure of the intensity matrix depends on whether the aircraft is outside the sensor areas (top), inside a sensor area (middle) or inside a weapon area (bottom). The arrows indicate the possible transitions. The notation, λ_{ij}^X , describes the intensity from i to j (e.g., Undetected to Detected, DU) for an area X , (e.g., inside the sensor area, S).

areas, the process can remain in its previous state or transit to a state to the left in Figure 1, but not to the right. Hence, if the process is in state Undetected, it will remain in this state. When the aircraft is inside a sensor area, the process can reach the states Detected or Tracked, but the states Engaged or Hit can only be reached when the aircraft is inside a weapon area. Hit is an absorbing state, meaning that the process can not leave this state.

In this model, $\Lambda(t)$ is not constant, but varies when the aircraft flies along the route, implying that (1) can not be applied directly. On the other hand, the intensities are piecewise constant, i.e.,

$$\Lambda(t) = \begin{cases} \Lambda_0, & t_0 \leq t < t_1 \\ \Lambda_1, & t_1 \leq t < t_2 \\ \Lambda_2, & t_2 \leq t < t_3 \\ \dots & \end{cases}$$

and for a time point t_n

$$\mathbf{p}(t_n)^T = \mathbf{p}(t_0)^T e^{\Lambda_0(t_1-t_0)} e^{\Lambda_1(t_2-t_1)} \dots$$

$$= \mathbf{p}(t_0)^T \prod_{k=0}^{n-1} e^{\Lambda_k(t_{k+1}-t_k)},$$

since $p(t_{k2})^T = p(t_{k1})^T e^{\Lambda_k(t_{k2}-t_{k1})}$ if Λ is constant during $t_{k1} < t < t_{k2}$.

The survivability model has been implemented with three different intensity matrices with structures according to Fig-

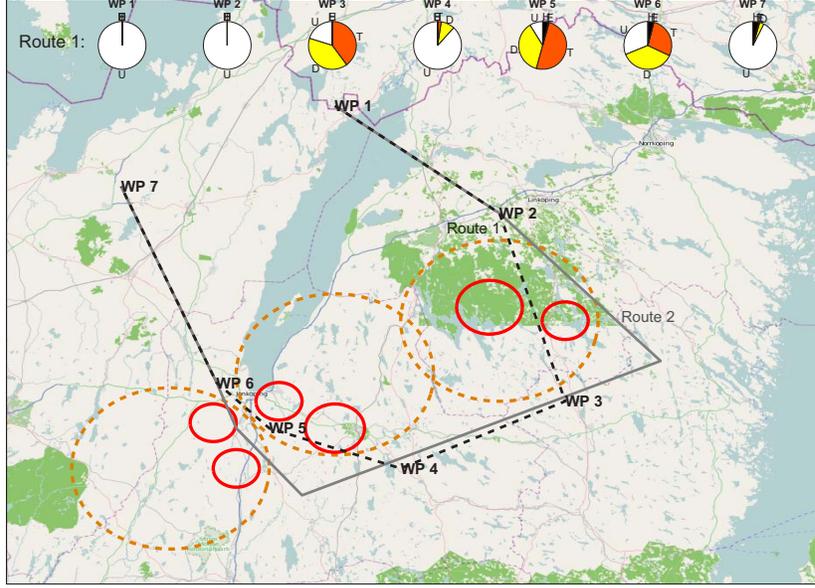


Figure 2: Two routes described by waypoints (WP1–WP7) that intersect the enemy’s sensor areas (dashed circles) and weapon areas (solid circles). The pie charts illustrate the state probabilities for Route 1 (black dashed line) when the aircraft has reached the waypoints. Map from ©OpenStreetMap contributors.

ure 1 and numerical values according to:

$$\Lambda^{Outside} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.2 & -0.2 & 0 & 0 & 0 \\ 0 & 0.2 & -0.2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Lambda^{Sensor} = \begin{bmatrix} -0.5 & 0.5 & 0 & 0 & 0 \\ 0.1 & -0.4 & 0.3 & 0 & 0 \\ 0 & 0.2 & -0.2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Lambda^{Weapon} = \begin{bmatrix} -0.7 & 0.7 & 0 & 0 & 0 \\ 0.1 & -0.5 & 0.4 & 0 & 0 \\ 0 & 0.1 & -0.4 & 0.3 & 0 \\ 0 & 0 & 0.1 & -0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Furthermore, it is assumed that the aircraft is undetected when the mission is started, i.e, the state vector is initialized with:

$$\mathbf{p}(0) = [1 \ 0 \ 0 \ 0 \ 0]^T.$$

The numerical values used in this paper have been selected for illustration only and do not correspond to any real sensor and weapon systems. However, a few implementation issues for selecting the values are worth commenting.

- Different kinds of sensor and weapon systems can be described in the model by selecting different values in the

rate matrices for describing their detection, tracking and hitting capabilities. However, in this paper, all sensors and weapons are described with Λ^{Sensor} and Λ^{Weapon} respectively. The reason is to ease the analysis of the model’s behavior.

- The transition probabilities λ_{TD}^O and λ_{DU}^O are smaller than λ_{ET}^O and λ_{ET}^S . Hence, the process quickly leaves the state Engaged when the aircraft leave the weapon areas. On the other hand, the sensors are likely to keep track of the aircraft for a while using, even though it is outside sensor areas. This can be done by predicting the future positions based on a model of the aircraft’s dynamics, see e.g., (Blackman and Popoli 1999).
- It is assumed that the air defense system has the capability of detecting and tracking the aircraft with higher accuracy if it is within the weapon area, compared to if it is only within the sensor area. This implies that $\lambda_{DT}^W > \lambda_{DT}^S$ and $\lambda_{UD}^W > \lambda_{UD}^S$.

Scenario

Route 1 depicted in Figure 2 consists of seven waypoints (WP1–WP7) and intersects with both sensor and weapon areas. The pie charts illustrate the state probabilities $\mathbf{p}(t)$ at the different waypoints if the aircraft follows this route. The aircraft is undetected at WP1 and WP2, since it has not passed any sensor or weapon areas. Before reaching WP3, the air-

craft must pass through a sensor area and the probabilities for the states Detected and Tracked are therefore high. Note that the implementation is such that state Tracked infers that the aircraft is both detected and tracked. The total probability that the aircraft is detected is therefore the sum of the probabilities for the states Detected and Tracked in this case. Even though WP3 is located outside the enemy’s sensor areas, the enemy is likely to still keep track of the aircraft. At WP4, the aircraft has been outside the sensor area for a long time and the probability that the enemy keeps track of the aircraft is low.

The aircraft has passed a weapon area before reaching WP5 and might have been hit. The state probability for Hit remains stable at WP6 and WP7 where the aircraft is outside the weapon area. The state probabilities for Detected and Tracked are quite high at WP6, but the state probability for Engaged is low. This is in accordance with the selection of larger numerical values for λ_{ET}^O and λ_{ET}^S , than for λ_{TD}^O and λ_{DU}^O . Hence, even though the enemy can keep track of the aircraft outside the sensor area, it is not likely that the aircraft is still engaged. Finally, at the last waypoint, the aircraft is far away from the sensor and weapon areas. The aircraft will here be either undetected or hit, i.e., not been able to fly unharmed to this point. The probability that the process is still in state Detected or Tracked is low and will decrease even more if the aircraft continues away from the dangerous areas.

Comparison of Routes

The aim of the survivability model is to allow the evaluation of different routes in order to determine which one to fly. This section discusses how the survivability model can be used for comparing different routes and illustrates the discussion by comparing the two routes in Figure 2.

Survivability for the Route

An air mission route usually ends outside the hostile area where the enemy is not able to track or engage the aircraft. Almost all probability mass is therefore allocated to either Undetected or Hit in the end of the route, as was indicated for WP7 in Figure 2. A natural way to evaluate the route is to consider the survivability at the last waypoint, i.e., the probability that the aircraft can fly the entire route without getting hit. The survivability at WP7 for the routes in Figure 2 are presented in Table 1, which shows that Route 1 is preferable, even though the difference is small. Hence, even though the intersection with the weapon area is slightly larger for Route 1, the survivability for Route 1 is higher.

Table 1: Survivability at WP7

	Route 1	Route 2
$1 - p_{Hit}(t_{end})$	96.0%	95.6%

The advantage with evaluating the route based on the survivability is that the evaluation is summarized into a single value, which allows for fast comparison of many routes. However, the survivability at the end of the route does not

show the risk of getting tracked. In low risk missions, only routes with survivability close to 100% will be accepted and the aim is to avoid being detected and tracked. It is difficult to compare these kinds of routes only based on the survivability values.

Probability State Vector over Time

Figure 3 shows how the state probabilities vary over time for the two routes. Route 1 has high state probabilities for

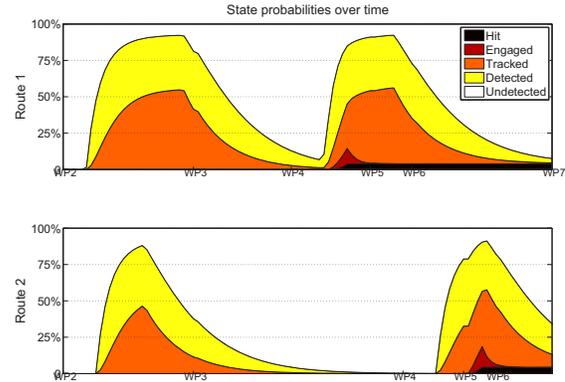


Figure 3: The state probabilities over the two routes depicted in Figure 2.

Detected and Tracked during the two parts where the route passes the sensor areas. The route intersects a weapon area between WP4 and WP5, which results in that the state probabilities for Engaged and Hit increase. At WP5, the aircraft has left the weapon area and the state probability for Engaged is almost 0. The state probability for Hit remains constant during the rest of the route.

Route 2 intersects less with the sensor areas and the state probabilities for Detected and Tracked are lower than for Route 1 during almost the entire route. However, when the aircraft enters the weapon area after WP5, the state probability for Tracked is quite high. Even though the intersection with the weapon area is smaller for Route 2 than for Route 1, the probability that the aircraft gets hit is (slightly) higher, since the enemy has a higher probability of tracking the aircraft when it enters the weapon area. This example shows that the model is able to capture the behavior that the risk of getting hit does not only depend on the time the pilot spends inside a weapon area, but also on the risk of getting tracked.

Analyzing a visualization of the probability state vector over the entire route, as shown in Figure 3, gives more information than only studying the survivability at WP7 as in Table 1. For instance, Figure 3 shows that there is a high risk that the aircraft will be detected and tracked. Furthermore, the critical parts of the routes are clearly shown in the visualization. When planning the air mission, these are the parts of the route that should be re-planned, if possible. It is valuable to identify the critical parts also in cases where re-planning is not possible. This can be used for identifying

when other actions of protection might be needed. A disadvantage of comparing routes with this kind of visualization is that it does not allow for automatic comparison of routes.

Probability for Reaching the States

There is only a small difference between the survivabilities at WP7 for Route 1 and Route 2 and it might therefore be interesting to also consider other risks. In order to calculate the total probability that the aircraft is detected, tracked or engaged at anytime during the route, several parallel Markov models can be used, see Figure 4. The parallel models are all

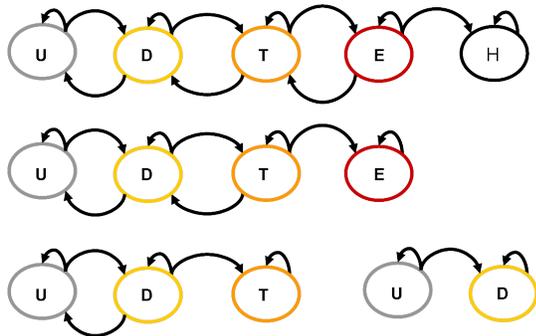


Figure 4: Markov models with different absorbing states. These are used for calculating the total probability that the process will reach those states anytime along the route.

versions of the original Markov model, but with other end states and their rate matrices are submatrices from the Λ s of the original model.

Figure 5 illustrates the probabilities that the process has reached the states when the aircraft has flown the two routes. The probability of getting hit is largest for Route 2 as was

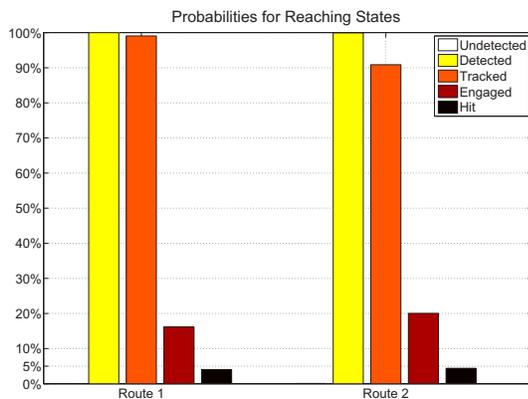


Figure 5: The white bars (too small to be visible) represent the probability of flying the entire route undetected. The other bars represent the probability that the process has reached the states at least once during the route.

shown also in the previous discussions. On the other hand, the probability that the aircraft gets tracked during the route

is smaller for Route 2, since it intersects less with the sensor and weapon areas.

The advantage of calculating the probability of reaching the states is that it summarizes the risk of the route in a few numbers and takes all states into account. This can be further summarized by assigning weights to the different states, i.e., values of how bad it is to reach the state, and add the probabilities multiplied with their weights. Table 2 shows the evaluation of the two routes based on three different sets of weights representing different risk preferences. As al-

Table 2: Added total risk multiplied with weights for the states Tracked and Hit. The weights for the other states are set to 0.

$W_{Tracked}$	W_{Hit}	Route 1	Route 2
0	1	3.97	4.40
0.5	0.5	51.5	47.6
0.1	0.9	13.5	13.0

ready noted, Route 1 is the best route if all weight is on state Hit. On the other hand, if state Tracked is also given some weight, Route 2 is better in this example, both when they are equally weighted and when the weight for Hit is 0.9 and the weight for Tracked is 0.1. Even though the state Hit is more dangerous than the state Tracked, it can be argued that there are situations when both these risks should be taken into account. First of all, in low risk missions, only routes with survivability close to 100% will be acceptable. The risk of getting tracked can then be used for selection between two routes with the same survivability. Secondly, if the enemy detects and tracks the aircraft, this might reveal the information regarding the intentions, plans and capabilities of the aircraft, which can make later missions more dangerous. Finally, the position information regarding the weapon areas is usually uncertain and the route might intersect more weapon areas than was planned for. However, decreasing the risk of getting tracked will increase the survivability.

Conclusions and Future Work

Planning an air mission route in hostile territory requires consideration of many factors, such as fuel consumption, mission accomplishment and survival. The enemy positions its weapons and sensors in order to protect its valuable assets and it is often not possible to accomplish the mission without exposing the aircraft to any risk. The scenario in Figure 2 shows that it is not always trivial to manually determine which route that is least risky. In a more complex scenario with different kinds of enemy sensor and weapon system, one can imagine that manual comparison of routes would be even more difficult and that automatic support for route comparison would speed up and improve the planning.

The survivability model presented in this paper can be used for evaluating a route both with respect to the probability of getting tracked and the probability of getting hit. In contrast to previous work, it also describes the dependency between these two risks. It can therefore describe that the enemy keeps track of the aircraft outside the sensor areas. It is also possible to model that the survivability for the route

does not only depend on the exposure time to the weapons, but also on the probability that the aircraft is tracked when entering a weapon area.

The paper also extends previous work by suggesting how visualizing the probability state vector over time can ease manual comparison between routes. Furthermore, it can be used for identifying critical parts of the route that might need to be re-planned. However, in an automatic route planning system it is desirable to compare routes based on more compact representations. The survivability for the last waypoint describes the probability that the aircraft can fly the entire route without getting hit. This evaluation summarizes the route into a single number, which is useful when many routes should be compared. However, it does not discriminate between routes with the same probability of getting hit, but different exposure to the risk of getting tracked. The paper suggests that this issue can be handled by calculating the probability of reaching the states at least once during the route. Finally it demonstrates how assigning weights for the different states allows the opportunity to take preferences regarding risk exposure into account. These preferences depend on, for instance, the importance of the mission and whether the aircraft is manned or unmanned. Contrary to previous work, the approaches suggested here enable routes to be compared based on other risks than the risk of getting hit.

Suggestions for Future Work

This work should be regarded as a first step towards a system that can aid the planning of air mission routes. In order to further investigate its applicability, it would be useful to implement the model in a more realistic environment and let the intended users test it. A remaining issue is how to assign the parameters in the rate matrices, i.e., the sensors detection and tracking rates as well as the hit rates of the weapons.

Future development of the survivability model could include incorporation of overlapping sensor and weapon areas. Furthermore, this paper has assumed that the locations and sizes of these areas are perfectly known. In practice, this kind of information is uncertain and further development of the model is needed in order to describe this uncertainty.

Markov models are often used to model stochastic phenomena that evolve over time, for instance in reliability theory, life and sickness insurance and medical decision making. In such situations, a decision maker needs to select actions that increase the probabilities that the Markov model remains in the suitable (healthy) states, e.g., medical treatment of an illness or maintenance of a critical component in a machine. It is also important to determine when such actions should be performed. This work has investigated how the outcome of actions describes as routes can be visualized and compared, but it would be interesting to investigate if the same comparing methods are applicable in other domains as well.

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