

Hybridizing Interval Temporal Logics: The First Step

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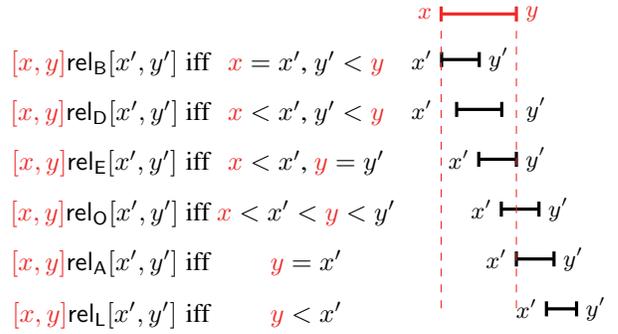
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Introduction

Temporal reasoning is one of the main topics investigated within the field of Artificial Intelligence. Formal methods for temporal reasoning arouse interest of researchers from both theoretical and practical point of view (2004). Such methods enable modelling and studying human-like reasoning mechanisms, thus constituting a valuable tool in cognitive science, philosophy, and linguistics. On the other hand, temporal reasoning formalisms have a number of potential practical applications, e.g., in task scheduling, action planning, and temporal databases. Temporal reasoning methods may be divided into *point-based* and *interval-based* depending on the type of the considered primitive ontological objects. My work revolves around the latter type of methods which seem to be more human-like and more suitable for such applications as continuous process modelling. My main result is that the satisfiability problem in the fragment denoted by $HS_{horn}^{\square, i, @}$ (which stands for a hybridized fragment of HS in which formulas are in a form of conjunction of *Horn* clauses and only box modal operators are allowed – diamond operators are disallowed) is NP-complete over reflexive, as well as over irreflexive and dense time frames. Before hybridization this fragment (denoted by HS_{horn}^{\square}) was P-complete over such time structures. The results mentioned in this abstract have been proved in my paper accepted to the 7th Indian Conference on Logic and its Applications (ICLA) (2016).

Halpern-Shoham Logic

One of the most elegant interval temporal logics is Halpern-Shoham logic (HS in short) (1991). In HS a time line is modelled as a partial order of timepoints. An interval is constituted by an ordered pair of timepoints, namely a beginning and an ending point. What is crucial is that HS introduces 12 binary relations between the current interval $[x, y]$ and a nonidentical interval $[x', y']$, which forms a pairwise disjoint and mutually exclusive set of relations: *begins*, *during*, *ends*, *overlaps*, *adjacent to*, and *later than* defined as follows:



and their inverses. In the language of HS, there are 12 possibility (diamond) and 12 necessity (box) operators corresponding to the abovementioned relations. The satisfaction relation for a model \mathcal{M} and an interval $[x, y]$ has the following definition:

$$\begin{aligned} \mathcal{M}, [x, y] \models \langle R \rangle \varphi & \text{ iff there is an interval } [x', y'] \text{ such that} \\ & [x, y] \text{rel}_R[x', y'] \text{ and } \mathcal{M}, [x', y'] \models \varphi; \\ \mathcal{M}, [x, y] \models [R] \varphi & \text{ iff for every interval } [x', y'] \text{ such that} \\ & [x, y] \text{rel}_R[x', y'], \mathcal{M}, [x', y'] \models \varphi; \end{aligned}$$

where R is one of B, D, E, O, A, L and their inverses, and satisfaction conditions for propositional variables and boolean connectives are defined in a classical way.

HS is very expressive but undecidable for the most interesting structures of time including \mathbb{N} , \mathbb{Z} , \mathbb{Q} , and \mathbb{R} (1991). One of the recently studied way to obtain decidable fragments of HS while maintaining expressive power high enough for practical applications consists in introducing syntactic restrictions on the formulas (2016). Two interesting fragments obtained in such a way are *Horn* and *core* fragments of the 'sub-propositional' language only allowing box modalities (excluding diamond modalities). Henceforth, they will be denoted as HS_{horn}^{\square} and HS_{core}^{\square} , respectively. To define the languages of these fragments, let a *positive temporal literal* λ be defined by the following grammar:

$$\lambda := \top \mid \perp \mid p \mid [R]\lambda.$$

where p is a propositional variable. An HS_{horn}^{\square} formula is

$$\varphi := \lambda \mid [U](\lambda \wedge \dots \wedge \lambda \rightarrow \lambda) \mid \varphi \wedge \varphi,$$

whereas an HS_{core}^{\square} formula is

$$\varphi := \lambda \mid [U](\lambda \rightarrow \lambda) \mid [U](\lambda \wedge \lambda \rightarrow \perp) \mid \varphi \wedge \varphi,$$

where λ is a positive temporal literal. The computational complexity of HS_{horn}^{\square} and HS_{core}^{\square} depends on the type of the underlying structure of timepoints – for details see Table 1. In the case of reflexive or dense structures HS_{horn}^{\square} is P-complete, whereas the complexity of HS_{core}^{\square} is not precisely known – it is only known that it is in P and NLOGSPACE-hard. In the case of irreflexive and discrete time structures HS_{horn}^{\square} is undecidable, while HS_{core}^{\square} is PSPACE-hard. Particularly interesting is HS_{horn}^{\square} which is tractable over reflexive and dense time structures and expressive enough to be applied, e.g., to ontology-based data access over temporal databases (2016).

Table 1: Computational complexity of HS fragments. My contributions are denoted by (\star).

	Irreflexive	Reflexive
Discrete	HS_{core}^{\square} : PSPACE-hard	HS_{core}^{\square} : in P, NL-hard
	HS_{horn}^{\square} : undecidable	HS_{horn}^{\square} : P-compl.
	$HS_{horn}^{\square, @, i}$: undecidable	$HS_{horn}^{\square, @, i}$: NP-compl. (\star)
Dense	HS_{core}^{\square} : in P, NL-hard	HS_{core}^{\square} : in P, NL-hard
	HS_{horn}^{\square} : P-compl.	HS_{horn}^{\square} : P-compl.
	$HS_{horn}^{\square, @, i}$: NP-compl. (\star)	$HS_{horn}^{\square, @, i}$: NP-compl. (\star)

Hybridization of Halpern-Shoham Logic

Restricting syntax of HS may lead to decidability (as pointed out in the previous section), however such limitations significantly decrease expressive power. Namely, HS_{horn}^{\square} , as well as HS_{core}^{\square} seems to lose the *referentiality*, i.e., the ability to label intervals and then to refer to a chosen interval with a concrete label. Importantly, the full HS is referential in this sense. The referentiality is a crucial construct in temporal knowledge representation (2000) and the most straightforward way to retrieve it is to *hybridize* a logic. Hybridization is obtained by adding the second sort of atoms called *nominals* that label single intervals, and *satisfaction operators* indexed by nominals that enable to refer to an interval in which this nominal is satisfied. As an example, the formula $@_i \varphi$ is true iff φ holds in the (single) interval labelled by the nominal i .

Hybridization usually increases expressive power of the logic, e.g., it enables to express identity of intervals labelled with i and j by $@_i j$. Surprisingly, although hybridization of interval temporal logics was already recognised as a promising line of research (2000), it has received only limited attention from the research community. One exception is an attempt of adding a very restricted reference property (that makes it possible to state that some propositional variable is satisfied in a particular interval (2015)).

My main contribution consists in proving computational complexity of a hybrid version of HS_{horn}^{\square} , denoted by $HS_{horn}^{\square, @, i}$. In particular, I have shown that in the case of reflexive or dense time structures $HS_{horn}^{\square, @, i}$ is NP-complete –

see Table 1. This result seems to be unexpected. Namely, in the case of the investigated HS fragment, hybridisation increases (assuming that $P \neq NP$) its computational complexity from P (or NLOGSPACE) to NP. Importantly, in classical modal logic hybridisation does not change the complexity of the logic, namely it is PSPACE-complete before and after hybridization.

Open Problems and Future Work

There is a number of open problems concerning the computational complexity of the described fragments of HS, e.g., it is not known what is the computational complexity hybridized HS_{core}^{\square} . The most interesting case is when the time structure is both irreflexive and discrete – it is not even known if in such a case HS_{core}^{\square} is decidable. If it turns out that HS_{core}^{\square} is decidable, it would be interesting to study if $HS_{core}^{\square, @, i}$ remains decidable. Another line of future research is to study expressiveness of the abovementioned fragments of HS and their potential practical applications. The next step would be to implement the reasoning algorithms involving the presented fragments of HS.

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