Online Search Cost Estimation for SAT Solvers

Shai Haim and **Toby Walsh**
{shai.haim,toby.walsh}@nicta.com.au
School of Computer Science and Engineering,
The University of New South Wales
Sydney, Australia
and
NICTA
Sydney, Australia

Abstract

We present two different methods for estimating the cost of solving SAT problems. The methods focus on the online behaviour of the backtracking solver, as well as the structure of the problem. Modern SAT solvers present several challenges to estimate search cost including coping with non-chronological backtracking, learning and restarts. Our first method adapt an existing algorithm for estimating the size of a search tree to deal with these challenges. We then suggest a second method that uses a linear model trained on data gathered online at the start of search. We compare the effectiveness of these two methods using random and structured problems. We also demonstrate that predictions made in early restarts can be used to improve later predictions. We conclude by showing that the cost of solving a set of problems can be reduced by selecting a solver from a portfolio based on such cost estimations.

Introduction

Estimating the cost of solving a NP-hard problem like propositional satisfiability (SAT) is a difficult task. Simple backtracking SAT solvers like DPLL unfold a proper-binary decision tree. The Weighted Backtrack Estimate (WBE) (Kilby et al. 2006), which is an adaptation of Knuth's offline sampling method (Knuth 1975) can generate good estimates of search cost for such solvers. However, more modern SAT solvers present several challenges for estimating their runtime. For instance, clause learning repeatedly changes the problem the solver faces. Estimation of the size of the search tree at any point should take into consideration the expected changes that future learning clauses will cause. As a second example, restarting generates a new search tree which again needs to be taken into account by any prediction method.

Our approach to these problems is to use an on-line method to estimate the cost of the search by observing the solver's *behaviour* in a small part of search. Our first method is an extension of an existing method. It adapts the Weighted Backtrack Estimator (Kilby et al. 2006) to support non-chronological backtracking. Our second method uses machine learning. We show that using machine learning, it is possible to achieve good estimates at a very early stage of the search, by exploiting data gathered from other instances from the same ensemble. These two methods are tied together, since the tree size estimated by the WBE method is a useful feature for the machine learning method.

Background

Knuth used probing sample to estimate the size of a backtrack search tree (Knuth 1975). If b_i is the branching rate observed at depth i of the probe, then $1 + b_1 + b_1 \cdot b_2 + \dots$ is an unbiased estimation for the size of the tree. Despite its simplicity, this method is strikingly effective. Unfortunately, random probing cannot be directly used during backtrack search. Inspired by Knuth's method, Kilby et. al. proposed two online methods to estimate the size of a search tree during backtracking search (Kilby et al. 2006): The Weighted Backtrack Estimator, which is discussed in depth in the next section, and the Recursive Estimator. The Recursive Estimator simply assumes that any unexplored right subtree is identical in size to the left subtree. Both methods are unbiased and independent of the problem or solver, but since they are both estimating the size of a complete binary search tree, they do not work directly in modern solvers and perform poorly for most satisfiable instances. Kokotov and Shlyakhter suggested some similar techniques to the RE. The Progress Bar for SAT Solvers (Kokotov and Shlyakhter 2000) estimates the remaining time to solving a SAT instance by observing previously visited nodes. The estimate is calculated using either *historical* or *predictive* heuristics. Historical estimators use the average observed for previous nodes at the same depth. The *simple average* estimator just uses a straight forward average, whilst the *weighted average* favours more recent subtrees. Predictive estimators, on the other hand, are based on the size of the subproblem (e.g. number and size of the clauses)

Machine learning has also been used to estimate search cost. By observing the solver as it solves the problem, we might be able to estimate how long it will take for the solver to solve it. Horovitz et al used a Bayesian approach to classify CSP and SAT problems according to their runtime (Horvitz et al. 2001). Whilst this work is close to that presented here, there are some significant differences. For example, they used SATz-Rand, which does not have some of the complex features tackled here such as clause learning. Xu et. al (Xu, Hoos, and Leyton-Brown 2007) used machine learning to tune empirical hardness models (Leyton-Brown, Nudelman, and Shoham 2002). Learning mostly used static features of the problem instance. The only exception was a group of features generated by probing the search space using DPLL and stochastic search. Their method gives a probabilistic estimate of runtime and not, as here, an estimate for a specific run. Their search cost estimates were used within a portfolio based SAT solver *SATzilla* (Xu et al. 2007).

Finally, an online machine learning method has been developed to speed up a QBF solver (Samulowitz and Memisevic 2007). Having solved different datasets of problems, a multinomial logistic regression model was built to classify each instance to its best heuristic. This model was used to suggest the best heuristic for new problem instances. Such a technique could also be used dynamically to change the heuristic used by a solver.

Weighted Backtrack Estimator

We begin by describing how the existing WBE algorithm (Kilby et al. 2006) can be adapted to cope with modern SAT solvers. At every point in search, the WBE algorithm estimates the search tree size as:

$$\frac{\sum_{d \in D} prob(d)(2^{d+1}-1)}{\sum_{d \in D} prob(d)} \tag{1}$$

Where $prob(d) = 2^{-d}$ is related to the probability that we visit such a depth using random probing, and D is the multiset of branches lengths visited. By storing the numerator and denominator, this estimate can be calculated in constant time and space at every backtrack. The resulting estimate is unbiased assuming we have a proper binary search tree.

Since WBE generate a tree size estimation it is significantly more effective for unsatisfiable instances. Moreover, WBE is not directly applicable to modern SAT solvers as they perform (conflict driven) backjumping. By backjumping over nodes, we no longer have a proper binary tree. A second problem is that on backtracking to a decision level, modern SAT solvers are not forced to branch on the negated decision. We can instead branch on a new variable. Another challenge for WBE is restarts. At every restart point, a new tree is generated. Any method to estimate search cost must take these factors into account.

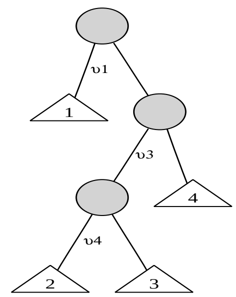
In order to construct a proper binary search tree representing the branching decisions of a SAT solver, and to compensate for backjumping, we observe the two atomic operations performed during search.

- *assign(v,b):* when v is a variable and b is a Boolean value. This action assigns the variable v the value b. This assignment will be kept in the next level of the search stack. After every assignment a unit propagation process takes place. The values that are assigned in this process are also considered to be assigned in this decision level.
- *backtrack(d):* backtrack back to decision level d. Unassign all variables assigned in any decision level equal to or greater than d. Any backtrack is also followed by unit propagation.

A binary tree can be generated as follows: we branch left from a node for every $assign$ operation, and we branch

$\langle stack, action, conflict \rangle$
$\langle \phi, assign(v_1, true), no \rangle$
$\langle \{v_1\}, assign(v_2, true), yes \rangle$
$\langle \{v_1, v_2\}, backtrack(1), no \rangle$
$\langle \phi, assign(v_3, true), no \rangle$
$\langle \{v_3\}, assign(v_4, true), no \rangle$
$\langle \{v_3, v_4\}, assign(v_5, true), yes \rangle$
$\langle \{v_3, v_4, v_5\}, backtrack(2), yes \rangle$
$\langle \{v_3\}, backtrack(1), yes \rangle$

(a) Sequence of actions



(b) Resulting binary search tree

Figure 1: Conversion of a DPLL trace into a binary tree. In 1(a), $stack$ is the assignment stack before the action, $action$ is the action taken, and $conflict$ denotes if this generates a conflict. In 1(b), conflicts are numbered, and edges labelled with assignments. Since decisions v_2 and v_5 are backjumped over, they do not appear as labels.

right when we $backtrack$ back to the node, even if the next assignment is at the same decision level. If we backjump over node n, this node is removed. Note that node depths in the binary tree no longer correspond with decision levels, Figure1 shows an example of this technique. In Figure 1(a), we see a list of $\langle stack, action, conflict \rangle$ tuples, representing a sequence of actions and the resulted assignment stack. The tree in Figure 1(b) is the explicit proper binary tree corresponding to the same sequence of steps. Note that in both cases there are 4 conflicts, but note that the node depths change.

WBE for Conflict Driven search

Every time a backjump occurs, WBE needs to update the depths of leaves beneath this backjump. This is not possible if we just store an accumulated sum for the denominator and numerator in the WBE estimation. Fortunately, the WBE estimation can be computed by observing two different parameters which are easy to adjust after backjumping. The first, C, is a simple counter of the nodes encountered so far in an in-order tree search (counting a node only after backtracking from its left subtree). The second, P, is the partial size of the tree explored assuming it is a complete binary tree. At any point in search, the WBE estimate can be generated by calculating:

$$\frac{C}{P} - 1 \tag{2}$$

Where C is the number of nodes encountered so far and:

$$P = \frac{1}{\sum_{n \in closed} (2^{d(n)+1})} \tag{3}$$

Where $d(n)$ is the depth of node n and $closed$ is the subset of nodes in the current branch whose left child has been closed. We can show this as follows:

$$\frac{\sum_{d \in D} prob(d)(2^{d+1} - 1)}{\sum_{d \in D} prob(d)}$$

$$= \frac{\sum_{d \in D} prob(d) 2^{d+1} - \sum_{d \in D} prob(d)}{\sum_{d \in D} prob(d)}$$

$$= \frac{\sum_{d \in D} prob(d) 2^{d+1}}{\sum_{d \in D} prob(d)} - 1 = \frac{2\,|D|}{\sum_{d \in D} prob(d)} - 1$$

$$= \frac{C}{\sum_{d \in D} prob(d)} - 1$$

Note that $C = 2\,|D|$ as C is increased by 2 for every conflict (once for the leaf and again for the node we backtrack to). Finally, we can show by induction on the depth of the tree that:

$$\sum_{d \in D} prob(d) = \frac{1}{\sum_{n \in closed} (2^{d(n)+1})} = P \tag{4}$$

where $d(n)$ is the depth of node n and $closed$ is the subset of the nodes in the current branch whose left child has been closed.

Both C and P can be computed incrementally as we branch and backjump. Since the search tree is not kept explicitly in memory, $closed$ is computed using a bit array. This increases the space and time complexity of calculating WBE by $O(d)$ where d is the maximum depth. We can avoid increasing the amortized complexity if we estimate search cost at only every $O(d)$ nodes.

Restarts create an extra challenge for WBE. Upon restarting, a new tree is generated. The search cost estimation therefore needs to change. Since WBE generate a *tree size* estimate, we can generate a cost estimation by adding the tree size estimated by WBE to the number of nodes explored until we reach a restart big enough to explore such a tree.

Linear model prediction (LMP)

To learn from more than just the size of previously explored search trees, we developed an online machine learning method. We estimate the runtime on a problem $\mathcal{P} \in E$, when E is an ensemble of problems after training a linear model using a subset of problems $\mathcal{T} \subset E$. For every training example $t \in \mathcal{T}$ a feature vector $x_t = \{x_{t,1}, x_{t,2}, \ldots, x_{t,k}\}$ is created from on observation window of the search tree. We

$Feature$	$init$	*Observation Window*				
		min	max	avg	SD	$last$
var	$\surd$					
cls	$\surd$					
cls/var	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
var/cls	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
FBC	$\surd$			$\surd$	$\surd$	$\surd$
FTC	$\surd$			$\surd$	$\surd$	$\surd$
ACS	$\surd$			$\surd$	$\surd$	$\surd$
SD			$\surd$	$\surd$	$\surd$	
BSD			$\surd$	$\surd$	$\surd$	
BS			$\surd$	$\surd$	$\surd$	
LCS		$\surd$	$\surd$	$\surd$	$\surd$	
CCS		$\surd$	$\surd$	$\surd$	$\surd$	
ABB		$\surd$	$\surd$	$\surd$	$\surd$	
AAB		$\surd$	$\surd$	$\surd$	$\surd$	
AAB/ABB		$\surd$	$\surd$	$\surd$	$\surd$	
ABB/AAB		$\surd$	$\surd$	$\surd$	$\surd$	
$LWBE$		$\surd$	$\surd$	$\surd$	$\surd$	$\surd$

Table 1: The feature vector used by linear regression to construct prediction models

selected features by removing the feature with the smallest standardised coefficient until no improvement is observed based on the standard AIC (Akaike Information Criterion). We then search for and eliminate co-linear features in the set.

Using ridge linear regression, we fit our coefficient vector w to create a linear predictor $f_w(x_i) = w^T x_i$. We chose ridge regression, since it is a quick and simple technique for numerical prediction, and it generally yields good results. We predict the log of the number of conflicts. Since the feature vector is computed online, it is important that it does not add significant cost to search. The feature vector therefore only contains features that can be calculated in constant time. We define the *observation window* to be that part of the search where data is collected. At the end of the observation window, the feature vector is computed and the model queried for an estimation.

The feature vector measures both problem structure and search behaviour. Since data gathered at the start of a restart tends to be noisy and less useful, we do not open the observation window immediately. To keep the feature vector a reasonable size, we use statistical measures of various parameters (that is, the minimum over the observation window, the maximum, the mean, the standard deviation and the last value recorded). The parameters collected are the number of variables (var), the number of clauses (cls), both the variable to clause ratio and its inverse, the fraction of binary and ternary clauses in the clause database (FBC and FTC respectively), the average clause size (ACS), the search depth as it appears in the assignment stack (SD) and as it appears in the binary tree generated for the WBE calculation(BSD), the learnt clauses size (LCC) and the conflict clause size (CCS), the fraction of assigned vars before backtracking (ABB) and after backtracking (AAB), the ratio between these two features and its inverse, and the log of the WBE prediction ($LWBE$). The full list of features used is shown in Table 1. All the features used can be calculated in constant time and space with the exception of the WBE which takes $O(d)$ time and space. We therefore only computed WBE every d conflicts where d is the depth recorded at the previous estimate.

To deal with restarts, we wait until the observation window is contained within a single restart. In addition, we exploited estimates from earlier restarts to help improve later estimates. To do this, we augmented the feature vector with all the search cost predictions from previous restarts.

Experiments

We ran experiments with these two methods using MiniSat (Een and Sorensson 2003). This is a state-of-the-art modern solver, which uses clause learning and clause deletion along with an improved version of VSIDS for variable ordering and a geometrical restart scheme. We used a geometrical factor of 1.5, which is the default for MiniSat. A geometrical factor of 1.2 yielded results of a similar quality. We used three different distributions of SAT problems.

- *rand:* An ensemble of 500 satisfiable and 500 unsatisfiable randomly generated 3-SAT problems with 200 to 550 variables and a clause-to-var ratio of 4.1 to 5.0.
- *bmc:* An ensemble of software verification problems generated by CBMC[1] based on a binary search algorithm coded in C. The different examples used different array sizes and different number of loop unwindings. In order to generate satisfiable problems, a faulty piece of code that causes memory overflow was added to the binary search code. These problems create a very homogeneous ensemble of problems. We used 250 satisfiable and 250 unsatisfiable problems.
- *fv:* An ensemble of hardware formal verification problems distributed by Miroslav Velev[2]. These problems were produced by the same technique but not for the same underlaying problem, and create an ensemble which is less homogeneous than the previous one. We used 56 satisfiable and 68 unsatisfiable problems.

Since training examples can be scarce, we restricted the size of our training set to no more than 500 problems. For the formal verification problems, we obviously had far less than that. In the first part of our experiments, when restarts were turned off, many of the formal verification problems were not solved. Our results in this part will only compare the other datasets. When restarts were enabled, all three data sets were used. In all experiments we used 10-fold cross validation, never using the same instance for both training and testing purposes. We measured the quality of the predictor by observing the percent of predictions which are within a certain factor of the correct cost (the *error factor*). For example, 80% for error factor 2, denotes that for 80% of the instances, the predicted search cost was within a factor of 2 of the actual search cost.

[1]http://www.cs.cmu.edu/ modelcheck/cbmc/

[2]http://www.miroslav-velev.com/sat_benchmarks.html

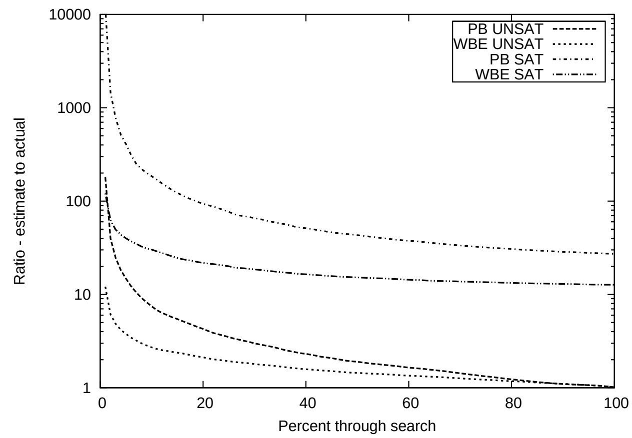


Figure 2: Mean ratio of WBE and PB estimates over time for the $rand$ dataset.

We compare our results with the ones obtained by the Progress Bar (PB) (Kokotov and Shlyakhter 2000). In order to make the comparison possible, we instrumented MiniSat with the Progress Bar. Kokotov and Shlyakhter proposed several different heuristics (Constant, Historical-Basic, Historical-Weighted and Clause Count). We observed similar results with all of these heuristics. We present results here for the Historical-Weighted heuristic since it performs slightly better for these data sets. We used the progress bar's default settings. Note that if the initial search is too deep, the Progress Bar may not provide any estimate.

Search Without Restarts

Figure 2 compares the quality of the WBE prediction and the Progress Bar prediction over time, for the $rand$ dataset. Both predictors return unbiased results for the unsatisfiable problems and converge to the correct value given enough time. WBE is generally more accurate than PB both for satisfiable and unsatisfiable instances. In all cases, both estimators start by over-estimating the search cost but their prediction improves with time as we backjump over nodes. Figure 3 presents the same data for the bmc dataset. For structured problems, WBE initially over estimates search cost by a large factor (in some cases with by a factor greater than 2^{1000}). During this period the Progress Bar does not make any prediction as the tree is too deep for it to work, and the "search space left" is estimated to be 100%. At some later point in search, we often observed a sharp improvement in the accuracy of both estimators. Typically this corresponds to search backjumping over an early mistake to a node very close to the root of the tree (or the root itself). For most instances PB starts returning run-time predictions at this point. The WBE also starts returning good prediction at this point. For unsatisfiable problems in the *bmc* dataset, this point occurs after 72% of the search (on average), but it appears to occur after a smaller percentage of the search for harder instances. We found a correlation coefficient of -0.45 between the total size of the search tree and the percent through search where this improvement occurs.

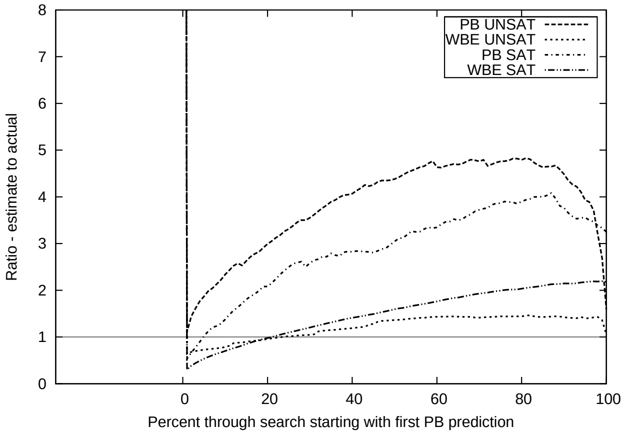


Figure 3: Mean ratio of WBE and PB estimates over time for the bmc dataset. Only starts when PB generates its first prediction.

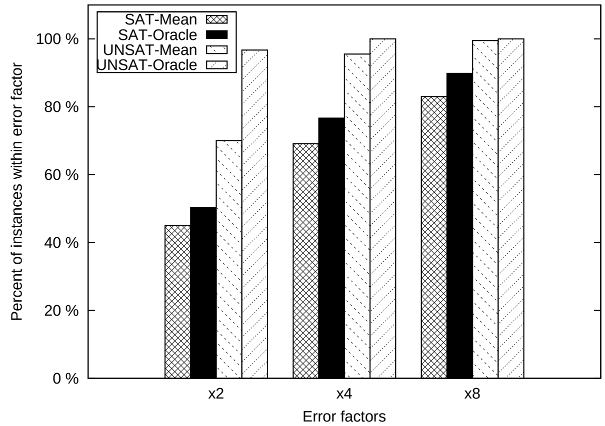


Figure 4: Quality of prediction when using an oracle to determine whether an instance is satisfiable or the geometric mean of satisfiable and unsatisfiable models (denoted *Mean*) - $rand$ dataset

			x2	x4	x8
sat	bmc	PB	3.8	6.8	8.9
		WBE	3.4	5.5	5.9
		LMP	40.7	68.7	85.8
	$rand$	PB	0.9	2.6	4.9
		WBE	2.2	7.8	14.4
		LMP	39.7	71.3	86.6
unsat	bmc	PB	4.9	10.3	12.8
		WBE	4.9	10.3	13.8
		LMP	36.9	68.5	93.6
	$rand$	PB	3.7	7.4	15.5
		WBE	12.7	29.4	47.3
		LMP	92.0	100.0	100.0

Table 2: Percentage of estimates within error factor after 2000 backtracks

			x2	x4	x8
sat	bmc	PB	24.5	36.2	47.3
		WBE	21.8	36.2	45.7
		LMP	49.1	78.9	95.0
	$rand$	PB	1.2	4.0	10.4
		WBE	4.0	12.0	22.5
		LMP	50.2	76.7	89.9
unsat	bmc	PB	22.0	35.4	43.8
		WBE	32.8	48.4	48.4
		LMP	78.1	98.4	100.0
	$rand$	PB	17.7	42.2	58.3
		WBE	38.9	67.0	81.6
		LMP	96.7	100.0	100.0

Table 3: Percentage of estimates within error factor after 35000 backtracks

In order to compare the quality of prediction of WBE, PB and LMP, we generated an estimate after a constant time, regardless of the true size of the problem. In all cases the estimate generated by LMP was superior to those generated by WBE and PB. Comparisons of the performance of those three methods after 2000 and 35000 backtracks are shown in tables 2 and 3 respectively. Satisfiable problems are harder to predict for all methods, due to the abrupt way in which search terminates with open nodes. The linear model deals better with random problems than crafted ones. We conjecture this is due to greater balance in the search tree. WBE performs better than PB for unstructured problems, while they perform similarly for structured instances. The significant improvement of both PB and WBE for structured problems after 35000 backtracks is due to the fact that easier instances are already converging rapidly on the correct answer.

Since we observe very different behaviour with satisfiable and unsatisfiable instances, we trained models on each type of instance separately. With a new (non-training) instance, we may not know if it satisfiable or unsatisfiable. Indeed, the point of search is often to decide this. Given a problem of unknown satisfiability, we therefore queried both models and returned the geometrical mean of the two estimates. Figures 4 and 5 compare using the geometric mean of the two models and using an oracle to decide which model to query for the $rand$ and bmc datasets respectively. We see that the geometric mean returns reasonable predictions. Alternatively we could train with just one model using both satisfiable and unsatisfiable instances. The performance is similar to the geometric mean of the two models (it is a bit better for sat problems and a bit worse for $unsat$ problems) but is sensitive to the proportion of satisfiable and unsatisfiable instances.

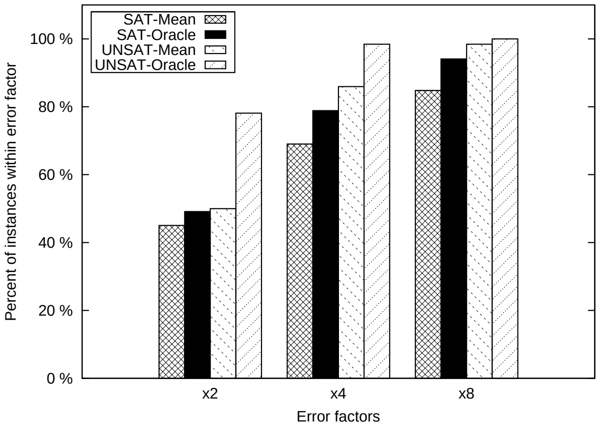


Figure 5: Quality of prediction when using an oracle to determine whether an instance is satisfiable or the geometric mean of satisfiable and unsatisfiable models (denoted *Mean*) - bmc dataset

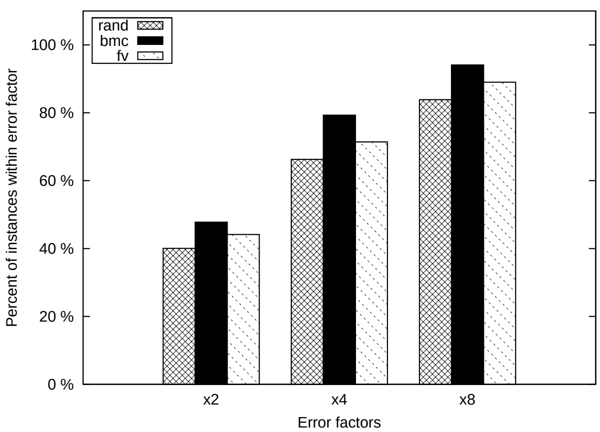


Figure 6: Quality of prediction for sat problems with restarts, (after 2000 backtracks in the *query restart*)

Search With Restarts

When restarts are used, we have to use smaller observation windows to give a prediction early in search as many early restarts are small. Figures 6 and 7 compare the quality of prediction of LMP for the 3 different datasets, for sat and $unsat$ instances respectively. The quality of estimates improves with the bmc data set when restarts are enabled. We conjecture this is a result of restarts before the observation window reducing the noise in the data.

In order to check our hypothesis that predictions from previous restarts improve the quality of prediction in the current restart, we opened an observation window at every restart. The window size is defined by $max(1000, 0.01 \cdot s)$ and it starts after a waiting period of $max(500, 0.02 \cdot s)$, when s is the size of the current restart. At the end of each observation window, two feature vectors were created. The first (x_r) holds all features from Table 1, while the second $(\hat{x}_r)$ is defined as $\hat{x}_r = \{x_r\} \cup \left\{f_{w_1}(x_1), f_{\hat{w}_2}(\hat{x}_2), \ldots, f_{\hat{w}_{r-1}}(\hat{x}_{r-1})\right\}$. A comparison of the two methods, for sat and $unsat$ instances, is given in Figures 8 and 9 respectively. We see that predictions from earlier restarts improve the quality of later predictions but not greatly.

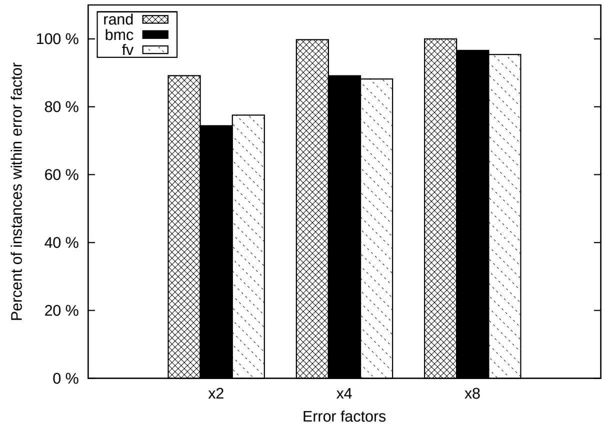


Figure 7: Quality of prediction for $unsat$ problems with restarts, (after 2000 backtracks in the *query restart*)

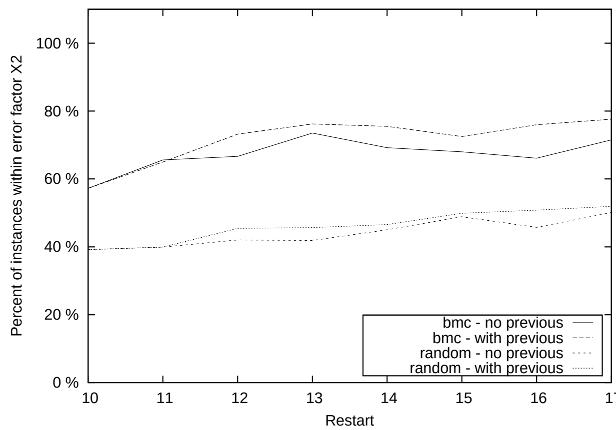


Figure 8: The effect of using predictions from previous restarts, for sat instances. Quality of predictions, through restarts, using two datasets (*bmc*,*rand*). The plots represent the percentage of instances within a factor of 2 from the correct size.

Solver selection using LMP

In our final experiment, we used these estimates of search cost to improve solver performance. We used two different versions of MiniSat. Solver A used the default MiniSat setting (geometrical factor of 1.5), while solver B used a geometrical factor of 1.2. The challenge is to select which is faster at solving a problem instance.

Table 4 describes the percentage improvement of the following strategies compared to the average run time for both solvers:

- *oracle:* Use an oracle to tells us which solver is better for

Dataset		Oracle	LMP (oracle)	LMP(AVG)
$rand$	sat	40.8	7.0	10.5
	unsat	7.5	-0.9	-1.4
fv	sat	66.7	17.2	16.8
	unsat	14.8	-0.6	-3.3
bmc	sat	59.6	13.3	13.6
	unsat	17.2	0.3	-0.4

Table 4: Percentage improvement over average run time for both solver A and B.

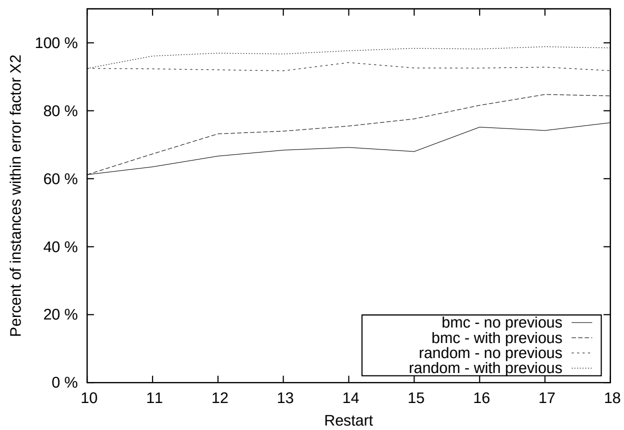


Figure 9: The effect of using predictions from previous restarts, for $unsat$ instances. We compare the quality of predictions, through restarts, using two datasets (*bmc,rand*). The plots represent the percentage of instances within a factor of 2 from the correct size.

the problem ($min(A, B)$).

- *LMP (oracle):* Use both solvers until each reaches the observation window (restart 9 for solver A, restart 19 for solver B), and generate a prediction, using an oracle that indicates which model should be queried. Terminate the one predicted to be worse.
- *LMP (AVG):* Same as *LMP (oracle)*, but without an oracle to determine whether the instance is *sat* or *unsat*. We instead query both models and use the geometric mean as the prediction.

These results show that for satisfiable problems, where solver performance varies more significantly, our method reduces the total cost. For unsatisfiable problems, where solvers performance does not vary as much, our method does not improve search cost. However, as performance does not change significantly on unsatisfiable instances, the overall impact of our method on satisfiable and unsatisfiable problems is positive.

Conclusions and Future Work

We have presented two different methods to generate estimates for the size of the search tree explored by modern day SAT solvers. The WBE method simply observes the search tree and requires no prior knowledge of the problem distribution. This method, like other tree-size based methods performs poorly for satisfiable instances. The LMP method, on the other hand, uses linear models which are trained on a problem set. We have shown that it is possible to train the model using a relatively small training set, which is of value when training examples are in short supply. We have demonstrated the effectiveness of both method on random problems, as well as on bounded model checking and hardware verification problems. We also proposed a simple way to use such predictions to select between different SAT solvers. There are many directions for future work. For instance, we conjecture it may be effective to use these methods to select between very different types of solver. We are currently using LMP to select between a geometric restart strategy and Luby's restart scheme.

Acknowledgements

This paper contains work that is to appear in (Haim and Walsh 2008). In particular, the LMP method is described in (Haim and Walsh 2008). However, the extension of the WBE method to conflict driven solvers, along with all results comparing WBE and LMP to the Progress Bar for SAT Solvers are presented here for the first time.

The second author is funded by the Department of Broadband, Communications and the Digital Economy, and the Australian Research Council.

References

Een, N., and Sorensson, N. 2003. An extensible SAT-solver. *SAT* 2919:502–518.

Haim, S., and Walsh, T. 2008. Online Estimation of SAT Solving Runtime. *Lecture Notes in Computer Science*. In Press.

Horvitz, E.; Ruan, Y.; Gomes, C.; Kautz, H.; Selman, B.; and Chickering, M. 2001. A bayesian approach to tackling hard computational problems. *Proceedings the 17th Conference on Uncertainty in Artificial Intelligence (UAI-2001)*.

Kilby, P.; Slaney, J.; Thiébaux, S.; and Walsh, T. 2006. Estimating search tree size. In *Proceedings of the Twenty-First National Conference of Artificial Intelligence, AAAI*.

Knuth, D. 1975. Estimating the Efficiency of Backtrack Programs. *Mathematics of Computation* 29(129):121–136.

Kokotov, D., and Shlyakhter, I. 2000. Progress Bar for SAT Solvers. In *Unpublished manuscript, http://sdg. lcs. mit. edu/satsolvers/progressbar.html*.

Leyton-Brown, K.; Nudelman, E.; and Shoham, Y. 2002. Learning the empirical hardness of optimization problems: The case of combinatorial auctions. In *CP '02: Proceedings of the 8th International Conference on Principles and Practice of Constraint Programming*, 556–572. Springer-Verlag.

Samulowitz, H., and Memisevic, R. 2007. Learning to Solve QBF. *In proc. of 22nd Conference on Artificial Intelligence, AAAI*.

Xu, L.; Hutter, F.; Hoos, H.; and Leyton-Brown, K. 2007. SATzilla-07: The Design and Analysis of an Algorithm Portfolio for SAT. *Lecture Notes in Computer Science* 4741:712.

Xu, L.; Hoos, H. H.; and Leyton-Brown, K. 2007. Hierarchical hardness models for sat. In *Principles and Practice of Constraint Programming (CP-07)*, volume 4741 of *Lecture Notes in Computer Science*, 696–711. Springer-Verlag.