Negotiation Games and Conflict Resolution in Logical Semantics

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Abstract

The purpose of this paper is to explore the extent in which the idea of using negotiation games in tandem with the theory of semantic games is relevant to the concept of meaning in logical semantics. The need for such negotiations is argued to arise when some formulas are logically non-coherent, which in turn may take place because of conflicts between the players playing the associated non-strictly competitive semantic language-games on these formulas.

Introduction

For the purposes of this paper, I will take *semantics* (or meaning) in this two-tiered topic to be submitted principally to philosophical and logical analysis, while *negotiation* is a phenomenon amenable principally to a game-theoretic analysis.

I shall argue, however, that there are some interesting and largely unexplored connections between the two, especially from the viewpoint of logical semantics. Therefore, the key question that I would like to ask is: Where do these two analyses meet? The prima facie connection seems to be that the concept of meaning can be given a logical explication in terms of certain games, namely in terms of the theory of semantic games introduced by Hintikka in the early 1970s (Hintikka, 1973) and investigated in a number of publications since (Hintikka & Sandu, 1991; Hintikka & Sandu, 1997; Pietarinen & Sandu, 1999).

In these games, the meaning of a proposition is truth-conditional in the sense that to know what a proposition means it suffices to know when the proposition is true in a given model and when it is false in it. That is, games lend the logical constants of the language their meaning in terms of the truth-conditions they obtain. The truth (and likewise the falsity) of the whole sentence or formula is spelled out by a certain existence claim, namely an assertion concerning the existence of a winning strategy for one of the players of the semantic game.

More precisely, to use games for logical meaning is to evaluate logical formulas according to the game rules prompted by the logical ingredients encountered in the formula. The evaluation starts with the outermost one. A game $\mathcal{G}$ involves a player V (Verifier, ∃loise, Myself) and a player F (Falsifier, ∀belard, Nature). The purpose of F is to falsify the formula (i.e. show that it is false in the structure $\mathfrak{A}$) and the purpose of the player V is to verify it (i.e. show that it is true in the structure $\mathfrak{A}$). For the sake of simplicity, it can be assumed that a first-order language $\mathcal{L}_{\omega\omega}$ does not contain $\rightarrow$ or $\leftrightarrow$. Now the symbols $\forall$ and $\wedge$ prompt a move by F, and $\exists$ and $\vee$ prompt a move by V. When players come across negation, they change roles, and winning conventions will also change. Each move reduces the complexity of a formula and hence an atomic formula is finally reached. The truth-value of an atomic formula, as established by a given interpretation, determines which player wins a play of a game. For a standard first-order language $\mathcal{L}_{\omega\omega}$ with $\{\vee, \wedge, \sim, \exists, \forall\}$, a strictly competitive non-cooperative game $\mathcal{G}(\varphi, g, \mathfrak{A})$ with an assignment g restricted to free variables of φ is defined by induction on the complexity of each $\mathcal{L}_{\omega\omega}$-formula φ between V and F:

(G.∼) If $\varphi = \sim\psi$, V and F change roles, and the next choice is in $\mathcal{G}(\psi, g, \mathfrak{A})$.

(G.∨) If $\varphi = \theta \vee \psi$, V chooses either Left or Right, and the next choice is in $\mathcal{G}(\theta, g, \mathfrak{A})$ if Left, and in $\mathcal{G}(\psi, g, \mathfrak{A})$ if Right.

(G.∧) If $\varphi = \theta \wedge \psi$, F chooses either Left or Right, and the next choice is in $\mathcal{G}(\theta, g, \mathfrak{A})$ if Left, and in $\mathcal{G}(\psi, g, \mathfrak{A})$ if Right.

(G.∃) If $\varphi = \exists x\, \psi$, V chooses an individual of the domain of the structure $\mathfrak{A}$, and the next choice is in $\mathcal{G}(\psi, g \cup \{(x, a)\}, \mathfrak{A})$.

(G.∀) If $\varphi = \forall x\, \psi$, F chooses an individual of the domain of the structure $\mathfrak{A}$, and the next choice is in $\mathcal{G}(\psi, g \cup \{(x, a)\}, \mathfrak{A})$.

(G.atom) If φ is atomic, the game ends, and V wins if φ is true, and F wins if φ is false.

Strict competitiveness means that if V loses then F wins, and if F wins, then V loses. Non-cooperation roughly means that players decide the action they take alone. Rather than choosing subformulas, the rules for connectives say that players choose elements from a domain of elements split into two. Furthermore, a strategy for each player in a game

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$\mathcal{G}(\varphi, g, \mathfrak{A})$ is a complete rule telling at every contingency where a player is required to move what his or her choice is. A winning strategy is a set of strategies by which a player can make operational choices such that every play of the game results in a win for him or her, no matter how the opponent chooses. Let $\mathcal{G}(\varphi, g, \mathfrak{A})$ be a game for $\mathcal{L}_{\omega\omega}$-sentences φ, and f a strategy. Then

- $(\mathfrak{A}, g) \models \varphi$ if and only if a strategy f exists which is winning for V in $\mathcal{G}(\varphi, g, \mathfrak{A})$;
- $(\mathfrak{A}, g) \not\models \varphi$ if and only if a strategy f exists which is winning for F in $\mathcal{G}(\varphi, g, \mathfrak{A})$.

The game-theoretic notion of truth invokes the key notion of strategies, which can be viewed as Skolem functions. For example, if Sxy is atomic, then $(\mathfrak{A}, g) \models \forall x \exists y\, Sxy$, if and only if there exists a one-place function f such that for any individual chosen by F (say, a), $Saf(a)$ is true in $\mathfrak{A}$.

Abstract versus strategic meaning and its game-theoretic relevance

Likewise, a prima facie conundrum in the above prima facie pleasant connection between games and meaning negotiation (conceived in its pre-theoretic sense) is that the meaning-establishing semantic games appear to manifest radically different kinds of games compared to those of negotiation. The former are games for material truth, not necessarily played by actual language users. They are abstract games that, at least in a quasi-Wittgensteinian sense, aim at establishing a link between language and reality. In contrast, negotiation games are close relatives of dialogical or *dialogue games* customarily taken to be played by the actual users of language (speakers and hearers, or utterers and interpreters) by sequences of alternating attacks and defences. Unlike semantic games, dialogues aim at logical truth for validity of the expressions of one's language.

In dialogue games for logic, briefly, there are two participants, the Defender D and the Attacker A (or the Proponent and the Opponent). The former proposes and defends a claim while the latter challenges it. The moves are made according to logical and procedural rules. Informally, the logical rules are: Conjunction prompts an attack by A, and the conjuncts are then available for the defence. Disjunction is not really attacked, but just defended by D choosing one of the disjuncts for the defence. Negation is, like in semantic games, a signal to change roles. That is, a negated statement is attacked by defending the statement stripped off of the negation. An existential statement is a request for a witness produced by D, instantiated as the value of the quantified variable to serve as a claim to be defended in the future. Likewise, an attack on universal quantification asks for an individual produced by A, and the result of the instantiation will be the next challenge. Furthermore, D loses if the claim can no longer be defended, and A loses if the claim can no longer be attacked. As in semantic games, the key concept in dialogue logic is the existence of winning strategies, which prescribes when a formula will be valid. An analogous result to that of the theory of semantic games is that a first-order sentence S can be deduced from the set of first-order sentences Γ ($\Gamma \vdash S$) if and only if $\Gamma \vdash S$ is valid in intuitionistic logic. For the foundations and applications of dialogue games in logic and language, see e.g. (Felscher, 2002; Lorenzen & Lorenz, 1978; Carlson, 1983; Mann, 1988; Rahman & Rückert, 2001).

Yet there is a residue of meaning not exhausted by the abstract character of truth-conditional meaning. Perhaps the most perspicuous places where this leaks out are to be found in a number of natural language phenomena. These include anaphora and the resolution of anaphoric links, syntactic cues in language interpretation, and presumptive meaning in the speaker/interpreter interaction. This residue actually amounts to a wide range of linguistic phenomena. Syntactically dominated inference relations in interpreting anaphoric pronouns and other expressions, background information and common ground in the context of an utterance, or knowledge about preferences and information of other participants are all such notions that can be subsumed under the notion of meaning that is not captured by the mere existence claim concerning the winning strategies in the semantic language-game. It is called *strategic meaning* within the game-theoretic framework of (Hintikka, 1987). Some prefer to call it (semantic) pragmatics. It essentially is intended to refer to all kinds of things the winning strategies of the players of a semantic game actually contain, how they are composed and amended as the game goes on, and what the players' knowledge of them during the play of the game is.

At this point, it may seem that it is this strategic meaning rather than the abstract, truth-conditional meaning that can in some sense be negotiated. For people draw slightly different syntactic inferences from sentences admitting of different readings or understandings, and they draw different inferences even from those sentences that seem unambiguous. Language-users' background information differs, and their cognitive repertoire has been framed or schematised differently, which inevitable amounts to disagreements in establishing common ground and drawing proper conclusions in situations constrained by presumptive meaning of the sentence on the part of the interpreter. This is to some degree true if we think of strategic meaning as something that can be confined to an actual social setting of language use. However, semantic language-games in the quasi-Wittgensteinian sense are not interpersonal, societal games. They are abstract games of meaning that the speakers and interpreters of the language have to grasp even before they are able to grasp all the rules that constrain such games. One learns the game gradually, by recognising its rules in different contexts over a period of time. As the meaning of natural language sentences varies from one context to another, different language-games are being construed that reflect this contextual change.

In a similar vein, strategic meaning is not something that is established interpersonally, although interpersonal language use may play some role in it. Like abstract meaning, it pertains to the realm of gradually learnable Wittgensteinian language-games whose rules have to be learned by varying games in different settings. Hence in order to understand what strategic meaning is and to use it in a language-game the presence of actual interlocutors is not presupposed.

Some further historical and philosophical evidence for this can be found in the Peircean perspective to semantic language-games. What Peirce noted was that one can well say that the utterer and the interpreter of a statement can subside within one person or one mind: "Thinking always proceeds in the form of a dialogue, — a dialogue between different phases of the *ego*, — so that, being dialogical, it is essentially composed of signs, as its Matter, in the sense in which a game of chess has the chessmen for its matter" (Peirce, 1967, MS 298, p. 6). Peirce even connected this dialogical idea with the notion of logical truth: "'The duality of the *ego* and *non-ego* is the chief constituent of the idea of the Truth" (Peirce, 1967, MS 515, p. 24).

Games and competitiveness

Therefore, in order to see what the role of the idea of negotiation in logical meaning is we need to look deeper into the logical and game-theoretical theories. One idea to begin with is that as it is well known in game theory, negotiation in the sense of bargaining commodity is not a zero-sum game. If surplus exists, it is some process of negotiation that is evoked to take control over how it is to be divided. The motivation for such a process is simply that if agreement is not reached, no one will benefit from there being some surplus.

Approaching the related idea from a logical perspective, it can be first noted that not all games for logic are determined (determinacy means that one of the players has a winning strategy). A well known example where determinacy does not hold is a logic of imperfect information (Hintikka, 1996). And if determinacy fails, then the law of excluded middle will fail too. In another nomenclature, one would say that such logics are partial, that is, the semantics is such that the formulas may have a truth-value of Undefined, or lack any truth-value altogether. The important thing nonetheless is that instead of being classical and behaving in a contradictory manner, the negation sign in partial or imperfect information logics is game-theoretic in nature, which as stated in the previous game rules means that the negation implements the exchange of the two roles between the two players.

Yet if this much is the case, why is not the law of non-contradiction invalidated? The reason is that the games have previously been assumed to be *strictly competitive*, that is, both players cannot come out as the winners. This means that in any semantic game, there can be no winning strategies for both players. However, as noted this holds only if the class of games is restricted to contain only those games that are strictly competitive. Yet it is perfectly feasible to relax this assumption and take some semantic games to be *non-strictly competitive*. This now means that the following no longer holds: If there exists a strategy f that is winning for the Verifier then there does not exist a strategy g that is winning for the Falsifier, and if there exists a strategy g that is winning for the Falsifier then there does not exist a strategy f that is winning for the Verifier.

Before proceeding, let us assume that semantic language-games are in their extensive form in the sense of the theory of games. In brief, extensive games capture the sequential structure of players' strategic decision problems. They can be represented as (finite) trees with histories (decision nodes) $h \in H$ and actions $a \in A$ labelling the edges departing from non-terminal histories. The game starts at the root of the tree and ends at the terminal histories in $Z \subseteq H$. At each non-terminal history k, a player has to make a decision as to what to choose. The outcome of this decision in a particular play of the game is a choice, and the set of all choices from a history determines a move. As related to logic, non-terminal histories of extensive games are labelled by non-atomic subformulas of a given formula φ, where φ is labelled at the root, and the terminal histories are labelled by the atomic formulas of φ. The terminal histories are also mapped to the outcome of the game by a payoff function $u_i(h), i \in \{V, F\}$ that assigns to each $k \in Z$ a value in $\{1, -1\}$ (see (Sandu & Pietarinen, 2002) for details, and (Pietarinen, 2002b) for some arguments as to why extensive games may after all be somewhat insufficient).

If the game is not strictly competitive, call it non-strictly competitive. In non-strictly competitive games, it may happen that there exist a winning strategy for both players. To implement this, one can for instance stipulate that there may be some terminal histories in Z that are winning for both players. This can be denoted game-theoretically by the payoff $u_i(h)$ that outputs the matrix $(1, 1)$ for some $h \in Z$, in addition to zero-sum matrices $(1, -1)$ or $(-1, 1)$. Consequently, given a literal p, it will be interpreted so that it has both the truth-value True and the truth-value False, and hence will have a truth-value of Over-defined.

The abundance of the classes of non-strictly competitive games in the game-theoretic literature suggests that in logic where game-theoretic concepts are quite commonplace, one should not rule such classes of games out offhand. Just as physical instances of games can be used as evidence for the claim that games in logic may encompass imperfect information and failures of the law of excluded middle, the other basic class of games in logic will spring into existence.[1] The other effect is that non-strict games may be determined even if their strict counterparts are not, and non-determinacy can in that case be restored only by introducing partial models into the language, see (Pietarinen, 2002a).

Logical conflict resolution by negotiation

But it is precisely here where the main thrust of this paper comes in. For if there are nonzero-sum payoffs that suggest some sensible interpretation of the "division of surplus", and if the terminal histories where these payoffs reside can be reached with a positive probability, there will be inevitably be a potential conflict between the players. In that case the commodity needs to be divided, which in logical terms means that the truth (and likewise the falsity) of the sentences of the underlying logic are unevenly agreed upon. Nonetheless, it is a *potential conflict* or potential non-coherence because it is not yet stated that there actually exist winning strategies for both the Verifier and the Falsifier

[1] See (Wright, 2000) for some vibrant stories concerning the role of non-zero-sum games in our societies, science and everyday life.

of the formula. But it already runs a risk of becoming an *actual conflict* or actual contradiction if the existence claim of abstract meaning referring to both participants comes to pass.

The difference between these two notions of conflict is that in case of potential conflicts, the only formulas giving rise to non-coherence are those composed of atomic constituents and their negations. If there are nonzero-sum payoffs and winning strategies in the non-strictly competitive semantic game exist for both players, the potential contradictions transmit to complex sentences and become actualised.

So let us then suppose that actual non-coherencies come to pass. That is, there are formulas of the form $S\wedge \sim S$, where S are non-atomic and "$\sim$" is game-theoretic negation. Surely this does not seem to be a welcomed feature of any logic. What I suggest is to call on a "negotiation process" in order to resolve this kind of conflict.

But here some fundamental questions concerning the resolution of contradictory statements in logic arise. For what in fact are the kind of negotiations that could be carried out in the game-theoretic framework concerning the meanings of a complex contradictory formula? Why should we not make our life easier and pre-empt such negotiations by sticking to the class of strictly competitive games in the first place? Further still, who plays the negotiation games? What are the basic characteristics of such games, and what, if anything, they have to do with semantic games?

To outline some partial answers to these fundamental questions (see (Pietarinen, 2002a) for further discussion), the commodity is over truth-values of complex sentences. Hence the conflict would be resolved even if we do not try to dispense with the nonzero-sum payoffs of atomic predicates (that is, with the statements "Myself wins at a terminal history" and "Nature wins at a terminal history"), provided that we would be able to dispense with non-strict winning strategies. It is nevertheless not clear how it is possible to dispense with the notion of existence of such strategies, because it is an objective property of the reality or the model in question, not pertaining to any epistemological questions of coming to know what such strategies are.

It would nonetheless be tempting to conclude that logical non-coherence may arise because of some cognitive or epistemological limitations, such as players' imperfect knowledge of the model or noisy information transmission between the parties. However, these informational limitations, at least as far as semantic language-games are concerned, give rise to partiality of the underlying logic rather than non-coherence, which of course may happen even if the language otherwise is completely interpreted. Admittedly, it is technically perfectly possible to map "over-defined truth values" to "truth-value gaps" (Langholm, 1988), but this is merely a technical roundabout tactics for substituting non-coherence in favour of partiality.

There is an alternative avenue that turns out to be vital in resolving logical conflicts of contradictory formulas. For we can take there being a negotiation process between the two participants involved in the dialogical interaction, or in the semantic game of the underlying language. We can take the *negotiation game* to be modelled by an alternating sequence of actions consisting of players' acceptances and rejections (see (Rubinstein, 1998) for a basic negotiation model in terms of alternating offers). More precisely, let $\mathcal{G}$ be a semantic game in its extensive form. Let X be a compact connected subset of a Euclidean space (that is, the set of agreements). The set of histories H of $\mathcal{G}$ is then one of the following six types ($a^j \in X, j \in \mathbb{N}$): (a) $\emptyset$ or $(a^0, R, a^1, R \ldots a^i, R)$; (b) $(a^0, R, a^1, R \ldots a^i)$; (c) $(a^0, R, a^1, R \ldots a^i, A)$; (d) $(a^0, R, a^1, R \ldots)$; (e) $(a^0, R, C, a^1, R, C \ldots a^i, R)$; (f) $(a^0, R, C, a^1, R, C \ldots a^i, R, B)$. A is taken to mean "accept", R "reject", C "continue" and B "negotiation breakdown". The set X may be viewed as a feasible division of a pie, and the set D denotes a "no agreement" situation.

In this game, the Verifier and the Falsifier make alternating offers according to some schedule of integers $I \in \mathbb{N}$. The first move in the schedule takes place when the first player in the team of either the Verifiers or the Falsifiers makes an offer in X, and the first player in the adversary team chooses either A or R. (The terminology "teams" here is just a generalisation of semantic games to accommodate imperfect information, see (Pietarinen & Sandu, 1999). If the game is one of perfect information, it is the same member of the team of one player who gets to choose repeatedly.) If the choice is A, the game ends, and if it is R, then the schedule moves to the next integer. This negotiation then repeats.

The game has infinite horizon, and if there is no acceptance the outcome is bound to be D (disagreement). Since there is a risk of breakdown, after each period $0 \ldots t, t \in \mathbb{N}$ there is a chance move that ends the game with a probability in $(0, 1)$.

We do not need to incorporate any specific notion of offers into this model. It suffices that the parties either stick to or throw away those winning strategies that lead to conflicting situations.

Furthermore, the Nash solution and other solution concepts are known only if $\mathcal{G}$ is a game of perfect information. It is thus an open question as to how such solution concepts that try to account for the players' beliefs and information under uncertainty (i.e. sequential equilibria) could be incorporated into the kind of negotiation games outlined here.[2]

Therefore, negotiation games are parasitic on semantic games for logics that may admit of nonzero-sum payoffs to be the interpretations of some atomic propositions. Moreover, the terminal histories where these payoffs are to be found have to be reachable by winning strategies in a non-strictly competitive semantic language-game. The essential import is that contradictory constituents may be passable in logic provided that not both players' winning strategies could lead to them. If that nonetheless is the case, then some such winning strategies have to be negotiated away.

[2]This does not affect the possibility of there being semantic games for a logic that are of imperfect information, because the outstanding question about the solution concepts concerns the negotiation phase of the overall game that takes place after the semantic game for sentence meaning has been completed to the point of revealing the existence of potential non-coherencies.

This latter phase in the overall language-game for meaning is not a version of any dialogue game between the Utterers and the Interpreters of language — contrary to the suggestions given in (Hulstijn, 2000), but an abstract language-game of conflict resolution where non-coherence results not from there being any contradictory game rules, but from strategic considerations in games that are members of a novel, non-strictly competitive class of semantic games among the totality of language-games. If a catch phrase is needed, we can say that negotiation in semantic sense aims at pulling up corrupt links between language and reality.

Wittgensteinian perspective to contradictions and language-games

Even Wittgenstein would have been satisfied by the kind of strategic outlook on contradictions outlined above. For the previous observations can be pushed further and complemented with yet another, and as far as I know, not previously noted but nonetheless fairly overt character of Wittgenstein's language-games, namely the potential competitiveness character in his conception of language-games. The places where this character can be found and contrasted with non-strictly opposed semantic games are where Wittgenstein puts emphasis on the "civil" nature of strategies needed in language-games (Wittgenstein, 1953, p. 125):

> We lay down rules, a technique, for a game, and that then when we follow the rules, things do not turn out as we assumed. That we are therefore as it were entangled in our rules. [...] It throws light on our concept of *meaning* something. For in those cases things turn out otherwise than we had meant, foreseen. That is just what we say when, for example, a contradiction appears: "I didn't mean it like that." The civil status of a contradiction, or its status in civil life: there is the philosophical problem.

An important thing to note here is that there does not have to be anything inconsistent in the rules of the language-game in order for one to end up with non-coherent formulas where both participants can claim success in their own purposes. Yet the great deal of the recent discussion concerning Wittgenstein's views on contradictories as a result of his way of setting up language-games presupposes that contradictories should somehow be the end products of contradictory game *rules* (see e.g. (Goldstein, 1989) for this line of argumentation). Such an assumption is unfounded and actually false, however, as already shown by the possibility of there being semantic language-games with characteristics different from the ordinary ones, resulting in inconsistencies simply by changing the class of games in question. Moreover, a swift refutation of the possibility of contradictory game rules comes from Wittgenstein himself: "Why may not the rules contradict each other? Because otherwise they wouldn't be rules" (Wittgenstein, 1978, p. 305).

In case of contradictories, the primacy of strategic thinking in language-games over and above focussing on the defining rules of the game is strongly emphasised by Wittgenstein also in the context of the foundations of mathematics. In attempting to assess the role of inconsistencies in mathematics, there would have been little point for Wittgenstein to try to argue that whenever faced with contradictories, different kinds of rules should be accused for such unfortunate cases. Hence, if by language-games we can see how some contradictory meanings of some mathematical statement or even of some natural language formulation may arise, it is by such games only that any inconsistency can take place in logical semantics.

Further remarks concerning some examples of real games resembling linguistic patterns characterised by their winning, losing, or competitiveness conditions, can be found in (Wittgenstein, 2000, item 226, p. 48).

Conclusion

It was argued that in case of there being certain non-coherencies in logic, semantic games for logical meaning need to be supplied with some sensible method of conflict resolution. It is possible to approach such resolution game-theoretically, one useful example being negotiation games with alternating offers.

Several further remarks and implications can be noted:

- Non-coherence in logic is related to paraconsistency, but unlike paraconsistent logics, in semantic games we have a theoretically motivated semantic reason for having it. Further, the one-place operation that functions as the negation in logic, being game-theoretic, is independently motivated genuine negative operator. After a fashion, our approach will answer to the old Jaŝkowski's problem (Jaśkowski, 1948), which says that any logic claiming the name of a paraconsistent one needs to satisfy three conditions. First, such a logic has to come with a one-place operator leading to a decent paraconsistent system (that is, to an inconsistent but non-explosive one). Second, its negation must be strong enough to be called negation. Thirdly, its semantics needs to be well motivated. The previous remarks are calculated to provide pointers to a comprehensive answer to this problem. For games are well-motivated and systematic for logical semantics, even for those that are non-classical. What is more, the previous attempts to solve this problem by paraconsistent systems in the literature remain to be based on some negative criteria — for example, they describe principles that must be rejected, such as ex falso, consistency, or triviality. Yet the game-theoretically defined negation is a genuine negation, what can be observed for example from the relation it has with negative constructions and negative operators in natural language (Pietarinen, 2002a).
- No dialetheic interpretation of contradictories is suggested, however. It does not make sense to assert $S \wedge \neg S$, where "$\neg$" is classical, not game-theoretic negation. For this would be an assertion about the game, saying that "there exists a winning strategy for V and there does not exist a winning strategy for F". Because of the game-theoretic behaviour of negation, it is not to be expected that the ensuing systems would yield to something like preservationist paraconsistent logic either.
- One question that has not been considered here concerns cooperation versus non-cooperation: In the negotiation

phase, does the adversary cooperate with us? One is reminded of the dictum that "Nature does not cooperate with us". This is no hindrance in developing the kind of game-theoretic models given above, however.

- Negotiation games are related to so-called cut-and-choose games that have been used in mathematical logic and computation (Hirsch & Hodkinson, 1997), and machine learning. In such games the Falsifier cuts by picking some element or a subset of elements, and the Verifier makes choices by accepting (repairing) the cut or refuting it. In learning environments, the game setting can be taken to consist of the Learner who is given random examples according to some probability distribution not known to her. The examples are shattered in a space, and her task is to select a hypothesis that would classify with high probability the new examples in a space under the same probability distribution. The other player is the Answerer whose task is to produce answers to the Learner's questions concerning the expected values of the distances within some tolerance rank. Since tolerance may encode noise in a learning environment, the Answerer has a power to control the amount of disturbance to the Learner. This can be seen as a game where 'You choose an error ϵ and I choose a confidence δ' (note the commonality with the $\epsilon - \delta$ definition of continuity in calculus).
- Non-coherent logics in the game-theoretic sense are in turn related to Peirce's understanding of vagueness and the ubiquity of vague signs in logic and his theory of semeiotics.

Looking away from the theory of semantic games, in classical game theory the idea of a negotiation has been taken to enjoy strong social connotations not only in the sense that negotiations take place among groups of actual participants in actual environments, but in the sense that it is typically assumed to be rational for actual players to resort to things like posturing, information concealing, exaggeration, threat, and deception in trying to achieve their goals. On the face of it, this seems far removed from the goals of meaning of expressions of one's language in its truth-conditional sense. For surely there are no such things in the medium that links language to the reality it aims to describe, one might argue. Yet how do we know that Nature does not deceive us? And if she does, why would I not engage in similar activities? As argued here, games for logic may be in a genuine need of some basic principles of negotiation between the contestants, and we just do not know yet what kind of complexities such principles might lead us to in the future.

References

Carlson, L., 1983. *Dialogue Games: An Approach to Discourse Analysis*, Dordrecht: Reidel.

Felscher, W., 2002. Dialogues as a foundation for intuitionistic logic, 115–146 in Gabbay, D. and Guenthner, F. (eds) *Handbook of Philosophical Logic* 5, (2nd ed.), Dordrecht, Kluwer.

Goldstein, L., 1989. Wittgenstein and Paraconsistency, in G. Priest, F.R. Routley and J. Norman (eds.), *Paraconsistent Logic. Essays on the Inconsistent*, Munich: Philosophia Verlag, 540–562.

Hintikka, J., 1973. *Logic, Language Games and Information*, Oxford: Oxford University Press.

Hintikka, J., 1987. Language understanding and strategic meaning, *Synthese* 73, 497–529.

Hintikka, J., 1996. *The Principles of Mathematics Revisited*, New York: Cambridge University Press.

Hintikka, J. and Sandu, G., 1991. *On the Methodology of Linguistics*, Oxford: Blackwell.

Hintikka, J. and Sandu, G., 1997. Game-theoretical semantics, in J. van Benthem and A. ter Meulen (eds), *Handbook of Logic and Language*, Amsterdam: Elsevier, 361–410.

Hirsch, R. and Hodkinson, I., 1997. Games in algebraic logic, in Dekker, P. et al., (eds.), *Proceedings of the Eleventh Amsterdam Colloquium*, Amsterdam: University of Amsterdam.

Hulstijn, J., 2000. Dialogue models for inquiry and transaction, Doctoral dissertation, University of Twente.

Jaśkowski, S., 1948. Rachunek zdań dla systemów dedukcyjnych sprzecznych, *Studia Societatis Scientiarum Toruensis* 5, 55–77. (Translated in English as: Propositional Calculus for Contradictory Deductive Systems, *Studia Logica* 24, 1969, 143–157.)

Langholm, T., 1988. *Partiality, Truth and Persistence*, Stanford: CSLI Publications.

Lorenzen, P. and Lorenz, K., 1978. *Dialogische Logic*, Darmstadt: Wissenschaftliche Buchgesellschaft.

Mann, W.C., 1988. Dialogue games: conventions of human interaction, *Argumentation* 2, 511–532.

Peirce, C.S., 1967. Manuscripts in the Houghton Library of Harvard University, as identified by Richard Robin, *Annotated Catalogue of the Papers of Charles S. Peirce* (Amherst: University of Massachusettes Press, 1967), and in 'The Peirce Papers: A supplementary catalogue', *Transactions of the C.S. Peirce Society* 7, 1971, 37–57.

Pietarinen, A., 2002a. Logic and coherence in the light of competitive games, to appear in *Logique et Analyse*.

Pietarinen, A., 2002b. A note on the structural notion of information in extensive-form games, to appear in *Quality & Quantity*.

Pietarinen, A. and Sandu, G., 1999. Games in philosophical logic, *Nordic Journal of Philosophical Logic* 4, 143–173.

Rahman, S. and Rückert, H., 2001. Dialogical connexive logic, *Synthese* 127, 105–139.

Rubinstein, A., 1998. *Modeling Bounded Rationality*, Cambridge, Mass.: MIT Press.

Sandu, G. and Pietarinen, A., 2001. Partiality and games: propositional logic, *Logic Journal of the IGPL* 9, 107–127.

Sandu, G. and Pietarinen, A., 2002. Informationally independent connectives, to appear in I. van Loon, G. Mints and R. Muskens (eds), *Logic, Language and Computation*, 9, Stanford: CSLI Publications.

Wittgenstein, L., 1953. *Philosophical Investigations*, (third edition 1967), Oxford: Blackwell.

Wittgenstein, L., 1978. *Philosophical Grammar*, Columbia: University of California Press.

Wittgenstein, L., 2000. *Wittgenstein's Nachlass, The Bergen Electronic Edition (Diplomatic Transcription)*, The Wittgenstein Trustees, The University of Bergen, Oxford University Press.

Wright, R., 2000. *Nonzero: The Logic of Human Destiny*, New York: Pantheon Books.