A Limitation of the Generalized Vickrey Auction in Electronic Commerce : Robustness against False-name Bids

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Introduction

The Internet provides an excellent infrastructure for executing much cheaper auctions with more sellers and buyers from around the world. However, we must consider the possibilities of new types of cheating, i.e., an agent may try to profit by submitting a false bid under a fictitious name (*false-name bid*). Such an action is very difficult to detect since identifying individual agents on the Internet is virtually impossible. As far as the authors know, the problem of false-name bids has not been previously addressed. Compared with the collusion problem that many researchers have discussed, a false-name bid is easier to execute. It can be done alone, while in collusion, a bidder has to seek out and persuade other bidders to join in the fraud.

In this paper, we examine the robustness of the Generalized Vickrey Auction (G.V.A.) against false-name bids. The G.V.A. is a generalized version of the well-known, widely advocated Vickrey auction, and it has proved to be incentive compatible, Pareto efficient, and individual rational, when no agents submit false-name bids.

In this paper, we first introduce some preliminaries and describe the G.V.A. protocol. Next, we describe settings where the G.V.A. is vulnerable against false-name bids. We then prove that there exists no single-round, sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.

Preliminaries

This paper will concentrate on private value auctions. In private value auctions, each agent knows its own preference and values goods independently of other agents. Although an agent's valuation may correlate with other agents' valuations, this restriction is reasonable for making a tractable analysis.

We primarily judge auction protocols based on whether these protocols fulfill incentive compatibility, Pareto efficiency, and individual rationality. In computational settings, agents can deal with huge amounts of data and infer other agents' preferences from the results of their bids. When knowing the preferences of others is profitable, each agent tends to waste its resources in order to keep its own preference secret and try to obtain the preferences of others. This situation can be avoided if an auction is incentive compatible.

The Generalized Vickrey Auction

Protocol

The G.V.A. is based on the Clarke-Groves mechanism, which induces each agent to tell the true value of public goods (Varian 1995). The G.V.A. protocol can be applied to various auctions, including auctions for multiple items with interdependent values, which are useful for auctions among computational agents.

The G.V.A. protocol is as follows: (1) Each agent i declares a valuation function $v_i(G)$ for the allocation G. (2) The G.V.A. chooses an optimal allocation G^* that maximizes the sum of all the agents' declared valuations, and then the G.V.A. announces the winners and their payment $p_i = \sum_{j \neq i} v_j(G^*_{\sim i}) - \sum_{j \neq i} v_j(G^*)$ ($G^*_{\sim i}$ is the allocation that maximizes the sum of all agents' valuations other than agent i's valuation).

A winner's payment on the allocated goods is considered to be the decrease in the sum of all the agents' utilities except for the winner's utility that results from the winner's participation. Therefore, the G.V.A. satisfies incentive compatibility.

Robustness of the G.V.A. for Multiple Units of a Single Item and Multiple Requirements of Agents

Vulnerable Example

Suppose that two agents denoted by agent 1 and agent 2 are bidding for two units of a single item.

- Agent 1's bid: ($6, $6)
  Agent 1 bids $6 for the first unit and $6 for the second unit, a total of $12 for both units.
- Agent 2's bid: ($3, $5)

The G.V.A. allocates the two units to agent 1. Agent 1 pays $8 and its utility is $12 − $8 = $4.

Now, suppose that agent 1 submits a bid (\$6, \$0) and a false-name bid (\$6, \$0) using the identity of agent 3.

- Agent 1's bid: (\$6, \$0)
- Agent 2's bid: (\$3, \$5)
- Agent 3's bid: (\$6, \$0)

The G.V.A. allocates a single unit to agent 1 and a single unit to agent 3. Agent 1's payment is \$9 − \$6 = \$3 and agent 3's payment is \$3. It turns out that agent 1 can get both units and its utility is \$6, since agent 3 is a fictitious name of agent 1.

The difference between agent 1's utility for a false-name bid and a truthful bid is \$2. Thus, submitting a false-name bid is profitable for agent 1.

Marginal utility

Our investigation found that the robustness of the G.V.A. depends on the marginal utility of a single item. The marginal utility of a single item means an increase in the agent's utility as a result of obtaining one additional unit[1] The following theorem shows one sufficient condition where the G.V.A. is robust against false-name bids.

Theorem 1 *The G.V.A. is robust, i.e., submitting false-name bids is not profitable, if the declared marginal utility of each agent is constant/diminishes.*

The proof appears in the full paper (Sakurai, Yokoo, & Matsubara 1999).

Non-Existence of Desirable Protocols

The next question is whether any auction protocol exists that is robust against false-name bids or not.

Theorem 2 *In auctions for multiple units of a single item and multiple requirements of agents, there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.*

Proof It is sufficient to prove just one instance in which no auction protocol satisfies the prerequisites.

We suppose that there are two units of a single item and that three agents are denoted by agent 1, agent 2, and agent 3.

- Agent 1's bid: $(a, 0)$
- Agent 2's bid: (b, a)
- Agent 3's bid: $(a, 0)$

We assume $a > b$. According to Pareto efficiency, agent 1 and agent 3 get one unit. Let P_a denote the payment of agent 1.

[1]One instance of goods in which the marginal utility diminishes is a book. One example where the marginal utility increases is in an all-or-nothing in which an agent needs a certain number of units, or the good is useless (one sock, etc.)

When agent 2 and agent 3 reveal their true valuations and if agent 1 submits a bid $a' = b + \epsilon$, then the allocation does not change. Let $P_{a'}$ denote agent 1's payment in this situation. According to individual rationality, the inequality $P_{a'} \leq a'$ should hold. Furthermore, according to incentive compatibility, $P_a \leq P_{a'}$ should hold. These assumptions lead to $P_a \leq b + \epsilon$. The conditions for agent 3's payment are identical to the conditions for agent 1's payment.

Next, we assume another case with two agents denoted by agent 1 and agent 2.

- Agent 1's bid: (a, a)
- Agent 2's bid: (b, a)

According to Pareto efficiency, the two units go to agent 1. Let us denote the payment of agent 1 $P_{(a,a)}$. If agent 1 submits a false-name bid using the identity of agent 3, the same result as in the previous case can be obtained. According to incentive compatibility, the following inequality must hold, otherwise agent 1 can profit by submitting a false-name bid: $P_{(a,a)} \leq 2 \times P_a \leq 2b + 2\epsilon$.

On the other hand, let us consider the case when there are two agents.

- Agent 1's bid: (c, c)
- Agent 2's bid: (b, a)

Let us assume $b + \epsilon < c < a$ and $a + b > 2c$. According to Pareto efficiency, the two units go to agent 2. Thus, agent 1 cannot gain any utility. However, if agent 1 replaces the bid (c, c) with (a, a), both units go to agent 1 and the payment is $P_{(a,a)} \leq 2b + 2\epsilon$, which is smaller than $2c$, i.e., agent 1's true value of these two units. Therefore, agent 1 can increase its utility by submitting a false bid (over-bidding a true valuation).2

We also obtained similar results when the G.V.A. was applied to multiple auctions (Sakurai, Yokoo, & Matsubara 1999).

Conclusions

We obtained a negative result on the problem of false-name bids. However, there are many situations where obtaining an optimal allocation is not necessary. In such situations, it is enough to design an auction mechanism that simultaneously satisfies individual rationality and incentive compatibility. Our future goal is to find an auction mechanism that can obtain reasonably good (even if not Pareto efficient) allocations.

References

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