# Auctions without Common Knowledge

(Extended Abstract)

Sviatosalv Brainov, Tuomas Sandholm[1]

Computer Science Department
Washington University
One Brookings Drive
St. Louis, MO 63130
{brainov, sandholm}@cs.wustl.edu

Most theoretical results on optimal auction design draw crucially on the revenue equivalence theorem (Vickrey, 1961). According to the theorem, the first-price sealed bid, second-price sealed bid, English and Dutch auctions are all optimal selling mechanisms provided that they are supplemented by an optimally set reserve price. The revenue equivalence theorem is based on the following assumptions: the bidders are risk neutral, payment is a function of bids alone, the auction is regarded in isolation of other auctions, the bidders' private valuations are independently and identically distributed random variables, every bidder knows only his own valuation and is uncertain about the other agents' valuations and there is common knowledge about the valuations' distribution. In this context common knowledge means that everybody knows the common prior distribution from where valuations are drawn, everybody knows that everybody knows, etc., ad infinitum.

In the paper (Brainov and Sandholm, 1999) the common knowledge assumption about prior beliefs is dropped, but all other classic assumptions are kept intact. In particular, the assumption that the agents' valuations are drawn from the same prior is kept. It is shown that without common knowledge the revenue equivalence theorem ceases to hold. The failure of revenue equivalence has significant practical importance since different auction forms lead to different expected revenues to the auctioneer.

In order to prove the failure of the revenue equivalence theorem, a simple auction setting is considered. The setting includes two risk-neutral bidders in an isolated auction for a single indivisible object. Each bidder knows his own valuation, but is uncertain about his rival's valuation. We assume that valuations are independent and that there exists some objective distribution from which valuations are drawn.

The analysis of optimal bidding in such auctions is usually conducted using the Nash equilibrium solution concept from noncooperative game theory (Nash, 1951), or a refinement thereof. In such an equilibrium, each agent bids in a way that is a best response to the other agents' bidding strategies. However, the Nash equilibrium solution concept relies heavily on the common knowledge assumption. Up to now there has been no satisfactory equilibrium concept for games without common knowledge. One cannot derive the optimal bids in the first-price auction without such a solution concept. Therefore, one cannot calculate the expected utility of the bidders either. Thus, we need a solution concept for an auction game without common knowledge.

In the paper we convert the auction game described above to a Bayesian decision problem with an infinite hierarchy of beliefs. We propose a solution to a such Bayesian decision problem. The solution is a generalization of the solution of Tan and Werlang (1988) and can be applied to finite as well as to infinite belief trees. The solution coincides with the standard Bayesian solution for finite trees and for trees representing common knowledge.

With each infinite belief tree we associate a strategy labeling that tells what the decision maker would do at each vertex of the belief tree if he were there. We identify a special class of strategy labelings, namely, balanced strategy labelings. The strategy labeling is balanced if the strategy associated with each vertex is a best response to the strategies associated with the successor vertices.

The notion of balanced strategy labeling serves as a solution concept for a Bayesian decision problem based on an infinite belief hierarchy. The concept of balanced strategy labeling preserves the central principle of consistency in the sense of Hammond (1988). The central principle of consistency says that the decision maker's decision at a vertex in a tree should depend only on the part of the tree that originates at that vertex. The central principle of consistency justifies the frequently used technique of backward (bottom-up) induction (recursion). The concept of balanced strategy labeling generalizes the backward induction to the case of infinite trees. If we have derived a strategy labeling for some level of a tree we can "cut" the belief hierarchy at that level and apply backward (bottom-up) induction starting

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from the cutting level. By doing so we do not lose any strategically relevant information, since the concept of balanced labeling guarantees that the strategies along the cutting line convey all the relevant information belonging to the infinite part of the tree.

With the help of the concept of balanced strategy labeling we obtain the following result.

**Theorem.** When there does not exist common knowledge about private beliefs, the revenue equivalence theorem ceases to hold, i.e., the bidder's expected utility is different for the different types of auctions.

Therefore, without common knowledge about prior beliefs the fundamental revenue equivalence theorem ceases to hold. The failure of the revenue equivalence theorem has significant practical importance. Since different auctions yield different revenues, auction designers should be careful when choosing auction rules. This opens promising prospects for comparative analysis of different auction forms using the solution concept presented in this paper.

Our approach is related to the work of Gmytrasiewicz, Durfee and Vidal (Gmytrasiewicz and Durfee, 1995; Vidal and Durfee, 1996). They presented a solution method based on finite hierarchies of beliefs. The recursive modeling method is based on the assumption that once an agent has run out of information his belief hierarchy can be cut at the point where there is no sufficient information. At the point of cutting, absence of information is represented with a uniform distribution over the space of all possible strategies. The beliefs of order higher than the order of cutting are ignored. This approach, however, cannot be applied for rational agents with perfect reasoning abilities. We cannot prohibit such agents from forming higher-order beliefs by applying a uniform distribution whenever there is no sufficient information. Once an agent has run out of information at some level of beliefs, he has also run out of information for higher-order beliefs while continuing to model further the belief tree. Unlike the method of Gmytrasiewicz, Durfee and Vidal, our method allows such extended modeling by applying a decision-making procedure based on infinite hierarchies of beliefs, and leads to different results. Put together, their method approximates an infinite belief tree by a finite one while our method solves the infinite tree via a finite tree without resorting to approximation.

The solution concept presented in the paper can be applied to any game based on infinite belief hierarchies. For auction games it can serve as a theoretical tool for analyzing expected revenue of alternative auction forms. Future work includes characterizing properties of infinite belief trees that guarantee that the solution exists and is unique.

## References


Brainov S. and Sandholm T. 1999. Auctions Without Common Knowledge, Technical Report 99-05, Computer Science Department, Washington University, St. Louis.

Hammond P. 1988. Consequentialist foundation for expected utility. *Theory and Decision*, 25:25--78.

Gmytrasiewicz P., Durfee E. 1995. A rigorous, operational; formalization of recursive modeling. *Proceedings of the First International Conference on Multi-Agent Systems*, pages 125--132.

Nash J. 1951. Noncooperative games. *Annals of Mathematics*, 54:28--295.

Tan T., Werlang S. 1988. The Bayesian foundations of solution concepts of games. *Journal of Economic Theory*, 45:370--391.

Vickrey W. 1961. Counterspeculation, auctions and competitive sealed tenders. *Journal of Finance*,16:8--37.

Vidal J., Durfee E. 1996. The impact of nested agent models in an information economy. *Proceedings of the First International Conference on Multi-Agent Systems*, pages 377—384.