Geographical constraints revisited through the basic relations of Contact, Share and Order

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ABSTRACT:

In the framework of relation algebra and their use to represent space-time constraints, we present a constructive approach of a lattice of "topological constraints algebras" based upon three basic relations: contact, share and order, as tools to incrementally improve "discernibility".
This scheme shows the articulations between Allen and RCC algebras, as well with intermediate and smaller algebras with 2, 3, 5 or 7 relations. Tractability values are estimated through the cardinality of convex and Ord-Horn classes for each of these algebras.
We will discuss how models of geographic objects can be related to these algebras, in particular: notions of partitioning (districting) and of territory (no crisp borders). The main and original idea behind, is to exhibit a range of structures which enables us to choose the smaller that fit the model of objects associated to the application. For GIS applications, several extensions are proposed, to help the choice for some specific spatial problems: nested partitioning, river basin, cadastre changes.

KEYWORDS: Relation Algebra, Allen's and RCC's relations, GIS, Geographic Information, constraints classes (convex, ord-Horn).

INTRODUCTION

The GIS know how to represent spatial objects as far as they are "mappables", i.e. defined by co-ordinates lists. Hence, the geographic knowledge is often limited to its numerical representation and constraints are merely those which are 'deducible' from the geometry.
The work of J.F. Allen, as published in 1983 ("Maintaining Knowledge about Temporal Intervals") founded a symbolical approach of time without numerical knowledge on interval ends. Several years after, the space received the same kind of attention.

The first section starts by recalling how the algebra of binary relations, with transposition and composition operators, provide a formal frame to Allen "temporal" relations, and how a similar work on spatial relations was tackled with the "mereology" and its associated axioms. Today people refer mostly to the 8 RCC relations and the 13 Allen's. Then it sketches an incremental bottom-up reading of the different topological constraints algebras. This approach shows the formal unity of this construction based on basic binary relations: equivalence, contact, parts, order.

We then build a 3 dimensional lattice of these algebraic structures, using contact, share and order as 'directions' and we show how it makes sense with their connections. The corresponding axioms systems show exactly what is required at each step.

The third section deals with the use of constraints with actual and present GIS.
Most of the time, spatial (spatio-temporal) objects have an explicit link to the geographic space (Earth co-ordinates) or an implicit link, like a pixel in a satellite image. This is very useful, even mandatory, to display them on a screen or map sheet. But many questions may be asked to them, which do not require the underlying co-ordinates. Likewise, a lot of geographic information is collected merely in terms of constraints, like "in town", "up-hill" ...
These are two major justifications for the explicit introduction of constraints into a GIS model.
We illustrate their use with several examples, from application domains of GIS, and in each case, we show how the lattice of constraints algebra can help us to choose the most adequate one.

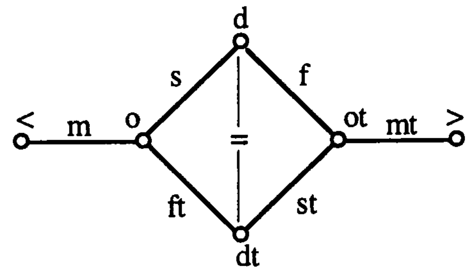
1. Allen's and RCC's algebras. Incremental approach of Spatio-Temporal Constraints Algebras

1.1. The theory of time intervals, from Allen.

What we pick out of the work of [Allen-83], is the mere topological approach of time, which releases intervals from their ends co-ordinates. Allen noted that these co-ordinates aren't necessary to the system he describes as follows: "an interval structure $(I,<,\subseteq)$ consists in a non empty set I and two binary relations < (precedence) and $\subseteq$ (inclusion) on I."
It has been noticed that this "Allen structure", is purely axiomatic and is not specific to "time".

Let's represent the 13 relations by the topological graph:

- the arcs of this graph are "punctual" relations
- the nodes are "general" relations
- the central lozenge represents the "equality" (equivalence relation connected to 4 general and 4 punctual relations)
- the vertical symmetry axis corresponds to the "wrapping" of the time axis (past ↔ future).



Algebraic point of view:
generally speaking, binary relations (R) over a set I, are parts of the Cartesian product IxI. One can build an "algebra of correspondences" by using the two operations:

- transposition : $R^t = \{ (i_1,i_2) \in IxI / (i_2,i_1) \in R \}$
- composition : $R_oS = \{ (i_1,i_2) \in IxI / (\exists i_3) (i_1,i_3) \in R, (i_3,i_2) \in S \}$

The composition must have a neutral element and there is a distributivity property: $(r_os)^t = r^t{}_os^t$.

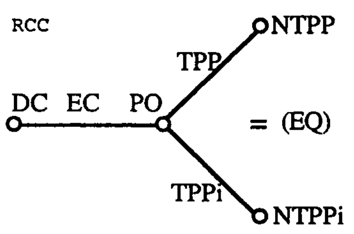
Moreover, with the 13 Allen's relations, there is a symmetry R→s(R), obtained when replacing < by its opposite > (backward time arrow). The 13^2 compositions must respect transposition and symmetry rules, accept "=" as neutral element and obey 43 more axioms, as listed in [Bestougeff-Ligozat-89]. An "Allen system" is a model, with a domain set, which may be N, Z, Q, R or any finite set I, given in extension, whose elements are called "intervals" and with 13 relations satisfying the above rules.

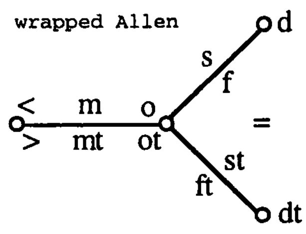
At that point we keep ourselves absolutely free from any reference to "points", "interval ends" or any numeric support for the order relation, as well as from any time or geographic co-ordinate axis.

1.2. Topological Relations in the two-dimensional Space.

The topological relations between parts of the 2D space ("regions") are well known, if the space model is R^2, dense, with neighbourhoods based on the notion of "limit". These relations are defined from opens o(i), closes c(i) and borders b(i) associated to regions. For example [Egenhofer-91] retrieves the 8 relations from intersections : $o(i_1) \wedge o(i_2)$, $o(i_1) \wedge c(i_2)$ ...
Research work has also been performed to manage regions without considering them as sets of points ("mereology"), this makes us able to build an algebra with 8 relations, which "captures" mostly the intuitive topological notions, ex. (following [RCC-92]):
DC (disconnected); EC (externally connected); PO (partially overlaps); TPP (tangential proper part); NTPP (non tangential proper part); EQ (equals); TPPi (tangential proper part inverse); NTPPi (non tangential proper part inverse).

We can figure these relations with the topological graph: the left graph (RCC) is formally identical to the right one, which is the Allen graph wrapped along the vertical symmetry axis (remember: turn back the time arrow)





Space objects, unlike of time intervals, have no (simple and total) order relation and the resulting graph wraps and merges the former symmetrical relations. Then we have 8 relations left from the 13.

The move from the Allen(13) algebra to the RCC(8), by wrapping the order relation has already been noticed as well as other moves to smaller algebras [Euzenat-94]. We may think that lessening the number of constraints should probably lead to weaker algebra, probably too general.

We will try instead, to show that it can lead to represent more specific situations, for it enables us to choose what is the minimal algebra for every case. To do that, we first present a bottom-up, incremental, approach, which adds constraints instead of removing them.

1.4. Minimal Structure: the equivalence relation (notion of discernment)

Being conscious of a surrounding world is the first step in the geographic cognition. Formally, this starts when the baby becomes able to distinguish a reference location from any other ('here & now' .vs. 'other place, other time'). This leads to structure time and space with an equivalence relation.

Let $I=\{i\}$ be a universe of "primitive features" who exist in space and time (without any more precision) and let Eq be a binary relation with axioms:

(A1): $\forall\, i,j \in I,\ Eq(i,j) \rightarrow Eq(j,i)$, symmetry of Eq,

(A2): $\forall\, i,j,k \in I,\ Eq(i,j) \wedge Eq(j,k) \rightarrow Eq(i,k)$, transitivity of Eq.

These 2 axioms infer the reflexivity $\forall\, i \in I,\ Eq(i,i)$. Eq is the relation of "indiscernibility", rather than merely an object identity, what is different from the identity of space-time location of these objects.

(D1): $Nq(i,j) \equiv \neg Eq(i,j)$, is the discernibility relation.

Relations {Eq, Nq} are mutually disjoint and give a first "in-out" algebra.

Discernment and extensionality.

The notion of discernment is linked to "extensionality" attached to a binary relation R, which follows the Leibniz' principle of the identity of indiscernibles [Bennett-95].

The "extensionality" formula: $\forall\, i,j \in I\ [\forall\, k \in I,\ R(i,k) \leftrightarrow R(j,k)] \leftrightarrow$ "i identical to j",

means that we can't discern i and j if we can't distinguish them in their relation R to any other object.

1.5. Contact Structure (notion of symbolic neighbourhood)

One more step to structure the space-time, is to add the constraint of "contact" (C). The discernibility is specified, ex.: among the communes different from Marseilles, there are peripheral ones. The C axioms:

(A1): $\forall\, i,j \in I,\ C(i,j) \rightarrow C(j,i)$, symmetry of C,

(A2): $\forall\, i \in I,\ C(i,i)$, reflexivity of C.

The extensionality formula ($[\forall\, k \in I,\ C(i,k) \leftrightarrow C(j,k)] \leftrightarrow$ i identical to j), is a theorem if we keep the axioms on Eq, but can - preferably - be used to define Eq. Nq may be re-defined from C and is called DC (disconnected). To obtain a complete set of mutually disjoint relations, we must add the definition of "externally connected":

(D1): $Eq(i,j) \equiv \quad [\forall\, k \in I,\ C(i,k) \leftrightarrow C(j,k)] \leftrightarrow Eq(i,j)$,

(D2): $DC(i,j) \equiv \quad \neg C(i,j)$,

(D3): $EC(i,j) \equiv \quad C(i,j) \wedge \neg Eq(i,j)$.

Relations {Eq, DC, EC} are mutually disjoint and give a "contact" algebra.

1.6. Parts Structure (notion of sharing, "mereology")

Another step is to augment our discernment ability in the space-time, by considering that objects may themselves be parts of objects. Let's introduce the relation "part of" (P) with axioms:

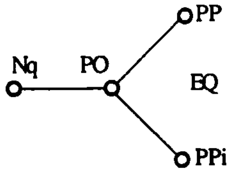
(A1): $\forall\, i \in I,\ P(i,i)$, reflexivity of P,

(A2): $\forall\, i,j,k \in I,\ P(i,j) \wedge P(j,k) \rightarrow P(i,k)$, transitivity of P.

In a way similar to §2.2, P can be seen as containing Eq, etc.... or an extensionality formula can be introduced to redefine Eq (now EQ) as the only possible symmetry for P (anti-symmetry). To obtain a complete set of mutually disjoint relations, we must add the definition of "proper part" PP, its transposed PPi and also "partial overlap" PO, i.e.: sharing space with an object as well as its outside.

(D1): $EQ(i,j) \equiv P(i,j) \wedge P(j,i)$
(D2): $Nq(i,j) \equiv \neg P(i,j)$
(D3): $PP(i,j) \equiv P(i,j) \wedge \neg Eq(i,j)$, (alternative definition: $PP(i,j) \equiv P(i,j) \wedge \neg P(j,i)$)
(D4): $PPi(i,j) \equiv PP(j,i)$, (PP, such as P, is non symmetrical)
(D5): $PO(i,j) \equiv (\exists\, k \in I,\ P(k,i) \wedge P(k,j)) \wedge \neg P(i,j) \wedge \neg P(j,i)$

Relations {Eq, Nq, PP, PPi, PO} are mutually disjoint and give algebra: RCC-5.



1.7. Topological Structure: combining contact and parts (RCC-8)

To combine now the 3 relations Eq (discernment), C (contact) et P (parts), there are several choices for axioms and definitions, to obtain a complete set of mutually disjoint relations. But the basic set is always the same: the 8 relations of the RCC algebra.

Axioms : (axioms A1 to A3 are coherent and sufficient with the definitions below)

(A1): $C(i,i)$ — reflexivity of C
(A2): $C(i,j) \rightarrow C(j,i)$ — symmetry of C
(A3): $C(k,i) \leftrightarrow C(k,j) \rightarrow Eq(i,j)$ — extensionality of C, constraint of transitivity on Eq

(then define P with C, or choose an other possible set of axioms, like)

(A'1): $P(i,i)$ — reflexivity of P
(A'2): $P(i,k) \wedge P(k,j) \rightarrow P(i,j)$ — transitivity of P
(A'3): $P(i,j) \wedge P(j,i) \rightarrow Eq(i,j)$ — extensionality of P, constraint of symmetry on Eq

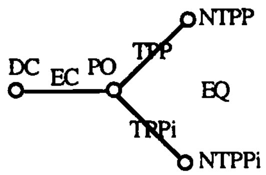
for example [Asher-Vieu-95] exhibits a set of axioms to build operators of inclusion, adjacency ...

The simple union of the 3 relations of the contact algebra and the 5 of RCC-5 gives 6 relations, because Eq and Nq are cited twice. To obtain a complete set, we must split the PP case into tangential PP (TPP) and non tangential PP (NTPP), and the same with PPi.

Definitions (of the 8 topological relations, equiv. to RCC, but different point of view):

- EQ — equivalence relation, after A3 et A'3 (reflexivity inferred)
- $DC(i,j) \equiv \neg C(i,j)$ — no contact (implies no share)
- $EC(i,j) \equiv C(i,j) \wedge (\forall\, k,\ P(k,i) \rightarrow \neg P(k,j))$ — contact without share
- $PO(i,j) \equiv \neg P(i,j) \wedge \neg P(j,i)$ — share with object and its complement
  $\wedge\ \exists\, k\ (P(k,i) \wedge P(k,j))$ — i is not in j neither j in i, i and j have a common part
- $TPP(i,j) \equiv P(i,j) \wedge \neg P(j,i)$ — share and contact with the complement
  $\wedge\ \exists\, k\ (C(k,i) \wedge \forall\, l,\ P(l,k) \rightarrow \neg P(l,j))$ — i is part of j, is not j itself, and i is in contact with a k, any part of whom can't be part of j (i.e.: outside j).
- $NTPP(i,j) \equiv P(i,j) \wedge \neg P(j,i)$ — share and no contact with the complement
  $\wedge\ \forall\, k,\ (C(k,i) \rightarrow \exists\, l,\ P(l,k) \wedge P(l,j))$ — i is part of j, is not j itself, and for any object in contact with i, there is a part of it which is part of j.
- $TPPi(i,j) \equiv TPP(j,i)$ — TPP transposed
- $NTPPi(i,j) \equiv NTPP(j,i)$ — NTPP transposed

Graph of the RCC-8 relations:



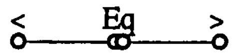
1.8. Order Structure (the "time arrow")

In the case of time, the order structure is obvious, linked to the relation of "precedence" noted b for "before". According to the already cited book [Bestougeff-Ligozat-89], the strict order is the best representation for it, then:

(A1): $\forall\, i,j,k \in I,\ \neg(b(i,j) \wedge b(j,i))$, asymmetry of b,
(A2): $\forall\, i,j,k \in I,\ b(i,j) \wedge b(j,k) \rightarrow b(i,k)$, transitivity of b (hence a-reflexivity: $\neg b(i,i)$).

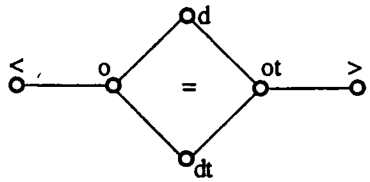
With a total strict order axiom (A3), then the relation Eq verifies: $\neg b(i,j) \wedge \neg b(j,i) \rightarrow Eq(i,j)$

With the transposed "after" a(i,j) ≡ b(j,i), the relations {Eq, b, a} are mutually disjoint and give a 3 relations algebra.

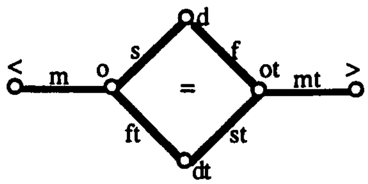


Add the relation C.
Let's call Allen-contact the 5 relations algebra {<, m, Eq, mt, >}.
The precedence relation is modified in order to accept to isolate the contact case.

Add the relation P.
Let's call Allen-mereo the 7 relations algebra {b, o, d, =, dt, ot, a}.
The equivalence relation is now modified (noted = instead of Eq).



Add both relations C and P.
We get now the full Allen algebra, with the 13 relations, noted Allen-13:
{<, m, o, s, ft, d, =, dt, st, f, ot, mt, >}



2. Geographical frameworks and the algebras lattice

2.1. The lattice of the eight "topological constraints algebras".

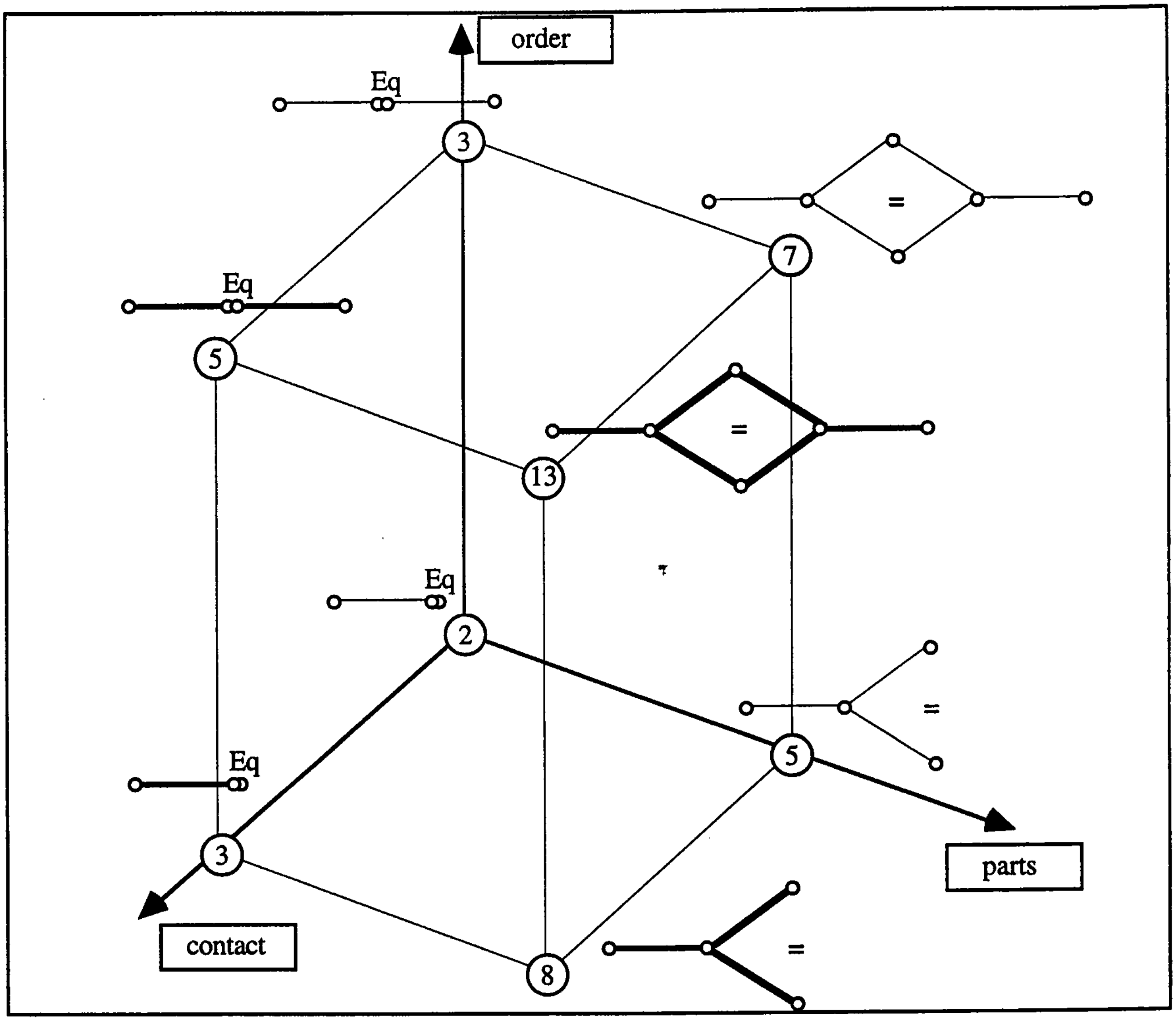
Let's examine this correspondence table between the eight different algebras:

in-out	RCC-contact	RCC-5	RCC-8	Allen-3	Allen-contact	Allen-mereo	Allen-13
Nq	DC	Nq	DC	b	<	b	<
				a	>	a	>
	EC		EC	b	m	b	m
				a	mt	a	mt
Eq	Eq	PO	PO	Eq	Eq	o	o
						ot	ot
		EQ	EQ			=	=
		Pp	TPP				s
							f
			NTPP			Pp'	d
		Ppi	NTPPi			Pp'i	dt
			TPPi				ft
							st

This table teaches us about the following groups:
- two "equalities": the basic equivalence relation Eq and the more complex EQ (or =), which merely needs the mereological structure, the contact doesn't influence.
- eight "exteriors": two without order (Nq et DC) distinguished by the contact, two with order without contact (labelled b[efore] and a[fter], to not confuse with < and >) and four with both contact and order. No influence from mereology.
- three "overlapping": PO without order, o and ot with. The contact doesn't interfere.
- six "interiors" (and their six dual): pp and ppi, without contact (doesn't care with order), tpp-ntpp and tppi-ntppi with contact but without order, d-s-f and dt-st-ft with both. Let's note here that the order can distinguish interior parts only combined with contact, hence d is equivalent to ntpp and dt to ntppi.

These remarks give us the power to choose, among possible axioms and definitions, the most appropriate for a given application.

Figure: the lattice of the 8 "topological constraints" algebras.
(the lattice vertices give the number of relations for every algebra: (n))



2.2. Operators related to each basic relation

- symmetrical discernment through order: ord(r)=(r',r") where {r'}= {r}∧{<}, {r"}= {r}∧{>}= s(r')
The relation is split into two halves of the same size;
wherever a relation is its own symmetrical, this symmetry can't help distinguish more. Let's see:
(1-5) Nq ↔ {b, a≡s(b)}
Eq unchanged
(2-6) DC ↔ {<, >≡s(<)}, EC ↔ {m, mt≡s(m)},
Eq unchanged
(3-7) Nq ↔ {b, a}
PO ↔ {o, ot≡s(o)},
EQ ↔ {=, =≡s(=)} identical (its own symmetrical), hence we can use notation = for EQ.
Pp ↔ {d, d≡s(d)} and Ppi ↔ {dt, d≡s(dt)} identical (see below)
(4-8) DC ↔ {<, >}, EC ↔ {m, mt}, PO ↔ {o, ot}, EQ unchanged (=),
TPP ↔ {s, f≡s(s)}, TPPi ↔ {st, ft≡s(st)},
NTPP ↔ {d, d≡s(d)} and NTPP ↔ {dt, dt≡s(dt)} identical because d and dt are their own symmetrical. Hence we can use notation d for NTPP and dt for NTPPi.

- asymmetrical discernment by contact: con(r)=(d,x) where {d}= {r}-{x} (default), x= exceptions
The relation is split into two parts of different size
(1-2) Nq ↔ {DC, EC},

Eq unchanged

(3-4) Nq ↔ {DC, EC}, PO and EQ unchanged,
Pp ↔ {TPP, NTPP}, Pp_i ↔ {TPPi, NTPPi}

(5-6) b ↔ {<,m}, a ↔ {>,mt},
Eq unchanged

(7-8) b ↔ {<,m}, a ↔ {>,mt}, = unchanged as well as o and ot,
Pp' ↔ {d, {s, f≡s(s)}}, Pp'i ↔ {dt, {st, ft≡s(st)}} in this case, parts and order combine immediately to provide a triple distinction.

- anti symmetrical discernment by share: sha(r)=(r',r",r"',=) where r' means part to all, r" means all to part, r"' part to part, and all to all is the equality (anti-symmetry axiom).

(1-3) Nq unchanged (no share at all) = (Ø,Ø,Ø,Ø)
Eq ↔ sha(Eq)={Pp≡r'(=), Ppi≡r"(=), PO≡r"'(=), EQ≡(=)}

(2-4) DC, EC unchanged
Eq ↔ sha(Eq)={ {TPP,NTPP}≡con(Pp), {TPPi,NTPPi}≡co(Ppi), PO, EQ}

(5-7) a, b unchanged
Eq ↔ sha(Eq)={ Pp', Pp'i, {o,ot}, =}

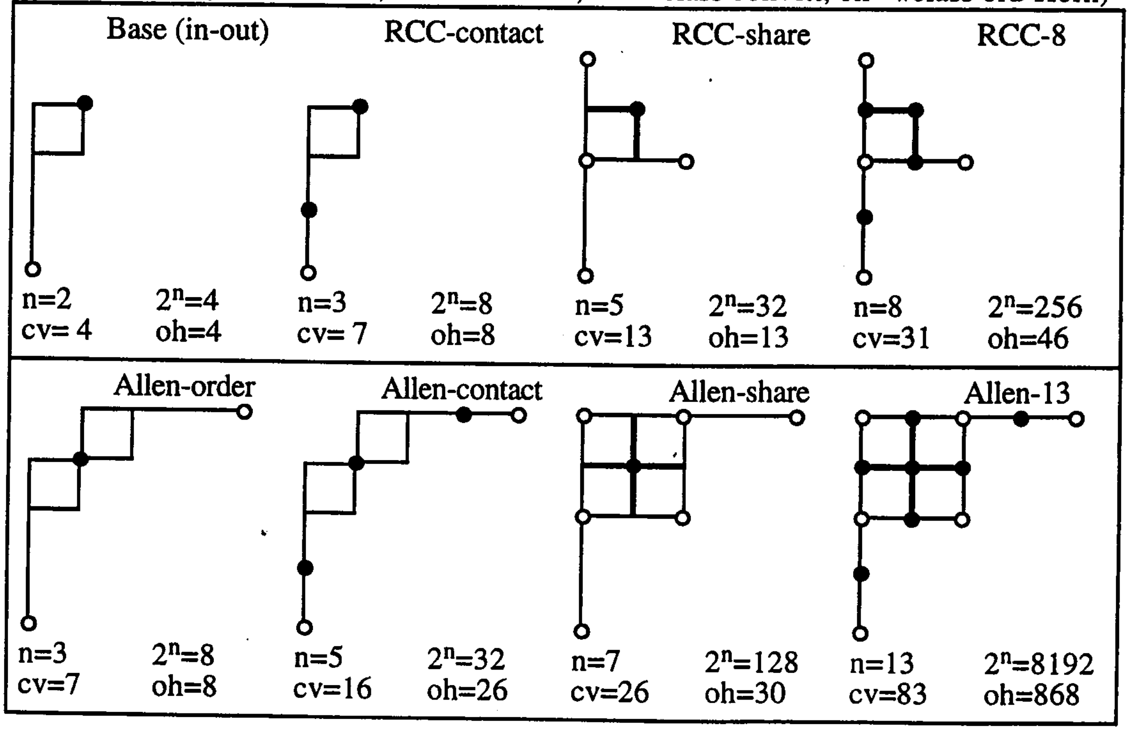
(6-8) <, >, m, mt unchanged,
Eq ↔ sha(Eq)={ {ord(con(Pp'))}, {ord(con(Pp'i))}, {o,ot}, =}, contact and order combine to provide a triple distinction.

2.3. Tractability estimates

Comparison of "sub-Allen" relation algebra de relations, based upon primary relations "contact", "share" and "order" : this gives the lattice of the eight algebra.
Let's count the cardinal of the different classes:
- convex
- pointizable (note: pointizable classes are still to be enumerated).
- ord-Horn,

according to the work of [Ligozat-96].
Synthesis.

Figure. Tractability values: classes of relations in the various sub-Allen algebras.
(white nodes are "stable" relations, black are "unstable" ones)
(n= #relations, 2^n= #class full, cv= #class convex, oh= #class ord-Horn)



3. The Use of Spatio-Temporal Constraints in GIS

3.1. Exhibiting constraints in the representation of geographic objects

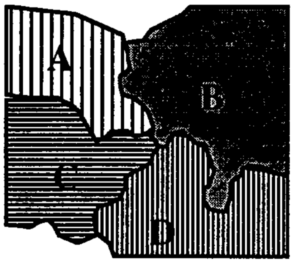
By now, the use of GIS is rapidly increasing among a very large variety of users. In the same time a considerable amount of geographic data is produced in a numeric format: vector maps, aerial or satellite images, geo-referenced statistics or census or measurements ...

These data are "spatial data" as soon as a (comprehensible) reference to space - and possibly time - is attached to them. Often, the reference is to an existing spatial object (a city, a county, a river...) but also - more directly - to geographic co-ordinates (a GPS mark, the corners of an image, hence every pixel of it ...). This is the "object and field" paradigm of GIS [Burrough-Frank-95] [Couclelis-92].
In both cases, the underlying co-ordinates of the points are required explicitly or implicitly.
This is mandatory for displaying geographic data on a screen or producing a final map sheet.
But many other queries **Do Not** require the underlying co-ordinates and are more or less constraint resolution problems.
Likewise, a lot of geographic information is collected merely in terms of constraints, like "in town", "along the river", "down stream", "up-hill" ...
These are two major justifications for the explicit introduction of constraints into a GIS model.

Two important cases allows us to illustrate how to choose the algebra which fits the best the application, instead of using RCC-8 in every condition: space partitioning and territories.

3.1.1. The spatial partitioning (exact boundaries)

Most of the explicit (manned) carvings of space, do not accept emptiness neither sharing: every spot on Earth belongs to one power (and only one at a time).This is a functional and bijective approach of space: it corresponds to the "contact" algebra {Eq, EC, DC}: reflexivity and symmetry.



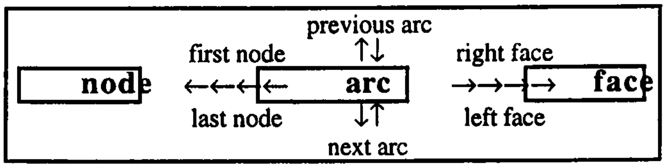
This partitioning determines 6 relations and 6 transposed

EC(A,C)	EC(C,A)
EC(A,B)	EC(B,A)
DC(A,D)	DC(D,A)
EC(B,C)	EC(C,B)
EC(B,D)	EC(D,B)
EC(C,D)	EC(D,C)

This case is the most frequent with the classical use of GIS.
To obtain the whole set of "contact" constraints, we need only the exhaustive list of EC's, because the Eq's are useless - under the assumption of unique reference addresses - and the DC's are defaults - under the close world assumption.

The "arc-centred" model with the information (right face, left face) can provide this list of EC.

ex: formats Edigéo or ArcInfo-Export

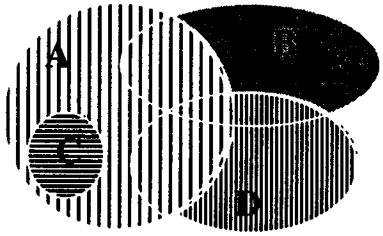


The total number of non trivial relations (not Eq(i,i)) is N = #face x (#face -1). The Euler equality [Laurini-Thompson-92] gives us an idea of the number of EC's:
But some of the arcs are on the outer contour, hence not informative for the topology, then #EC < #arc. For example, in the partitioning above: #node = 6, #face = 4, #arc = 9 (Euler verified) but 4 arcs are useless and we get only 5 EC's, as explicitly listed above. Symmetry gives 5 transposed, the last 2 relations (because N=12) are then DC's.

3.1.2. The notion of territories (undetermined or vague boundaries)

Among many possible definitions of territories, we will refer to a very simple version of the "territorial process" as cited in [Offner-Pumain-96]. This is a non functional approach of the space,

i.e.: the space may be shared for several uses. Hence, several territories may overlap in some part of space. This is an injection because any use needs one territory and only one, for this is the use which defines its territory (and this use may change over time).
This approach leads us to the RCC-5 algebra {EQ, Nq, PP, PPi, PO}: reflexivity and transitivity.



The territories on the left determine 6 relations and 6 transposed

PO(A,B)	PO(B,A)
PPi(A,C)	PP(C,A)
PO(A,D)	PO(D,A)
Nq(B,C)	Nq(C,B)
PO(B,D)	PO(D,B)
Nq(C,D)	Nq(D,C)

This situation is frequent, quasi-permanent with environmental, geophysical, social, economical data: ex. settlement zones for such vegetal or animal, soil type, economical influence, cultural areas ...

The common way to manage this kind of information is to draw frontiers - more or less arbitrarily - and (over-) lay them on objects, in order to fall back in the partitioning case, which behave more easily with present GI tools. The traditional cartography also liked "choropleth" maps, as now the display software do, because of the difficulty to correctly show overlaid parts (ex. figure above).
The next step is to divide the objects according to a whole-spread intersection: ex. the 4 objects A, B, C and D above, produce 8 new objects that form a partitioning (A, C, AB, AD, ABD, B, BD, D).
We can then use a 3 relations "contact" algebra with this set of divided objects.

Another way is to explicitly collect the relations, as it is often possible in the real world:

- the PP information comes from the explicit inclusion of one object (or landuse) within another: this is a "spatial dependency" (ex. such bird lives only where buffalo lives);

- the PO information comes from the sharing of territories between two uses (German and Dutchmen share the Côte d'Azur with Frenchmen).

Like in the previous §, EQ's are useless, Nq are defaults, and here, PPi's are given by transposition. This solution is weaker than the first one because it can answer intersection predicates but cannot exhibit where the intersection is (neither the original objects). But this solution is more robust in that it doesn't depend on the precision of laid on frontiers (see §3.3.2).

3.2. Applications with Geographic Information

Several works have been undertaken on the translation of numeric (quantitative) relations, associated to the geometry of the regions in the 2D space, into symbolic (qualitative) relations.
For example, distance from a reference, elevation (above sea level), cardinal orientation ... are metric measurements that can be turned into symbolic properties relative to rough classes, as listed below.
Remark (global versus local): these geometric measurements are stationary in space, hence global on Earth, but their use in term of constraints is local: it doesn't matter that Lyon is higher than Vienna or not, but it does that Lyon is downstream Geneva.

3.2.1. Elevation

The elevation data may produce locally useful constraints. Ex.: the "river basin" notion induces an order relation ("up-stream/downstream"), which is a right-linear order (we avoid the case of delta), i.e. for every t, the set of its successors is totally ordered:

$$\forall t, \exists u, \exists v, R(t,u) \wedge R(t,v) \rightarrow u=v \vee R(u,v) \vee R(v,u)$$

This can be combined with topological algebras "in-out" and "contact" like previous cases, but also with the full RCC-8 algebra.
When restricting to a successors set, like mentioned above, we can even use the Allen-13 algebra.

3.2.2. Nested partitionings

In several cases, the hierarchical structure of power has lead to hierarchical partitionings of the administrative space. This gives another order relation that can be used together with topology.
Ex.: in the USA: states, counties... , in Europe: the Eurostat zones, or in France:

"Commune" $\subseteq$ "Département" $\subset$ "Région" (administrative entity)
(the equality commune = département, is verified only for Paris).

Each map being a partitioning, can be represented by the "contact" algebra (3 relations).
The order relation (inclusion) between the regions from two different hierarchical levels, can be represented by an "Allen-contact" algebra (5 relations).

3.3.3. Integrated use by combining constraints: Spatio-temporal Composition

In the most general case and under the assumption of a unique spatio-temporal addressing, we must consider the direct product between (I,{RCC-8}) and (T,{Allen-13}).
In several practical cases, it's probably enough to use Allen-contact: for example, many administrative processes perform full updates at regular time intervals, like census. Such a process registers data for all objects, even if no change has occurred since the last update. Under this assumption (no share between time intervals: time partitioning), a "lexicographic" product can be interpreted this way:

- either two (spatio-temporal) regions aren't on the same time interval and we skip the spatial relation;
- either they have the same life-time and we must look at the space domain to know how they relate.

Example: cadastre (land registry) changes:
The space and the time are both partitioned then we can use $(I,\{RCC\text{-}3\})_{\chi_\rho}(T,\{Allen\text{-}contact\})$,
where denotes: $\forall (i,t),(j,u) \in I x T,\ r \in R\text{-}\{\rho\},\ s \in S,\ (r_{\chi_\rho} s)((i,t),(j,u)) \leftrightarrow r(i,j) \vee \rho(i,j) \wedge s(t,u)$
and choose ρ = mt in order to distinguish spatial relations in the only case where the lifetime j of the reference region immediately precedes the lifetime i of the considered region.
This can be very useful to control the topology between regions of two successive versions.

4. CONCLUSION

The use of spatial constraints in GIS is very promising. A lot of different directions are currently investigated by a lot of people from the AI and Data Base, research communities.
The ability to store new geographic knowledge (localisation by constraints) as well as to run new reasoning process (constraint satisfaction in decision making), can greatly help the geographers to turn a new look at the GIS.

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