A TENTATIVE NEW LOGICAL APPROACH FOR DEALING WITH KBS's

Luis M. Laita, Luis de Ledesma, Aurora Pérez, Angélica de Antonio
Facultad de Informática, Universidad Politécnica de Madrid
Boadilla del Monte, 28660 Madrid, Spain
laita@fi.upm.es

Abstract

Expert systems rules express relevant relations between their antecedents and their consequents. If the rules are manipulated by a logic (usually classical logic) in such a way that they give rise to new formulas with the form of rules (generated rules), these formulas may not express relevant relations between atecedents and consequents, and then become useless. We propose in this paper, a logic that partially avoids this problem, and in addition deals with both non-monotonicity and uncertainty, A description of the verification concept of consistency in forward and backward reasonings is given in terms of this logic.

1. Motivations and purpose of the paper

Expert systems rules express relevant relations between their antecedents and their consequents. Considering these rules together with the given facts as axioms of a theory, if its underlying logic is classical logic, the generation of new formulas of the form $\alpha \rightarrow \beta$ or $\neg(\alpha \rightarrow \beta)$ (where α and β may in their turn be complex formulas containing $\neg, \rightarrow, \wedge$), from the theory and the logic, may not involve a relevant relation between α and β, due to the so-called "implication and conjunction-implication paradoxes". An example of implication paradox is the following. If the expert propposes a fact α, as the formula $\alpha \rightarrow (\beta \rightarrow \alpha)$ is a theorem in classical logic, it results that $\beta \rightarrow \alpha$ is a valid generated formula. But such a formula means that $\beta \rightarrow \alpha$ for any β, which makes it useless to be used in the KBS. It would then be of interest to use as the logic underlying the generation of new formulas from a given KBS, an entailment logic that could keep that idea of relevant relation in the sense that they would be paradox-free. Moreover such a logic would acquire more interest if it would accept both non-monotonicity and degrees of certainty.

In this paper, we will propose a system, based on parts of Cheng's entailment logic (Cheng & Ushijima 1990, Cheng 1992), Gabbay's non-monotonic logic (Gabbay, 1982) and Rasiowa's regular many-valued logic (Rasiowa 1974), that, partially, captures all together the notions of relevant relation, non-monotonicity, and uncertainty.

The study will be mainly based on pragmatic and semantical concepts, leaving the syntactical aspects for a future work. The semantic approach consists in that our reasoning will be based on the consideration of a particular model. The pragmatic aspect will appear in the definition of the concept of "relevant relation" between an antecedent and a consequent of an entailment.

The main purpose of the paper is to propose some ideas for discussion about the convenience of using and mixing non-classical systems of logic for dealing more conveniently with KBS's problems, rather than to propose a completed system. One reason is that this is as yet a work in progress, so that there are in it quite a number of items that are at want of a rigorous study.

2. This paper's concept of relevant relation

2.1. Definition. Relevant relation is expressed by the following recursive definition. (1) If $\alpha \rightarrow \beta$ is an entailment expressing a KB-rule, then there exists a relevant relation between α and β (the pragmatic justification is that the expert has judged that such is the case). (2) If an entailment formula $\alpha \rightarrow \beta$, that results from application of G-C logic (to be defined below) from a set of other entailment formulas which express relevant relations between their antecedents and consequents, is implication and conjunction-implication paradox-free, then it expresses a relevant relation between its antecedent and its consequent. An example of implication paradox has been given above in 1; a characterization of conjunction-implication paradox will be given in terms of the model in 5.

2.2. Remark. This concept of relevant relation is a limited one, but at least it avoids the many cases of irrelevance that are produced by the mentioned paradoxes.

3. Cheng´s approach to implication paradox-free logic

3. 1. Outline

Axioms

1. $(\alpha \to (\beta \to \gamma)) \to ((\beta \to (\alpha \to \gamma)$,
2. $(\beta \to \gamma) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$,
3. $\alpha \to \alpha$,
4. $\alpha \wedge \beta \to \beta \wedge \alpha$,
5. $(\alpha \to \beta) \wedge (\alpha \to \gamma) \to (\alpha \to \beta \wedge \gamma)$,
6. $\alpha \to \neg\neg\alpha$
7. $(\neg\alpha \to \beta) \to (\neg\beta \to \alpha)$,
8. $\alpha \wedge \neg\beta \to \neg(\alpha \to \beta)$

Inference rules

1. Modus Ponens, "from α , $\alpha \to \beta$, infer β".
2. And-introduction, "from α and β , infer $\alpha \wedge \beta$"

The primitive connectives in this calculus are $\to$, $\wedge$, and $\neg$. A special disjunction is defined: "$\alpha \vee \beta$ iff $\neg\alpha \to \beta$ ": note that by axiom 7, $\alpha \vee \beta \to \beta \vee \alpha$.

3.2. Our use of Cheng´s calculus.

3.2.1. Language.
In addition to the propositional variables, let us introduce three constant propositions F (absolute falsity), CI (conjunction-irrelevance), and T (absolute truth).

3.2.2. Axioms.
The same as above with the following addenda or changes.
-Add as axioms 1'. $\neg$(F$\to$CI), 1''. $\neg$(CI$\to$F), 1'''. $\alpha \to$ T.
-Delete axioms 1 and 4.
-Change axiom 7 by the axiom, $(\alpha \to \beta) \to (\neg\beta \to \neg\alpha)$.

3.2.3. Rules.
Change rule 2 by the rule, "from $(F \to \alpha)$ and $(F \to \beta)$, if α and β, infer $\alpha \wedge \beta$ and $F \to \alpha\wedge \beta$ ".
-Add rule, "from $\alpha \wedge \beta$, infer $\beta \wedge \alpha$".

3.3. The generation of paradox-free entailments in this calculus.
We will see by using the model of section, that implication paradoxes as $\alpha \to (\beta \to \alpha)$ that allows, given α, finding $\beta \to \alpha$ for any α, and conjunction-implication paradoxes as $(\alpha \to \beta) \to (\alpha\wedge \gamma \to \beta)$ are not allowed in our version of Cheng´s calculus.

4. An approach to non-monotonicity

4.1. Outline.

Gabbay defines a forming formulas operator G which in a few words may be described as follows. Gα is asessed to be true at a time t if it is accepted at t that at some future moment $s \geq t$ it will be known that α was true at t (it has previously been defined an order relation among moments of time). Such an asessment is based on the usual behaviour of the universe to which the system refers to and no evidence on the contrary. G embodies an increment of knowledge, not the occurrence of new events.

Non-monotonicity appears when, given a set of formulas hypotheses, an expert adds a formula of the form Gα to that set, in order to find some conclussion. But if some new hypothesis is added to the first mentioned set, the expert may now judge convenient to change Gα by, say, Gβ, so that the conclusion previously found does not hold any more and it should be changed by another one.

5. G-C logic

5.1. Introduction.

We suggest trying a logic consisting of:

(i) The axioms and inference rules of Cheng's calculus, modified with the changes and addenda in 3.2.

(ii) The Gabbay's two logic axioms $\alpha \to G\alpha$, and $GG\alpha \to G\alpha$,

(iii) An axiom $\neg G\alpha \to \neg\alpha$.

(iii) $\alpha \vee \beta$ is defined as $\neg\alpha \to \beta$.

We will call this logic "G-C" logic (G and C respectively stand for Gabbay´s operator and for Cheng´s calculus) KBS's expressed in terms of this logic will be called "G-C-KBS's".

5.2 Interpretation of G-C logic.

5.2.1. Definition. An entailment-preorder (e-p) L consists of a meet semilattice L1 with eight vertices together with a five elements meet semilattice L2, L1 and L2 having four common elements as described in 5.2.2. below. Three operators N, E, G, three distinguished elements F, CI, and T, and two distinguished subsets Tt and Ff of L, are added, satisfying the following conditions.

(0) We define separatedly in L1 and L2 a preorder relation "a ≤ b iff a divides b". We also define separatedly in L1 and L2, "g.l.b.(a, b) = a ∧ b = g.c.d (a,b)". Note that considering L as a whole, not all pairs of elements will have a meet.

(1) N is a mapping N: L→ L that satisfies for all a, b ∈ L:

(i) Na ≠ a,

(ii) a ≤ NNa,

(iii) if a ≤ b, then Nb ≤ Na.

(2) Tt is a distinguished subset of L, of "true values" that satisfies:

(i) for all a∈ L, a ∈ Tt iff Na ∉ Tt,

(ii) for all a,b ∈ L, a ∧ b ∈ Tt iff a ∈ Tt and b ∈ Tt,

(iii) there exists no a, b ∈ L such that a ∈ Tt, Nb ∈ Tt and a ≤ b.

(3) E is a mapping L × L→ L that satisfies for all a. b. c ∈ L:

(i) E(a,b) ∈ Tt iff a ≤ b, in particular, E(a,a) ∈ Tt

(ii) E(b,c) ≤ E(E(a,b),E(a,c)),

(iii) E(a,b) ∧ E(a,c)≤ E(a,b3 c),

(iv) E(a,b) ≤ E(Nb,Na),

(v) E(a, NNa) ∈ Tt

(vii) E(a∧Nb, NE(a,b))

(The arrow corresponding to E will herafter be called "G-C-entailment")

(4) G is a mapping L→ L that satisfies,

(i) Ga = GGa

(ii) E(a, Ga) ∈ Tt for all a∈ L

(iii) NGa → Na

5. 2. 2. An example. Consider the particular p-e of figure 1.

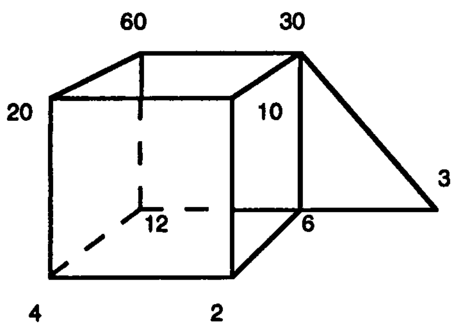

Figure 1

(1) The vertices are labeled 2, 4, 6, 12, 10, 20, 30, 60, 3. { 2, 4, 6, 12} is the set Ff of *false values*,. {10, 20, 30, 60} is the set Tt of *true valuess*. 3 is the representation of CI , 2 that of F, and 60 that of T. The role of 3 is to indicate the non existence of α ∧ 3, for α ∈ {2, 4, 10, 20}.

(2) N is defined as follows. N2 = 60, N4 = 20, N6 = 30, N12 = 10, N10 = 12, N20 = 4, N30 = 6, N60 = 2, N3 = 30.

(3) G is defined as follows: if Ga = 60 then a = 60, if Ga = 20, then a =20 or 10, if Ga = 10, then a = 10, if Ga = 30 then a = 30 or 10.

(7) Definition of E.

E	2	4	6	12	10	20	3 0	60	3
2	60	60	60	60	60	60	60	60	2
4	2	20	2	20	2	20	2	60	2
6	2	2	30	30	2	2	30	60	2
12	2	2	2	10	2	2	2	60	2
10	2	4	6	12	10	20	30	60	2
20	2	4	2	4	2	20	2	60	2
30	2	2	6	6	2	2	30	60	2
60	2	2	2	2	2	2	2	60	2
3	2	2	30	30	2	2	30	60	30

5.3. Interpretation..

An interpretation I of G-C logic is the p-e L described along this paragraph together with a mapping v_a:{simple propositions} → L, and a mapping v_f: {formulas} → L which satisfies for all simple proposition A and all formulas B, C, (i) $v_f(A) = v_a(A)$, $v_f(\neg B) = Nv_f(B)$, $v_f(B \wedge C) = v_f(B) \wedge v_f(C)$, $v_f(B \rightarrow C) = E(v_f(B), v_f(C))$. I maps formulas into elements of L which are in Tt or not. We denote I(A)= t iff in L, A∈ Tt, I(A)= f iff A⊆ Ff. I(A) = 3 means that A is not always able to form a conjunction. A formula A is valid in L iff I(A) = t. All axioms of G-C logic are valid in L. If the premises of the inference rules are valid, the conclussions are also valid.

Note that as for instance 12 ≤ (4 ≤ 12) has value 2, the implication paradox $\alpha \rightarrow (\beta \rightarrow \alpha)$ is not valid in this logic. Also, (20≤ 60), but it is not the case that 3∧20 ≤ 60 because 3∧20 does not exist, thus if $\alpha \rightarrow \beta$, then $\alpha \wedge \gamma \rightarrow \beta$ is not valid. Then the implication and conjunction-implication more frequent paradoxes are not valid in this logic.

5.4. A total order relation can be defined among the labels of the vertices of the lattice: the total order provided by the indexes i = 0, 1, ..., 5, of the e_i considered as a total order of *certainty degrees*. The e_i are defined as as follows: e_0 = 2 or 3, e_1 = 4 or 6, e_2 = 12, e_3 = 10, e_4 = 20 or 30, e_5 = 60. Statements as e_0 =2 or 3, mean that 2 and 3 indiferently indicate a same truth value, so that if an expert proposes 3 as some certainty degree, and for practical purposes is more convenient to take 2 instead, it is allowed to do so.

6. Expression of G-C-KBS's

6.1. Literals. A *literal* is any proposition appearing in the KBS's rules preceded or not by ¬. A *G-literal* is a literal preceded by the symbol G.

6.2. KBS bases of facts. We consider two different sets of facts: a set of *established* known now to hold with total certainty (which is defined as having value 60) *facts*, and a set of *associate facts*, of the form $G\alpha$ that should be added to the set of the established facts on which grounds those $G\alpha$'s are supposed to hold.

The associated facts should be given a certainty value in {10, 20, 30, 60} in the list $e_0, ..., e_5$

For instance, *facts* can be (δ) Flight AA740 with destination Chicago left New York at 5 A.M.", (β)"No plain can land in Chicago because of athmosferic bad conditions", (ε) "Radio message sent at 7:02 A.M. to pilot of AA740 informing of fact β ".

An example of associated fact to those four facts is (according to some accepted policy of the Company), ($G\alpha_1$) "The airplain will be deviated to a close to Chicago safe airport"(certainty {30, 60}).

Established facts are literals, associated facts are G-literals.

We thus take as a convention the following definition of set of facts.

6.3. Definition. A set of facts is (in addition to the usual requirements in KBS's, as for instance being the IF conditions of rules which are not THEN conditions) a set of established facts together with *all* the facts which are associated to all the *conjunctions* of the members of that set (there may be different associated facts to, say, $\alpha \wedge \beta$ than to α and β alone). The associated facts should be assigned a range of certainty in {10, 20, 30, 60}.

6.4. Remark. In order to justify our definition of rule to be given below, let us simulate the process that an expert would pursue in order to write a KBS rule having G-C logic as its underlying logic.

(i) The expert chooses as premises some literals which he considers are now absolutely true as for example δ , β , ε of the example in 6.2.

(ii) The expert proposes some G-literals as $G\alpha_1$ which are associated facts to the conjunction of the literals of (i). He then proposes the conjunction of the mentioned literals and G-literals, in our example $\delta \wedge \beta \wedge \varepsilon \wedge G\alpha_1$, as the antecedent of the rule. He also assigns to $G\alpha_1$ a linearly ordered certainty range in {10, 20, 30, 60}, as for instance, {30. 60}.

(iii) The expert proposes a disjunction of literals, as $\beta_1 \vee \beta_2$ as the consequent of the rule. The disjuncts are written from left to right according to some criterium of precedence. In the example, β_1 is judged more likely to be the consequence than β_2. Let for instance β_1 be "AA740 will land at South Bend" and β_2 "AA740 will land at Kalamazoo".

(iv) β_1 is given a certainty range that must include the range given to $G\alpha_1$, and β_2 is given a certainty range that must include the certainty range of β_1.

(v) In order to show the possibility of non-monotonicity, suppose that in addition to $\delta \wedge \beta \wedge \varepsilon$, we know at 7:15 (φ): "there is an extense high wind area between New York and Chicago that makes the planes with destination Chicago fly around New York and wait". We may now have as associated fact to $\delta \wedge \beta \wedge \varepsilon \wedge \varphi$, ($G\alpha_2$) "The AA740 flight is still at 7:50 both not far from New York and short of fuel" (range of certainty {30, 60}). Then, according to some policy of the company, the rule may be now:

$$\delta \wedge \beta \wedge \varepsilon \wedge \varphi \wedge G\alpha_2 \rightarrow \beta_3,$$

where β_3 is now "AA740 returning to New York".

These considerations lead to the following definition of rule.

6.5. Rules. A rule is a G-C entailment between a *conjunction* of literals and G-literals and a *disjunction* of literals. Each conjunction of literals in the left side of the implication must be accompanied by all the G-literals which are associate to the conjunction. The literals in the disjunction must be given by the expert or KB-builder a precedence order: if β_1 appears written at the left side of β_2, it means that β_1 is considered by the expert more likely to be the conclussion of the rule than, β_2, even though β_2 ought to be taken under consideration for the case that β_1 does not hold. Accordingly, a rule is a G-C logic entailment of the form:

$$\alpha_{11} \wedge \alpha_{12} \ldots \wedge \alpha_{1m} \wedge G\alpha_{21} \wedge \ldots \wedge G\alpha_{2q} \rightarrow \alpha_{31} \vee \alpha_{32} \ldots \vee \alpha_{3s}$$

where $G\alpha_{2i}$ (i=1,..., q) are the associated fact to the conjunction $a_{11} \wedge \alpha_{12} \ldots \wedge \alpha_{1m}$, and $\alpha_{31} \vee \alpha_{32} \ldots \vee \alpha_{3s}$ is a G-C logic disjunction of literals that have given an order of precedence by writting them from left to right in the disjunction. The literals $\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1m}, \alpha_{31}, \ldots, \alpha_{3s}$, may be preceded by the symbol of negation. Moreover, the expert assigns to each G-literal in the premise a range of certainty beginning at some value ei. On their turn, α_{31} is given a certainty range that must include the range given to G-literal having the largest range (that is less certainty) α_{32} is given a certainty range that must include the certainty range of α_{31}. and so on.

7. Treatment of uncertainty

7.1. Introduction.

KBS's expressed in the proposed G-C logic language would be difficult to manipulate because too many aspects (a special disjunction, associate facts, mutiple values, etc) ought to be considered. Nevertheless there is a way to simplify this manipulation by using Rasiowa's regular logic, which allows manipulating multiple certainty degrees by using bivalued logic techniques.

7.2. Regular many-valued logic

7.2.1. Description.

Let us consider a chain of certainty degrees, e_0, e_1, e_2,...,e_{m-1}, with $e_i < e_j$ for $i < j$. Rasiowa suggested a simplification of the use of multiple values in formulas by defining unary-connectives σ_i (i=1,...,m-1) which assign truth values in {F, T} as follows. Let $/\alpha/$ be the certainty degree (one of the e_i's above) of a formula α. $\sigma_i(\alpha)$ = T if $/\alpha/ \geq e_i$, and $\sigma_i(\alpha)$ = F otherwise. In this context a regular many-valued logic is a system which in addition to the axiom schemas of classical bivalued logic, it includes the following axiom schemas (the index i varies from 1 to m-1)

(M1) $\sigma_i(\alpha \vee \beta) \leftrightarrow \sigma_i(\alpha) \vee \sigma_i(\beta)$
(M2) $\sigma_i(\alpha \wedge \beta) \leftrightarrow \sigma_i(\alpha) \wedge \sigma_i(\beta)$,
(M3) $\sigma_i(\alpha \rightarrow \beta) \leftrightarrow \wedge j=1...i(\sigma_j(\alpha) \rightarrow \sigma_j(\beta))$,
(M4) $\sigma_i\sigma_j(\alpha) \leftrightarrow \sigma_j(\alpha)$,
(M5) $\sigma_{m-1}(\alpha) \rightarrow \alpha$,
(M6) $\sigma_i(\alpha) \rightarrow \sigma_j(\alpha)$ $j \leq i$
(M7) $\sigma_1(\alpha) \vee \neg\sigma_1(\alpha)$,
(M8) $(\sigma_i(\alpha) \rightarrow \sigma_j(\beta)) \rightarrow ((\sigma_j(\beta) \rightarrow \gamma) \rightarrow (\sigma_i(\alpha) \rightarrow \gamma))$,
(M9) $\sigma_i(\alpha) \rightarrow \sigma_i(\alpha) \vee \sigma_j(\beta)$,
(M10) $(\sigma_i(\alpha) \rightarrow \sigma_k(\gamma)) \rightarrow$
$((\sigma_j(\beta) \rightarrow \sigma_k(\gamma)) \rightarrow (\sigma_i(\alpha) \vee \sigma_j(\beta) \rightarrow \sigma_k(\gamma)))$,
(M11) $\sigma_i(\alpha) \wedge \sigma_j(\beta) \rightarrow \sigma_i(\alpha)$,
(M12) $\sigma_i(\alpha) \wedge \sigma_j(\beta) \rightarrow \sigma_j(\beta)$,
(M13) $(\sigma_k(\gamma) \rightarrow \sigma_i(\alpha)) \rightarrow$
$((\sigma_k(\gamma) \rightarrow \sigma_j(\beta)) \rightarrow (\sigma_k(\gamma) \rightarrow \sigma_i(\alpha) \wedge \sigma_i(\beta)))$,
(M14) $(\sigma_i(\alpha) \rightarrow (\sigma_j(\beta) \rightarrow \gamma)) \rightarrow$
$((\sigma_i(\alpha) \wedge \sigma_j(\beta)) \rightarrow \sigma_k(\gamma))$,
(M15) $(\sigma_i(\alpha) \wedge \sigma_j(\beta) \rightarrow \gamma) \rightarrow$
$(\sigma_i(\alpha) \rightarrow (\sigma_j(\beta) \rightarrow \sigma_k(\gamma)))$,
(M16) $\sigma_i(\alpha) \wedge \neg\sigma_i(\alpha) \rightarrow \sigma_j(\beta)$,
(M17) $\sigma_i(\alpha) \rightarrow (\sigma_i(\alpha) \wedge \neg\sigma_i(\alpha) \rightarrow \sigma_j(\beta))$,
(M18) $\alpha \rightarrow \sigma_1(\alpha)$,
(M19) $\sigma_i(\alpha) \rightarrow \sigma_j(\beta) \vee \sigma_i(\alpha)$,
(M20) $\neg\sigma_i(\alpha) \leftrightarrow \sigma_{m-i}(\neg\alpha)$.

The inference rule is *Modus Ponens.*

Suppose we have the sequence of the p-e labels (see 5.2.2), e_0 = 2 or 3, e_1 = 4 or 6, e_2 = 12, e_3 = 10, e_4 = 20 or 30, e_5 = 60.

Let A be any literal or G-literal. Let us denote by v(A) the p-e label assigned to A. We make /A/ range over the indexes 0, 1, ..., 5 in such a way that /A/= i iff v(A) = e_i. For instance, v(A) = 10 iff /A/ = 3.

Applying to the indexes 0, 1, 2, ..., 5 the operators σ's one obtains:

/A/	0	1	2	3	4	5
σ_1(A)	F	T	T	T	T	T
σ_2(A)	F	F	T	T	T	T
σ_3(A)	F	F	F	T	T	T
σ_4(A)	F	F	F	F	T	T
σ_5(A)	F	F	F	F	F	T

The consideration of the table for the given example (for any other sequence the argument is similar) suggests the following interpretation of the results of application of operators σ_i's to formulas. σ_{m-1} makes true the literals or G-literals A such that v(A) is exactly 60 (/A/ = 5), σ_3 makes true any formula A such that v(A) $\in$ {e_3, e_4, e_5} ($/\alpha/ \in$ {3, ..., 5}), ..., in general σ_i makes true the literals or G-literals A such that v(A) $\geq e_i$ (that is $/A/ \geq$ i).Then σ_i(A) is a measure of the range of certainty of A because to state that σ_i(A) is true means that A has a certainty degree ranging over the set {e_i,..., e_5} . On the other hand, stating for instance that v(A) is exactly e_i (/A/ = i), it means that both formulas in the pair (σ_i (A),$\neg\sigma_{i+1}$(A)) are true; v(A) = e_0 (/A/ = 0) means $\neg\sigma_1$(A) is true.

7.2.3. Facts.

In this context, to state for instance that an associated fact $G\delta$ is given a certainty value v($G\delta$) = 10 (e_3), means having $\sigma_3(G\delta)$ and $\neg\, \sigma_4(G\delta)$. We agree in giving (non-associate) facts the p-e value 60.

7.2.4. Rules.

Similarly to it has been explained for facts, G-C- rules must be changed into σ_i-expressions. For the sake of simplicity we work with the rule in 6.4 $\delta \wedge \beta \wedge \varepsilon \wedge G\alpha_1 \rightarrow \beta_1 \vee \beta_2$ which, by definition of the connective $\vee$, results in:

$$\delta \wedge \beta \wedge \varepsilon \wedge G\alpha_1 \rightarrow (\neg\beta_1 \rightarrow \beta_2).$$

Now, let {30, 60} be the certainty range for $G\alpha_1$, let also {30, 60} and {10, 30, 60} respectively be the certainty ranges of β_1 and β_2.

The final reformulation of the rule above has then the form $\delta \wedge \beta \wedge \varepsilon \wedge \sigma_4 G\alpha_1 \rightarrow (\sigma_4\neg\beta_1 \rightarrow \sigma_3\beta_2)$, which is a formula of classical bivalued logic We can apply to this formula the axiom of classical bivalued logic $(a \wedge b \rightarrow g) \leftrightarrow (a \rightarrow (b \rightarrow g))$, obtaining as a final result the rule:

$$\delta \wedge \beta \wedge \varepsilon \wedge \sigma_4 G\alpha_1 \wedge \neg\sigma_4\beta_1 \rightarrow \sigma_3\beta_2$$

If the rules have more than two disjuncts, the same process must be applied. We obtain then a set of rules of bivalued classical logic, subject to the axioms M1 to M20 above, and with just one consequent.

8. Forward reasoning inconsistency

Once the transformations described in 7 for both facts and rules have been performed, one has both the base of facts and the KB transformed into σ_i-expressions. These can be now manipulated using Modus Ponens as for bivalued logics. *If trough this manipulation, the pair $(\sigma_i(\alpha), \neg\sigma_j(\alpha))$ for $i \geq j$ is found, a contradiction (better said, a conflict) exists.* For instance, for m=6, to state $\sigma_2(\alpha)$ means that $v(\alpha)$ ranges over the set $\{e_2, e_3, e_4, e_5\}$, and to state, say, $\neg\sigma_2(\alpha)$, that is $\sigma_{6-2}(\neg\alpha)$, means that $v(\neg\alpha)$ ranges over the set $\{e_4, e_5\}$; this embodies an overlapping of values for $v(\alpha)$ and $v(\neg\alpha)$; in other words, α and $\neg\alpha$ are being assigned some common values in the model.

Other conflicts may arise from constraints as declaring $\sigma_i(\alpha)$ and $\sigma_j(\beta)$ to be incompatible.

It has to be remarkeed that, by using G-C logic, we have in addition to the given KB rules, a set of generated rules, which "have sense" because they keep expressing relevant relations between the literals that appear in the original KB set of rules. All the transformations described in 7 should be applied to both the set of given KB rules and the set of generated rules, and consistency refers to the union of both sets.

Applying forward reasoning to the set of given and generated rules is a larger process than the mere application to just the given rules. Nevertheless such a process may bring to light inconsistencies that migh be hidden in the ordinary process.

There is also a theoretical advantage when dealing with the augmented set of rules, that consists in the possibility of constructing more propoer algebraic models for Verification than those arising from the consideration of just the set of given rules .

9. Backwardard reasoning inconsistency

(vi) *Backward reasoning conflict for a given set of facts* is a question of inadequacy rather than of logical inconsistency. It consists in that, once a goal q and a set Γ of facts expressed as σ_i -formulas, the conjunction of which is p, are proposed, at least one of the next two conditions hold; (1) the goal is not reachable by forward reasoning from the facts in Γ , (2) the following two conditions hold *simultaneously*: (2-1) $p \rightarrow \alpha$ for any formula or σ_i -formula α formed using the language that has as predicates just but all the predicate symbols appearing in the KB, (2-1) for all formula or σ_i -formula β of the language mentioned in (2-1), $\beta \rightarrow q$ holds. The two conditions in (2) imply that q is reachable from Γ through any sequence of implications starting in the conjunction of the elements of Γ. So, backward reasoning conflict means that q can be reached from Γ following conflicting paths in the sense that there are at least two chains of implications beginning in the conjunction of the elements of Γ and ending in q, such that an arrow in the first chain has a formula α as antecedent or consequent of the implication and another implication in the second chain has $\neg\alpha$ also as antecedent or consequent. This is similar to proving some mathematical theorem by means of proofs that contradict each other.

Acknowledgements.

This work is supported by the Spanish Comission for Science and Technology CICYT

References

Cheng, J., Ushijima, K., 1990. Nonmonotonic Reasoning Based on Entailment Logic., in Proc. Fourth International Symposium in Knowledge Engineering, pp 192-198, Barcelona.

Cheng, J., Ushijima, K.,1992. Conditional Relation Calculus as the Logical Basis of Reasoning Rule Generation in Knowledge Based Systems, in Proc. Fifth International Symposium in Knowledge Engineering, pp 313-320, Sevilla.

Gabbay, D. M., 1982. Intuitionistic basis for non monotonic logic. in Proc. Conf. on Automated Deduction, Springer Lecture Notes in Computer Science. Vol 138, 260-273.

Rasiowa, H., 1974.. An algebraic approach to non-classical logics. *Studies in Logic and the Foundations of Mathematics*, vol. 78, North Holland