DIAMOND: Diagrammatic Reasoning System Demonstration*

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Abstract

We demonstrate an interactive diagrammatic reasoning system DIAMOND, which proves theorems of natural number arithmetic. The user constructs concrete proofs of ground instances of a theorem by applying geometric operations to a diagram. DIAMOND then automatically derives from these example proofs a generalised proof, called a schematic proof, and checks that this is indeed a proof of the theorem.

DIAMOND (**Diagrammatic** Reasoning and Deduction) is a diagrammatic proof system implemented in the functional programming language Standard ML of New Jersey version 109. For detailed information on DIAMOND, the reader is referred to (Jamnik, Bundy, & Green 1997) and our paper entitled *Verification of diagrammatic proofs* published in this volume.

In DIAMOND we exploit the property that diagrams can be drawn for concrete instances of theorems. Instead of using abstractions to express general diagrams, DIAMOND captures the generality of the diagrammatic proof with a recursive program which when instantiated for each value of a parameter generates a proof for the corresponding instance of a theorem. The extraction of the recursive program consists of three steps:

- the *interactive* construction of example proofs,
- the *automatic* extraction of a schematic proof,
- the *automatic* verification of this schematic proof.

Ground Instances of a Diagrammatic Proof

An example proof is constructed interactively with the user. It consists of a sequence of geometric operations that need to be applied to the diagram. The geometric operations capture the inference steps of the proof. This sequence in some way justifies, *i.e.* proves, some ground instance of the theorem. (Jamnik, Bundy, & Green 1997)

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Schematic Proof

DIAMOND abstracts the concrete, interactively constructed example proofs in order to extract a schematic proof that will be applicable to any ground instance. A schematic proof captures the generality using the general number of applications of geometric operations on a diagram. This number of applications is some function of a parameter n, where n is a natural number. If two instantiations of a proof procedure have a common structure, then this structure is automatically extracted and abstracted by DIAMOND. An existing proof rule of logic, namely the constructive ω-rule is used to justify that such general schematic proofs do indeed form a formal proof (Jamnik, Bundy, & Green 1997)

Verification

The schematic proof is an educated guess of a general proof induced by DIAMOND's abstraction mechanism. It needs to be formally verified that the schematic proof proves the theorem at hand. In particular, we need to show that for any instance n a schematic proof generates a correct proof of a proposition $P(n)$. DIAMOND automatically checks the correctness of a schematic proof by carrying out a meta-level verification proof in a theory of diagrams which models the processes in DIAMOND. (see our paper in this volume)

Results

Currently, we have diagrammatically proved with DIAMOND approximately 15 theorems or varying depth and range. We are working on extending DIAMOND to prove more theorems. This proves to be a promising approach to emulating an informal human reasoning with diagrams on machines.

References

Jamnik, M.; Bundy, A., and Green, I. 1997. Automation of diagrammatic reasoning. In Pollack, M., ed., *Proceedings of the 15th IJCAI*, volume 1, 528–533. International Joint Conference on Artificial Intelligence. Also published in the Proceedings of the 1997 AAAI Fall Symposium on Reasoning with Diagrammatic Representations II.