Relating Decision under Uncertainty and MCDM Models

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Abstract

We point out the striking similarity between decision under uncertainty and multicriteria decision making problems, two areas which have been developed in an almost completely independent way until now. This pertains both to additive and non-additive (including qualitative) approaches existing for the two decision paradigms. This leads to emphasize the remarkable formal equivalence between postulates underlying these approaches (like between the "sure-thing principle" and mutual preferential independence of criteria), and to emphasize the benefit of importing uncertainty-originated notions to multicriteria evaluation (for weighting the importance of (coalitions of) criteria especially). Eventually, we also take advantage of the relation between possibilistic logic and multicriteria evaluation in the qualitative setting for representing and handling context-dependent,"nonmonotonic", aggregation attitudes.

Introduction

For a long time, Artificial Intelligence had not been much concerned by decision issues. However, many reasoning tasks are more or less oriented towards decision or involve decision steps. In the last past five years, decision under uncertainty has become a topic of interest in AI. The application of classical expected utility theory to planning under uncertainty and the algorithmic issues raised by its implementation have been specially investigated (e.g., [7], [5]) as well as a search for more qualitative approaches [4]. or the use of game theoretic paradigms [24]. It is worth noticing that AI in these works does not always play the role of an enduser with respect to decision theory. AI brings also new, more qualitative, frameworks for describing the incomplete and uncertain knowledge about the world, via nonmonotonic logics for instance, or for representing the decision maker's preferences. It.should be noted that uncertainty and preferences are not then directly assessed by means of probability or utility functions, but are rather expressed through collections of pieces of information which implicitly constrain such functions.

Interestingly enough, AI research has not much considered multicriteria decision making (MCDM) until now. One reason may be that the two topics, decision under uncertainty and MCDM, have been studied by two different schools, in a completely independent way, in spite of the formal similarity of the standard tools for these two paradigms (expected utility vs. weighted average of the degrees of satisfaction of the criteria). It suggests some analogy between the two types of problems, and raises the question of the similarity of the postulates which lead to these additive models, as well as the one of the parallel between non-standard, i.e non-additive models which have been developed more recently for both decision under uncertainty and MCDM.

Besides, the multicriteria decision problem is usually viewed in these models as the joint satisfaction of the set of criteria, with or without compensation between the levels of satisfaction, taking into account the levels of importance of the criteria. The aggregation has thus a "conjunctive" flavor even if the conjunction is not really of a logical type when compensation is allowed. However, the subjective evaluation, of a possible decision, of an object, according to several criteria, is often in practice, not as simple as such a conjunctive attitude. One criterion, or a set of criteria may have a veto effect if they are not all satisfied, or on the contrary, it or they may be enough for favoring a high evaluation (whatever the level of satisfaction of the other criteria is) in case it or they are satisfied to a sufficient level. The description of an evaluation procedure may also involve conditional statements, and exhibits some form of nonmonotonicity if, for instance, when a criterion cannot be satisfied, the alternatives are evaluated according to some other criterion not considered in the general situation when the first criterion is satisfied. The modelling of such aggregation attitudes may then benefit of AI methods.

The paper is organized as follows. Section 2 makes a parallel between decision under uncertainty and MCDM. Section 3 presents theoretical results for each of the two decision problems, some classical ones for the additive settings, and some newer ones in non-additive or qualitative frameworks. Section 4 suggests some logical modellings of complex multicriteria evaluations in the qualitative setting based on max and min operations.

The two decision paradigms

Let us first consider the decision under uncertainty problem. Let $S = \{s_1,...,s_n\}$ be a set of possible states of the world, and X be a set of possible consequences x. An act, or a decision f is then viewed, following Savage [20] as a mapping from S to X, which leads to a consequence $x = f(s)$ when performed in state s. Starting from postulates that a relation of preference in X^S, the set of potential acts, should fulfill, Savage has shown that choosing between acts amounts to choosing the one(s) maximizing an expected utility of the form (in a finite setting):

$$U(f) = \sum_i p(s_i)u(f(s_i)) \qquad (1)$$

where p is a probability distribution over S and u a real-valued utility function over X. More recently, other types of integrals have been justified (under other postulates) for ranking the acts (see Section 3) by a purely ordinal setting, (where scales for uncertainty and preferences are sets of linearly ordered levels). The following estimate has been proposed for ranking acts

$$U_\pi(f) = \min_i[\max(u(f(s_i)),(1-\pi(s_i)))] \qquad (2)$$

where the utility function u and the possibility distribution π are mapped into the same ordinal scale (commensurability assumption of the preference and of the uncertainty scales). 1- (.) is just here for denoting the order-reversing map on the scale. Clearly (2) favors decisions for which there does not exist a state which is both highly possible and leads to poor consequences.

Let us turn now towards multiple criteria decision, Let C be a set of n criteria. We furthermore assume that each decision f can be mapped, according to criterion i, by means of a function u_i from the set D of decisions to the same evaluation scale. Then, examples of multicriteria evaluations are

$$U(f) = \sum_i \alpha_i u_i(f), with \sum_i \alpha_i = 1 \qquad (3)$$

$$U(f)=\min_i[\max((1-\alpha_i),u_i(f))], \text{ with } \max_i \alpha_i=1 \quad (4)$$

where (3) is the classical weighted average aggregation while (4) is a weighted conjunction (if the level of importance α_i is 0, the bottom element of the scale of criterion i is not taken into account, while if for all i, $\alpha_i = 1$, the top element in the scale, we recover the logical, conjunctive, min aggregation). See section 3 for justification of (3) and (4). Note that (4) can be viewed as the degree of inclusion of the fuzzy set of important criteria into the set of more or less satisfied ones.

The resemblance between (1) and (3), and between (2) and (4) is striking. *Each criterion* in (3) or (4) *corresponds to a state* in (1) or (2), and the levels of importance α_i are analogues to probability (or possibility) weights over S. Dubois and Prade [10] have pointed out that (3) and (4) can be viewed respectively as the probability and the necessity of a fuzzy event made by the set of criteria satisfied by f, viewing the α_i's as a (probability or possibility) distribution. Then, α_i can be viewed as the level of probability (or possibility) to be in a state s_i where criterion i is used to evaluate act f. A multiple criteria evaluation problem is then equated to the problem of satisfying an uncertain criterion. It leads to interpret $U(f)$ in (3) as the expected utility to satisfy the "right" criteria, to read (4) as the uncertainty of satisfying the right "criteria". In latter case, it amounts to prefer the acts d which satisfy all the criteria which have a high possibility to be the right one (i.e. all the important/highly possible criteria, according to the chosen interpretation, should be satisfied as much as possible). The next section investigates the correspondence between MCDM and decision under uncertainty problems, in a broader and more theoretical way.

Theoretical issues

Non-additive measures

It has been known for a long time that additive set functions (e.g probability measures) were not well-adapted to represent all the facets of human behavior. Thus, we are naturally led to introduce non-additive set functions to deal with uncertainty. In the following, non-additive set functions are either used for uncertainty modelling, or for weighting coalitions of criteria.
In this section $X = \{ x_1,\dots, x_n \}$ will denote a finite set and $P(X)$ the set of subsets of X.

Definition 1 *A non-additive measure (also called fuzzy measure) on $(X,P(X))$ is a set function μ: $\mathcal{P}(X) \to [0,+\infty]$ such that $\mu(\phi) = 0$ and if $A \subset B$, then $\mu(A) \le \mu(B)$ that is μ is a non decreasing set function.*

Note that this definition encompasses the notions of probability measures, possibility and necessity measures, the belief functions of Shafer [22] that were already known and used in the AI community. Let us give the definitions of the main integrals (in the finite case) of non-additive measures. See [6] and [26] for the original articles.

Definition 2 *Let μ be a non-additive measure on (X, $\mathcal{P}(X)$) and an application f: $X \rightarrow [0,+\infty]$.*
The Choquet integral of f w.r.t μ is defined by:

$$(C)\int f d\mu = \sum_{i=1}^{n} (f(x_{(i)}) - f(x_{(i-1)}))\mu(A_{(i)})$$

where the subscript (.) indicates that the indices have been permuted in order to have $f(x_{(1)}) \leq \ldots \leq f(x_{(n)})$, $A_{(i)}=\{x_{(i)}, \ldots, x_{(n)}\}$ and $f(x_{(0)}) = 0$ by convention.

Definition 3 *Let μ be a non-additive measure on (X, $\mathcal{P}(X)$) and an application f: $X \rightarrow [0,+\infty]$.*
The Sugeno integral of f w.r.t μ is defined by:

$$(S)\int f \circ \mu = \max_{i=1}^{n}(\min(f(x_{(i)}), \mu(A_{(i)})))$$

with the same notations and conventions as above.

From a formal point of view, the Choquet integral and the Sugeno integral differ only with the operators used in their definition; respectively + , × and max, min. Nevertheless, they are very different in essence, since the Choquet integral is more adapted to numerical problems and the Sugeno integral is better suited for qualitative problems (note that 2 and 4 are Sugeno integrals).

Additive measures in decision theory

Decision with uncertainty: Savage [20] has shown that under a set of seven conditions on the preference relation $\succeq$ on the acts, there exists a unique probability measure P on the states of the world S and a unique utility function u: $X \rightarrow$ R that represent the preference relation in the sense that: $f \succeq g \Leftrightarrow \int_S u(f)dP \geq \int_S u(g)dP$ where f and g are two acts. Since we are mostly interested in the additivity property, we present only the second condition of the seven that is responsible for the additivity of P.

Definition 4 *We say that $\succeq$ verifies the independence with respect to equal subalternatives iff for every acts f,g, f',g' such that, on $B \subset S$ we have $f = f'$ and $g = g'$ and on B^c we have $f = g$ and $f' = g'$, then $g \succeq f$ implies $g' \succeq f'$.*

This property is known as the sure-thing principle. Anscombe and Aumann [1] have proposed a less general but simpler concept where the set of consequences considered is only the set of money lotteries. In this framework, the independence property is defined as follows:

Definition 5 *A preference relation $\succeq$ is said to be independent iff for all acts f,g, h, and every real $\alpha \in [0,1]$, $f \succ g \Rightarrow \alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$.*

Then, there exists a unique probability measure P on S and a unique utility function u that represent $\succeq$ iff, $\succeq$ is a weak order that satisfies some continuity and monotonicity properties and is independent.

Multicriteria decision making: We denote by ($X = \times_{i \in C} X_i$, $\succeq$) a MCDM problem. where C is the set of criteria. X the set of consequences, and $\succeq$ a preference relation on X. Acts (or decision) are simply elements of X, denoted $x,y\ldots$ For brevity, we define $X_J = \times_{i \in J} X_i$ for any $J \in C$, and x_J denotes an element of X_J. A verv impoltallt notion in multicritcria decision making is that of preferential independence.

Definition 6 *$J \subset C$ is said to be preferentially independent of J^C iff for every c $x_J, y_J \subset X_J$ for every $x_{JC}, y_{JC} \in X_{JC}$* we have $(x_J, x_{JC}) \succeq (y_J, x_{JC}) \Leftrightarrow (x_J, y_{JC}) \succeq (y_J, y_{JC})$ *that is, the preference on the criteria of J is not influenced by the other criteria. If every $J \subset C$ is preferentially independent, the criteria are said to be mutually preferentially independent.*

We suppose now we have utility functions u_i: $X_i \rightarrow$IR and we want to define a global utility function U: $X \rightarrow$IR representing $\succeq$. Murofushi [19], has shown that if U is expressed as a Choquet integral w.r.t a non-additive measure μ on C (more on this in section 3.3), then μ is additive iff the criteria are mutually preferentially independent if there are at least three attributes for which the preference relation is non-trivial. In fact, Debreu [8] had given a first result concerning additive representations of preference relations in multicriteria decision making before Murofushi that was not restricted to a Choquet integral representation but had the drawback of having topological assumptions difficult to verify in practice.

In both cases, additivity of the utility function was implied by similar axioms: the sure-thing principle and the mutual preferential independence. The similarity becomes clear if we identify S and C, $f(s_i)$, $s_i \in S$ and x_i ,$i \in C$, B and J, taking formally $f=(y_J, x_{JC})$, $g=(x_J, x_{JC})$, $f'=(y_J, x'_{JC})$, $g'=(x_J, x'_{JC})$.

Recently, Bacchus and Grove [2] have extended preferential independence to goals described by formulas of propositional logic, in the scope of describing qualitative information on multiple attribute utility functions.

Using non-additive measures

Decision under uncertainty: We now come to the non-additive refinement of the previous results and see how non-additive measures can avoid the problems encountered in the additive case like Ellsberg or Allais paradoxes. Using the previous result of Savage, we know that representing the preference relation in a non-additive way reduces to weakening the independence hypothesis, that is the sure-thing principle. In the additive case, even in the less general approach of Anscombe and Aumann the independence hypothesis is not always verifed. Schmeidler [21] has proposed a non-additive version of Anscombe-Aumann theorem. With the same continuity and monotonicity hypothesis, the result holds if independence is replaced by co-monotonic independence that is independence restricted to co-monotonic acts (acts inducing the same ordering on the states of the world) and probability measure is replaced by non-additive measure. This result is the starting point of a non-additive expected utility theory giving the opportunitv to understand the human processes involved in decision making more deeply. It has been refined by Wakker [28], who has weakened the independence hypothesis of co-monotonicity to what he called max-min independence, and has established a similar representation theorem.

Multicriteria decision making: Non additive measures and integrals have been used as well in multicriteria decision making (MCDM) [12], giving much more flexibility than additive approaches. Unfortunately, there is no corresponding theorem to the one of Murofushi, giving a characterization of non- additive measures or even particular cases such as belief functions and the Choquet integral in MCDM. However, recent results and formalisms about non- additive measures have shed some light on their interpretation in MCDM [13,15].We develop this point below.

Let us denote X a set of n criteria. By analogy witll cooperative game theory, a non additive measure μ defined on X assigns to every *coalition* $A \subset X$ of criteria a number in [0,1] giving the importance of the coalition A for the decision problem in consideration. A well-known concept in cooperative game theory is the one of Shapler value of a game [23]. expressing the average importance of each player in the game. Murofushi has applied this concept to MCDM, obtaining a convenient and theoretically well founded notion of importance of criteria.

A second key concept here is the one of interaction. Although it was felt from a long time that non additive measures can model some kind of interaction between criteria, this was not formalized until Murofushi proposed a definition of an interaction index I_{ij} for a pair of criteria i, j, borrowing concepts from multiattribute utility theory:

$$I_{ij} = \sum_{K \subset X \setminus \{i,j\}} \xi(|K|)[\mu(K \cup \{i,j\}) - \mu(K \cup i) - \mu(K \cup j) + \mu(\{i,j\})]$$

with

$$\xi(k) = \frac{(n-k-2)!k!}{(n-1)!}$$

Recently, Grabisch [13] extended this definition to any number of criteria, leading to what was called an *interaction representation* of non additive measures, encompassing Shapley value and I_{ij}. This representation through interaction indices happens to be much closer to the decision maker's mind than the usual measure representation.

It has been shown that if two criteria i, j have a positive interaction I_{ij}, then it means that they act in a *conjunctive way*, i.e. *both* of them have to be satisfied in order to have some impact on the decision (complementary criteria). By contrast. if I_{ij} is negative, criteria *i,j* act in a *disjunctive* way, which means that one of the two is sufficient (redundant or substitutive criteria). Another effect which can be modelled by interaction indices is the *veto* effect. A criterion i is said to be a veto if the utility function can be written as follows

$$u(x_1,\ldots,x_n)=u_i(x_i) \wedge u'(x_1,\ldots,x_n)$$

It means that the utility u cannot exceed the one of criterion i. The dual effect, where $\wedge$ is replaced by v, is called *the favor* effect. If u is expressed under the form of a Choquet integral (as in the theorem of Murofushi above [19], see 3.2, it is possible to model these two effects by a suitable choice of the non additive measure μ. It has been shown that non additive measures whose corresponding I_{ij} are positive (resp. negative) for any j model a veto effect (resp. a favor effect) on criterion i.

These results, along with other related to the interaction representation, lay the foundations to a comprehensive use of non additive measures in MCDM.

A question arises about the interpretation of such results in decision under uncertainty, namely:
What is the meaning of interaction in the framework of decision under uncertainty? We already know that the Shapley value corresponds to the pignistic transformation of belief functions (by taking the center of gravity of the set of probabilities compatible with the belief function) proposed by Smets [25]. It remains to find the corresponding transformation for I_{ij} and higher order indices.

Qualitative decision

We turn now to decision making where only qualitative (ordinal) information is available. In decision making under uncertainty, a qualitative equivalent of the Von Neuman Morgenstern (VNM) utility model has been proposed by Dubois and Prade [11]. We present it briefly, in order to clarify its analogy with qualitative MCDM.

We consider the set S of states of the world, X the set of consequences, and a given act f, thus a mapping from S to X . The original probabilistic VNM approach tends to model the preference of the decision maker on the different probability distributions on S (called "lotteries"). We deal here with "possibilistic lotteries" on S, taking values on a ordinal scale L. Thus for any π in L^s, π (s) expresses the belief that the true state of the world is s. We denote by $\succ$ the preference relation of the decision maker on the different possibilistic lotteries, and we consider that utilitvy functions on the consequences are also qualitative, taking values on the same ordinal scale L (commensurability assumption) .

Under a set of 6 conditions on $\succ$, which can be said to be the qualitative counterpart of the VNM model. Dubois and Prade have shown that there exists a utility function u: $X \to L$ which represents the preference of the decision maker, i.e. $\pi_1 \succ \pi_2 \Leftrightarrow U(\pi_1) > U(\pi_2)$ with

$$U(\pi) = \min_{s\in S}[\max((1-\pi(s)), u(f(s)))] \qquad (5)$$

It is easy to show that U can be written as well as a minimum on the set of consequences:

$$U(\pi) = \min_{x\in X}[\max((1-\prod(f^{-1}(x))), u(x))] \qquad (6)$$

where $\prod$ is the possibility measure generated by π .

Let us turn now to qualitative MCDM. We consider as before that a non additive measure μ, defined on S the set of criteria, is able to model the importance of coalitions of criteria on a qualitative scale L. We consider an act f, described by a vector of consequences $[x_i]_{i\in S}$ on $\times_{i\in S}X_I$, and qualitative utility functions u_i: $X_I \to L, i\in S$. Of course, the Choquet integral is of no use here, but we can use instead the Sugeno integral, which can be said to be the qualitative counterpart of the Choquet integral, possessing very similar properties [16]:

$$U(f)=\max \min[u_i x_{(i)}, \mu(A_{(i)})]$$

with notations of Sec. 2. It can be shown that if μ is a necessity measure generated by a possibility distribution π, then $U(f) = \min_{i\in S}[u_i(x_i)\ \max(1-\pi(\{i\}))]$, an expression which is the same as (5) in the case of decision under uncertaintv. Thus again. the parallel has been established between multicriteria decision making and decision making under uncertainty. An important question in the qualitative approach is how to define Shapley value and the interaction indices, since original definitions are obviously suitable only for the quantitative case. Based on the qualitative coun- terpart of the founding axioms of Shapley value, Grabisch [14] has recently shown that the only reasonable definition of Shaplev value which estimates the contribution of criterion i to a coalition, seems to be

$$s_{\max(i)} := \max_{A\subset X\setminus\{i\}} [\mu(A\cup\{i\} \neg_v \mu(A)].$$

where $a \neg_v b$ = a if a>b, and 0 otherwise. It defines a possibility distribution; indeed, when μ is a possibility rneasure $s_{\max(i)}$ is nothing but the possibility π({i}) (in the same way as the classical Shapley value gives back the probability of i when μ is a probability). This permits to define interaction indices as in the numerical case.

Lastly, we mention that veto and favor effects. which could be modelled as the Choquet integral w.r.t. some suitable non additive measure, can be modelled as well by thc Sugel1o integral w.r.t. the *same* measures.

Towards a logical setting

The weighted min combination considered in the previous 2 sections can be easily adapted in order to handle context-dependent specifications by turning the weights into degrees of truth which are equal to 1 if the context condition is fulfilled and are 0 otherwise. Thus, if criterion C has to be considered only if proposition p is true, the evaluation function μ_C is turned into max ($\mu_C(f)$, 1 - $v_f(p)$) where $v_f(p)$ denotes the true value of p when f is applied. This enables us to handle conditional prioritized requirements of the form "C_1 should be satisfied, and among the solutions to C_1 (if any) the ones satisfying C_2 are preferred, and among satisfying both C_1 and C_2, those satisf'ying C_3 are preferred and so on", where C_1, C_2, C_3 … are here supposed to be classical constraints (i.e $\mu C_i = 0$ or 1); such requirements have been considered in the database setting by Lacroix and Lavency [17]. Thus, one wish to express that C_1 should hold (with importance or priority $\alpha_1 = 1$), and that if C_1 holds, C_2 holds with priority α_2, C3 holds with priority α_3 (with $\alpha_3 < \alpha_2 < \alpha_1$). Using the representation of' conditional requirements presented above, this nested conditional requirement can be represented by the expression:

$$U(f)=\min(\mu_{C1}(f), \max(\mu_{C2}(f), 1-\min(\mu_{C1}(f),\alpha_2)) \quad \max(\mu_{C3}(f), 1-\min(\mu_{C1}(f), \mu_{C2}(f), \alpha_3)) \qquad (7)$$

It reflects that we are completely satisfied if C_1, C_2 , C_3 ($U(f)$ = 1) are completely satisfied, we are less satisfied ($U(f)$ = 1 - α_3) if C_1 and C_2 only are satisfied, and we are even less satisfied if only C_1 ($U(f) = 1 - \alpha_2$) is satisfied. For instance, the requirement "if thev are not graduated, they should have professional experience, and if they have professional experience, they should preferably have communication abilities", is an example where only conditional constraints, organized in a hierarchical way occur. It will be represented by an expression of the form $U(f)=\min[\max(\mu_{prof.exp}(f), \mu_{grad}(f)), \max(\mu_{com.ad}(f), 1-\min(1-\mu_{grad}(f), \mu_{prof.exp}(f), \alpha)]$ so that if f has the professional experience and communication abilities, f completely satisfies the request, as well as if d is graduated; f satisfies the request to the degree 1- α if f is not graduated and has professional experience only. $U(f) = 0$ if d is neither graduated nor has professional experience (even if f has communication abilities).

The expression (7) can be interpreted as the semantics of the possibilistic propositional logic knowledge base $K = \{(C_1, 1) ; (\neg C_1 \vee C_2, \alpha_2); (\neg C_1 \vee \neg C_2 \vee \neg C_3, \alpha_3)\}$. Indeed , the semantics of a possibilistic logic base $K = \{(p_i, \alpha_i); i = 1, \ldots, n\}$, where p_i is a classical proposition and α_i, a level in a totally ordered scale V, is given by the function from the set of interpretations V, defined by

$$\mu_K(f) = \min_{i=1,\ldots,n} \max(v_f(p_i), 1-\alpha_i) \qquad (8)$$

Possibilistic logic has been developed for handling formulas pervaded with uncertainty, and can be used for encoding default knowledge (of the form "if C_1 generally C_2"). See [9] for details. Here, the practical interpretation is rather in terms of priority, and K reads: a good decision d should satisfy C_1 imperatively (otherwise $U(f) = 0$), also C_2 preferably (priority α_2), and if possible C_3 (with priority level $\alpha_3 < \alpha_2$) This provides a logical description of the more or less satisfying decisions.

Expressions (7) or (8) are Sugeno integrals. They correspond to conjunctive normal forms (i.e. it is a min of max). They can be turned in disjunctive normal forms (max of min) which provides a description of the different classes of decisions ranked according to their level of preference. Let us consider, for instance, the following 3-criteria based evaluation: "if f satisfies A and B, f is completely satisfactory, if A is not satisfied, worse solutions should satisfy C". Such a "nonmonotonic" evaluation function is common in MCDM (see [27] for a practical example). It can be directly represented by the disjunctive form $U(f)=\max(\min(\mu_A(f), \mu_B(f), \min(\mu_C(f), 1-\mu_A(f), 1-\alpha))$ with $\alpha < 1$. Turned into a conjunctive form, it corresponds to the base $K = \{(\neg C \rightarrow A, 1); (\neg C \rightarrow B, 1); (A \rightarrow B, 1); (A, \alpha); (B, \alpha)\}$ where $\rightarrow$ is the material implication, which provides a logical equivalent description of the requirement.

Lastly, let us illustrate the above ideas by providing the multicriteria decision counterpart of the penguin example of the nonmonotonic literature. The three corresponding defaults can be encoded by the possibilistic logic base $K; = \{(p \rightarrow b, 1); (p \rightarrow \neg f, \alpha); (b \rightarrow f, \beta)\}$ with $0 < \beta < \alpha < 1$, and p =penguin, b =bird, f =fly. Putting the semantical counterpart of K (obtained by applying (8)) in the disjunctive form, we compute the class of items ranked according to the level to which they obey the prioritized constraints in K. Namely, the models of $(f \wedge \neg p) \vee (\neg f \wedge \neg p \wedge \neg b)$ are completely satisfying (at level 1), those of $b \wedge \neg f$ at level $1 - \beta$, those of $b \wedge f \wedge p$ at the smaller level $1 - \alpha$, and those of $p \wedge \neg b$ at the level 0 (they are not acceptable at all).

In this section,we have only considered binary constraints for the sake of simplicity. However, the possibilistic framework can accommodate intermediary levels of satisfaction, by representing a flexible constraint with a set of formulas (C_j, α_j) for $j = 1 \ldots, k$, where binary constraints are nested $C_k \subset \ldots \subset C1$, with $\alpha_1 > \ldots > \alpha_k$ (i.e we must at least satisfy C_1 and should satisfy C_2 if possible and so on). Besides, we have assumed that the priority levels are given. However, as in default reasoning, they can be obtained from the constraints modelling conditional statements, which are then encoded as possibilistic logic formulae as in [3]. It is also directly related to the modelling of conditional desires as proposed by [18].

The paper has suggested that the systematic investigation of the similarities betwen MCDM and decision under uncertainty may prove fruitful for a cross-fertlization of both fields, and seems to accommodate the requirements of AI methodologies.

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