Evidence and Belief

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Abstract

We discuss the representation of knowledge and of belief from the viewpoint of decision theory. While the Bayesian approach enjoys general-purpose applicability and axiomatic foundations, it suffers from several drawbacks. In particular, it does not model the belief formation process, and does not relate beliefs to evidence. We survey alternative approaches, and focus on formal model of case-based prediction and case-based decisions.

A formal model of belief and knowledge representation needs to address several questions. The most basic ones are: (i) how do we represent knowledge? (ii) how do we represent beliefs? (iii) how is belief updated in light of new evidence?

Decision and economic theory pose additional questions. First, how are knowledge and belief reflected in decision making? Second, can one derive the model axiomatically? That is, can we characterize the model by a set of conditions on observable data, in a way that would provide an observable definition of theoretical constructs, and would also help judge the reasonability of the model from descriptive and normative viewpoints?

The Bayesian paradigm has provided answers to all these questions already in the 1950's. In this paradigm, what one knows is described by a set of possible states of the world, endowed with an algebra of events. Belief is represented by a probability measure over this algebra. The acquisition of knowledge is represented by restricting the set of states to those compatible with new evidence, and belief is updated via Bayes's rule.

This very elegant approach to knowledge and belief representation was supported by axiomatizations (Ramsey (1931), de Finetti (1937), Savage (1954), Anscombe and Aumann (1963)) that derived this epistemic model, coupled with expected utility maximization, from in-principle observed behavior. Thus the model was supported by an axiomatic decision theory that made it clear how it would be applied in economic theory, and what observations would falsify it. Moreover, the axioms underlying the model (mostly Savage's derivation) are elegant and highly compelling. No wonder that the Bayesian paradigm has become the dominant, and for many decade practically the only approach in economic theory.

Yet, the Bayesian paradigm suffers from two related drawbacks. First, in many cases it is not cognitively plausible. It does not seem to conform to the way people actually think, and it may not be practicable even as a normative theory. Second, the Bayesian approach, as practiced in the economic profession, does not provide any hint as to where the prior beliefs come from. In its pure form, the Bayesian paradigm does not leave room for a theory of generation of beliefs. In particular, it cannot describe how prior beliefs are based on evidence, because any piece of evidence that one might obtain should be used for the update of a Bayesian prior in a large enough state space, which allowed for the possibility of obtaining this evidence as well as others. Thus, the prior is, by definition, based on no evidence whatsoever. It also follows that one cannot tell a "rational" prior from an "irrational" one based on the way they are derived from evidence.[1]

The 1980's witnessed several models of non-Bayesian decision making that departed from the Bayesian representation of beliefs, though not from its representation of knowledge. These models of multiple priors (see Gilboa and Schmeidler (1989), relying on Schmeidler (1989), and Bewley (2003)) could be conceptualized as axiomatic decision theories that correspond to classical statistics: knowledge is still represented by events, but belief can here be modeled as a set of probability models, rather than a single such measure. This allowed a more realistic description of how people think about and how they behave in the face of uncertainty. It also made the choice of beliefs less arbitrary, because lack of information could be represented in these models by a larger set of measures that are considered possible.

[1] Observe that we discuss the Bayesian approach in its pure form, as it is taught in economic theory. The Bayesian approach to statistics, computer science, machine learning, and related fields is not as extreme.

Qualitative decision theory (see Brafman and Tenneholtz (1997), Dubois, Prade, and Sabbadin (1998), Dubois, Godo, Prade, and Zepico (1998)) offers another approach to modeling uncertainty, which frees the representation of belief from the numerical constraints, while retains the representation of knowledge via a state space.

A more radical departure from the Bayesian model was suggested by Gilboa and Schmeidler's case-based decision theory (1995, 2001). This work was inspired by case-based reasoning (Schank (1986), Riesbeck and Schank (1989)), and attempted to provide a formal, axiomatically-based decision theory that is based on reasoning by analogies. In this paradigm, all that one knows are cases that actually occurred. Belief is not explicitly modeled. To an extent, belief is implicit in the similarity function (which is axiomatically derived from behavior). New evidence takes the form of additional cases, which, in turn, affect future behavior.

In recent years we have started to work on formal case-based models that allow for an explicit representation of belief. Such models (Gilboa and Schmeidler (2003), Billot, Gilboa, Samet, and Schmeidler (2003), and Gilboa, Lieberman, and Schmeidler (2004)) aim to model the relationship between evidence and belief, and thereby to provide a possible account of the way in which beliefs are generated. In these models, as in case-based decision theory, knowledge is represented by cases that actually occurred, and beliefs can take the form of probability vectors or of likelihood rankings. The main results are based on a combination principle, which states, roughly, that a conclusion that is supported by two databases would also be supported by their union. These results and the limitations of the combination principle are discussed.

A major paradigm for the representation of knowledge and belief that is notable in its absence from decision and economic theory is the propositional logic paradigm. Whereas there are vast bodies of literature on the representation of knowledge and beliefs by propositions, and on the way such beliefs should be updated and revised, there is relatively little general-purpose, axiomatically-based theory on how such knowledge and belief are reflected in decision making, and what patterns of decision making are equivalent to these representations.

It appears that decision and economic theory might greatly benefit from a general-purpose decision theory that is based on propositional logic, and even more so from a decision theory that would combine the three modes of reasoning: probabilistic, logical, and analogical. It is also possible that the study of knowledge and belief representation may benefit from addressing the question of representation in the context of an axiomatic decision theory.

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