

Concurrent Actions in the Situation Calculus

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Abstract

We propose a representation of concurrent actions; rather than invent a new formalism, we model them within the standard situation calculus by introducing the notions of *global actions* and *primitive actions*, whose relationship is analogous to that between situations and fluents. The result is a framework in which situations and actions play quite symmetric roles. The rich structure of actions gives rise to a new problem, which, due to this symmetry between actions and situations, is analogous to the traditional frame problem. In [Lin and Shoham 1991] we provided a solution to the frame problem based on a formal adequacy criterion called “epistemological completeness.” Here we show how to solve the new problem based on the same adequacy criterion.

Introduction

In [Lin and Shoham 1991] we proposed a methodology for formalizing the effects of actions in the situation calculus. In this paper we extend this methodology to a framework which allows concurrent actions.

Recall that traditional situation calculus [McCarthy and Hayes 1969] is a many-sorted first-order logic with the following domain independent sorts: situation sort (s), propositional fluent sort (p), and action sort (a). There is a domain independent function $Result(a, s)$, which represents the resulting situation when a is performed in s , and a domain independent predicate $H(p, s)$, which asserts that p holds in s .

It is clear that there is an asymmetry between actions and situations in this picture. While situations are ‘rich’ objects, as manifested by the various fluents that are true and false in them, actions are ‘poor,’ primitive objects. In this paper, we propose to model concurrent actions in the situation calculus by correcting this asymmetry. We introduce the notions of *global actions*, *primitive actions*, and the binary predicate In , whose roles will be completely analogous to those of situations, fluents, and the predicate H , respectively. Intuitively, a global action is a set of primitive ac-

tions, and In expresses the membership relation between global actions and primitive actions. When a global action is performed in a situation, all of the primitive actions in it are performed simultaneously.

Formally, the extended situation calculus is a multi-sorted first-order logic with four domain-independent sorts: situation sort (s), propositional fluent sort (p), global action sort (g), and primitive action sort (a). We have a binary function $Result$ with the intuitive meaning that $Result(g, s)$ is the resulting situation of performing the global action g in the situation s . We have two binary predicates, H and In . Intuitively, $H(p, s)$ means that the fluent p is true in the situation s , and $In(a, g)$ means that the primitive action a is one of the actions in g .

For any finite set of primitive actions, A_1, \dots, A_n , we assume that $\{A_1, \dots, A_n\}$ is a global action satisfying the following properties:

$$\forall a.(In(a, \{A_1, \dots, A_n\}) \equiv (a = A_1 \vee \dots \vee a = A_n)), \quad (1)$$

and

$$\forall g(\forall a.(In(a, g) \equiv (a = A_1 \vee \dots \vee a = A_n)) \supset g = \{A_1, \dots, A_n\}). \quad (2)$$

If A is a primitive action, then we shall write $\{A\}$ as A . Most often, whether A is a primitive action or the corresponding global action $\{A\}$ will be clear from the context. For example, A is a global action in $Result(A, s)$, but a primitive action in $In(A, g)$. We shall make it clear whenever there is a possibility of confusion.

To be sure, there are other proposals in the literature for extending the situation calculus to allow expressions for concurrent actions. Most introduce new operators on actions [cf. Gelfond *et al.* 1991, Shubert 1990]. For example, in [Gelfond *et al.* 1991] a new operator “+” is introduced, with the intuitive meaning that $a + b$ is the action of executing a and b simultaneously. This approach is common also in the programming languages community. The relationships between our formalism and those with new operators for concurrent actions are delicate. It seems that our formalism is more convenient in expressing complicated

actions such as the global action where every agent makes a move.

Another way to extend the situation calculus is to think of *Result* as a relation, rather than a function. For example, we can introduce $RES(a, s_1, s_2)$ with the intuitive meaning that s_2 is one of the situations resulted from executing a , along with possibly some other actions, in s_1 . This is essentially the approach taken in [Georgeff 1986, Peleg 1987, and others]. The drawback of this approach is that it does not explicitly list the additional actions that cause the transition from s_1 to s_2 . Thus Georgeff (1986) had difficulty in formalizing the effect of a single action when performed exclusively.

A more radically different approach to concurrency is via temporal logic [Allen 1984, McDermott 1982]. For a comparison between the situation calculus approach to reasoning about action and that of temporal logic, see [Pelavin and Allen 1987, Shoham 1989].

The rest of the paper is organized as follows. In section 2 we introduce a formal criterion, called *epistemological completeness*, to evaluate theories of actions. This criterion was first introduced in [Lin and Shoham 1991] in the context of traditional situation calculus with respect to primitive actions; the extension to global actions is straightforward. In section 3, we show that this criterion not only help us clarify the traditional (fluent-oriented) frame problem, as we have done in [Lin and Shoham 1991], it also clarifies the new *action-oriented* frame problem. In section 4, we illustrate our solution to these problems using the well-known Stolen Car Problem. The solution is extended to a class of causal theories in section 5, and compared to others in the context of conflicting sub-actions in section 6. Finally, we conclude in section 7.

Epistemologically complete theories of action

In the situation calculus we formalize the effects of actions using first-order logic. However, as is well-known, adopting classical semantics leads to problems such as the frame problem, the ramification problem, *et cetera*. In [Lin and Shoham 1991] we argued that the ultimate criterion against which to evaluate theories of (primitive) actions is what we called *epistemological completeness*. The various problems amount to achieving this completeness in a precise and concise fashion. We shall see that, when we introduce concurrency, new problems arise. However, the notion of epistemological completeness is relevant also to their definition and solution. This section therefore repeats some key definitions from [Lin and Shoham 1991], appropriately modified to the context of global actions.

In that paper, and here as well, we concern ourselves only with deterministic actions, which map a situation into a unique other one. Intuitively, a theory of a (deterministic) action is epistemologically complete

if, given a complete description of the initial situation, the theory enables us to predict a complete description of the resulting situation when the action is performed. In order to formalize this intuition, we first introduce the notion of *states*, which are complete descriptions of situations with respect to the set of the fluents we are interested in.

In the following, let \mathcal{P} be a fixed set of ground fluent terms in which we are interested. This fixed set of fluents plays a role similar to that of the *Frame* predicate in [Lifschitz 1990].

Definition 1 *A set SS is a state of the situation S (with respect to \mathcal{P}) if there is a subset \mathcal{P}' of \mathcal{P} such that*

$$SS = \{H(P, S) \mid P \in \mathcal{P}'\} \cup \{\neg H(P, S) \mid P \in \mathcal{P} - \mathcal{P}'\}.$$

Therefore, if SS is a state of S , then for any $P \in \mathcal{P}$, either $H(P, S) \in SS$ or $\neg H(P, S) \in SS$.

Thus we can say that a first-order theory T is epistemologically complete about the global action G (with respect to \mathcal{P}) if it is consistent, and for any ground situation term S , any state SS of S , and any fluent $P \in \mathcal{P}$, either $T \cup SS \models H(P, Result(G, S))$ or $T \cup SS \models \neg H(P, Result(G, S))$, where \models is classical first-order entailment.

However, as is well-known, classical entailment is not the only choice, we may also interpret our language nonmonotonically according to a nonmonotonic entailment. Indeed, the notion of epistemological completeness is not limited to monotonic first-order theories. In general, for any given monotonic or nonmonotonic entailment \models_{\square} , we can define epistemological completeness as follows:

Definition 2 *A theory T is epistemologically complete about the action G (with respect to \mathcal{P} , and according to \models_{\square}) if $T \not\models_{\square} \text{False}$, and for any ground situation term S , any state SS of S , and any fluent $P \in \mathcal{P}$, there is a finite subset SS' of SS such that either $T \models_{\square} \bigwedge SS' \supset H(P, Result(G, S))$ or $T \models_{\square} \bigwedge SS' \supset \neg H(P, Result(G, S))$.*

We note that for any sets T , SS , and formula φ , $T \cup SS \models \varphi$ if there is a finite subset SS' of SS such that $T \models \bigwedge SS' \supset \varphi$. Thus if we replace \models_{\square} in Definition 2 by classical entailment \models , we get the same definition we have earlier for monotonic first-order theories.

The frame problems

There are a number of well-known problems in formalizing the effects of actions. The most famous one is the frame problem [McCarthy and Hayes 1969], which is best illustrated by an example. Consider a primitive action *Paint* which paints *Block10* blue:

$$\forall s. H(C(Block10, Blue), Result(Paint, s)). \quad (3)$$

The axiom tells us nothing about what happens to the color of a red block next to *Block10* after *Paint* is

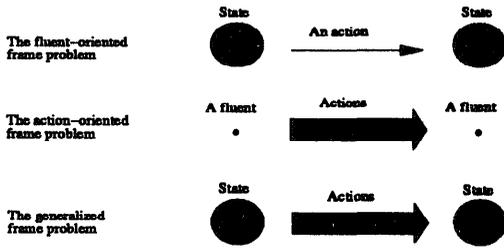


Figure 1: The frame problems

performed. For that, we need a so-called frame axiom which says that the neighboring block will still be red:

$$\forall s.(H(C(Block9, Red), s) \equiv H(C(Block9, Red), Result(Paint, s))). \quad (4)$$

The frame problem is that of succinctly summarizing the frame axioms. Formally, we can say that the frame axiom is needed because, although $\{(3)\}$ is a complete theory about *Paint* w.r.t. $\{C(Block10, Blue)\}$, it is not so w.r.t. $\mathcal{P} = \{C(Block10, Blue), C(Block9, Red)\}$. It is easy to see that $\{(3), (4)\}$ is complete w.r.t. \mathcal{P} . The frame problem, then, is concerned with achieving epistemological completeness in a convenient way for a given set of fluents.

In [Lin and Shoham 1991], we proposed a solution to the frame problem for a wide class of causal theories. However, for concurrent actions, in addition to the traditional frame problem, there is a closely related problem.

Suppose now that we have a new primitive action *Close* which closes the door. Consider the global action $\{Paint, Close\}$. Since (3) tells us nothing about $\{Paint, Close\}$, we need an “inheritance axioms” which says that $\{Paint, Close\}$ can inherit the effect of *Paint*:

$$\forall s.H(C(Block10, Blue), Result(\{Paint, Close\}, s)). \quad (5)$$

It is clear that, for any global action that includes *Paint*, and does not include a subaction that “interferes” with *Paint*, we need a similar axiom. Then, like the frame problem, we have a problem of how to succinctly summarize these “inheritance axioms.”

Again, in terms of epistemological completeness, we can say that the axiom (5) is needed because, although $\{(3)\}$ is an epistemologically complete theory about *Paint* w.r.t. $\{C(Block10, Blue)\}$, it is not so about $\{Paint, Close\}$. The new problem, then, is again about how to achieve epistemological completeness in a convenient way for a given set of global actions.

It is clear that the two problems are symmetric (Fig.1). The first one, called the *fluent-oriented frame problem*, involves a rich structure of propositional fluents, but only a single primitive action. The second one, called the *action-oriented frame problem*, involves

a single fluent, but a rich structure of actions. The symmetry is exactly the same as the one between situations and actions in our framework.

In [Lin and Shoham 1991] we argued that a useful way to tackle the fluent-oriented frame problem is to consider a monotonic theory with explicit frame axioms first, and then to show that a succinct and provably equivalent representation using, for example, a nonmonotonic logic, captures the frame axioms concisely. We shall follow the same strategy here for the generalized frame problem. Let us illustrate it using a version of Kautz’s stolen car problem [Kautz 1986].

The Stolen Car Problem revisited

The scenario is as follows. Initially, the car is present, but after two waiting periods it is gone. When was it stolen?

Suppose we have two propositional fluents *Stolen* (the car is gone) and *Returned* (the car owner is back), and two primitive actions: *Steal* and *Return*. After *Return*, the car owner returns:

$$\forall s.H(Returned, Result(Return, s)). \quad (6)$$

If the owner of the car has not returned, then after *Steal*, the car would be stolen:

$$\forall s.(\neg H(Returned, s) \supset H(Stolen, Result(Steal, s))). \quad (7)$$

If *Return* and *Steal* are performed simultaneously, then the effect of *Steal* would be canceled:

$$\forall s.(\neg H(Stolen, s) \supset \neg H(Stolen, Result(\{Steal, Return\}, s))). \quad (8)$$

Let $\mathcal{P} = \{Stolen, Returned\}$. Then, of course, $\{(6), (7), (8)\}$ is not an epistemologically complete theory of *Return*, *Steal*, and $\{Steal, Return\}$.

Following the strategy in [Lin and Shoham 1991], we shall provide two ways to complete the theory. One is to stick to first-order logic, and supply the necessary frame axioms and inheritance axioms explicitly. The other one is to use a nonmonotonic logic. It is important that the two completions are equivalent.

A monotonic completion

We first explicitly supply necessary inheritance axioms. This will give us a causal theory for each global action as defined in [Lin and Shoham 1991].

In this example, there is only one global action, $\{Steal, Return\}$, to consider. Because of (8), $\{Steal, Return\}$ can only inherit the effect of *Return*:

$$\forall s.H(Returned, Result(\{Steal, Return\}, s)). \quad (9)$$

Now we have a causal theory, $\{(6), (7), (9)\}$, about the three actions. We can use the technique in [Lin and Shoham 1991] to generate necessary frame axioms.

For *Steal*, we have the frame axioms:

$$\begin{aligned} \forall s.(H(\textit{Returned}, s) \equiv \\ H(\textit{Returned}, \textit{Result}(\textit{Steal}, s))), \end{aligned} \quad (10)$$

$$\begin{aligned} \forall sp.(H(\textit{Returned}, s) \supset \\ [H(p, s) \equiv H(p, \textit{Result}(\textit{Steal}, s))]). \end{aligned} \quad (11)$$

For *Return*, we have:

$$\forall s.(H(\textit{Stolen}, s) \equiv H(\textit{Stolen}, \textit{Result}(\textit{Return}, s))). \quad (12)$$

For $\{\textit{Steal}, \textit{Return}\}$, we have:

$$\forall s.(H(\textit{Stolen}, s) \equiv H(\textit{Stolen}, \textit{Result}(\{\textit{Steal}, \textit{Return}\}, s))). \quad (13)$$

Let T_1 be the set of axioms (6) – (13). Then it is clear that T_1 is an epistemologically complete theory of *Steal*, *Return*, and $\{\textit{Steal}, \textit{Return}\}$. Furthermore, if we assume that these are the only global actions:

$$\forall g.(g = \textit{Steal} \vee g = \textit{Return} \vee g = \{\textit{Steal}, \textit{Return}\}), \quad (14)$$

then

$$\begin{aligned} T_1 \cup \{(14)\} \vdash \forall g_1 g_2 s. (&\neg H(\textit{Stolen}, s) \wedge \\ &H(\textit{Stolen}, \textit{Result}(g_2, \textit{Result}(g_1, s))) \\ &\supset \neg H(\textit{Returned}, s) \wedge \\ &\neg H(\textit{Returned}, \textit{Result}(g_1, s)) \wedge \\ &(g_1 = \textit{Steal} \vee g_2 = \textit{Steal})). \end{aligned}$$

That is, the owner never returned, and that *Steal* must have happened during one of the waits.

A nonmonotonic completion

Although T_1 gives the right answers to the stolen car problem, it suffers from the frame problems since it appeals to the explicit inheritance and frame axioms. We now provide an equivalent nonmonotonic theory that avoids them.

We introduce two auxiliary predicates. We have $ab(p, g, s)$ which is true if the truth value of p changes after g is performed in s :

$$\forall pgs(\neg ab(p, g, s) \supset (H(p, s) \equiv H(p, \textit{Result}(g, s)))). \quad (15)$$

We also have $\textit{Canceled}(g_1, g_2, s)$ which is true if the “normal” causal effect of the global action g_1 is canceled by some other actions in g_2 when they are performed simultaneously in the situation s . Thus the causal rules (6) and (7) are rewritten as:

$$\begin{aligned} \forall gs.(In(\textit{Return}, g) \wedge \neg \textit{Canceled}(\textit{Return}, g, s) \supset \\ H(\textit{Returned}, \textit{Result}(g, s))), \quad (16) \\ \forall gs.(In(\textit{Steal}, g) \wedge \neg \textit{Canceled}(\textit{Steal}, g, s) \wedge \\ \neg H(\textit{Returned}, s) \supset H(\textit{Stolen}, \textit{Result}(g, s))), \quad (17) \end{aligned}$$

and (8) is replaced by

$$\forall gs.(In(\textit{Return}, g) \supset \textit{Canceled}(\textit{Steal}, g, s)). \quad (18)$$

Now let T_2 consist of the axioms (15) – (18), the following instances of (1):

$$\forall a.(In(a, \textit{Steal}) \equiv a = \textit{Steal}), \quad (19)$$

$$\forall a.(In(a, \textit{Return}) \equiv a = \textit{Return}), \quad (20)$$

$$\begin{aligned} \forall a.(In(a, \{\textit{Steal}, \textit{Return}\}) \equiv \\ a = \textit{Steal} \vee a = \textit{Return}), \end{aligned} \quad (21)$$

and the unique names assumption:

$$\textit{Return} \neq \textit{Steal} \wedge \textit{Returned} \neq \textit{Stolen}. \quad (22)$$

Then the circumscription of *Canceled* in T_2 with H allowed to vary, written $\textit{Circum}(T_2; \textit{Canceled}; H)$, implies the causal rules (6), (7), and (9). Thus, in a sense, $\textit{Circum}(T_2; \textit{Canceled}; H)$ solves the action-oriented frame problem for the Stolen Car Problem.

We solve the fluent-oriented frame problem using the solution proposed in [Lin and Shoham 1991]. Specifically, we circumscribe ab in $\textit{Circum}(T_2; \textit{Canceled}; H)$ according to the policy in [Lin and Shoham 1991]. The results in [Lin and Shoham 1991] show that the circumscriptive theory is equivalent to $T_1 \cup \{(22)\}$ in the sense that for any sentence φ in the language of T_1 , φ is a first-order consequence of the circumscriptive theory iff it is a first-order consequence of $T_1 \cup \{(22)\}$. In particular, T_2 is a nonmonotonic theory that is epistemologically complete about *Steal*, *Return*, and $\{\textit{Steal}, \textit{Return}\}$.

We notice that the axiom (18) does not take into account the fact that for *Return* to override the effect of *Steal*, *Return* itself must not be overridden by something else such as “*Murder*.” Thus a more appropriate axiom might be:

$$\begin{aligned} \forall gs.(In(\textit{Return}, g) \wedge \neg \textit{Canceled}(\textit{Return}, g, s) \\ \supset \textit{Canceled}(\textit{Steal}, g, s)). \end{aligned} \quad (23)$$

But if we simply circumscribe *Canceled* in the above axiom, we shall have two minimal models, one in which $\textit{Canceled}(\textit{Return}, \{\textit{Steal}, \textit{Return}\}, s)$ is true, and the other in which $\textit{Canceled}(\textit{Steal}, \{\textit{Steal}, \textit{Return}\}, s)$ is true. It is clear that we should prefer the second one. Formally, this can be done by using prioritized subdomain circumscription [Lifschitz 1986]. But we suspect that the formal theory will be complicated, witness the result in [Lifschitz 1987]. Notice that axioms of the form (23) resemble rules in logic programs with negation-as-failure. Thus it would be natural that we use default logic to capture *Canceled*, and pipe the result to circumscription. Although this is perfectly well-defined, some may find it odd to use two nonmonotonic logics at the same time. An alternative formulation of (18) in light of *Murder* is the following axiom:

$$\forall g.(In(\textit{Return}, g) \wedge \neg In(\textit{Murder}, g) \supset \textit{Canceled}(\textit{Steal}, g)). \quad (24)$$

Although (24) is not as good as (23), it should suffice in many applications.

Causal theories

We notice that both our monotonic and nonmonotonic solutions to the Stolen Car problem are adequate in the sense that they are epistemologically complete. Furthermore, they are provably correct with respect to each other. In this section, we show how the solutions can be generalized to a class of causal theories.

In the following, let \mathcal{P} be a fixed set of propositional fluents, and \mathcal{G} be a fixed set of global actions. Let Def be the instantiations of (1) and (2) to the global actions in \mathcal{G} . Thus, for example, if $\{A_1, A_2\} \in \mathcal{G}$, then

$$\forall a.(In(a, \{A_1, A_2\}) \equiv a = A_1 \vee a = A_2)$$

will be an axiom in Def .

A causal theory of \mathcal{G} consists of a domain constraint of the form

$$\forall s.C(s), \quad (25)$$

a set of causal rules of the form

$$\forall s.(R(s) \supset H(P, Result(G, s))), \quad (26)$$

and a set of cancellation axioms of the form

$$\forall s.(K(s) \supset Canceled(G_1, G_2, s)), \quad (27)$$

where $C(s)$, $R(s)$, and $K(s)$ are formulas with s as their only free variable, $P \in \mathcal{P}$, and $G, G_1, G_2 \in \mathcal{G}$. We assume that for any pair of global actions $G_1, G_2 \in \mathcal{G}$, there is at most one cancellation axiom (27) for it.

As with the Stolen Car problem, a causal theory is usually not epistemologically complete. In the following, let T be a fixed causal theory of \mathcal{G} .

A monotonic completion

For any global action $G \in \mathcal{G}$, if there is a global action G_1 , a causal rule about G_1 in T :

$$\forall s.(R(s) \supset H(P, Result(G_1, s)))$$

such that

$$\{\forall s.C(s)\} \cup Def \vdash \forall a.(In(a, G_1) \supset In(a, G)),$$

and a cancellation axiom about G_1 in G :

$$\forall s.(K(s) \supset Canceled(G_1, G, s)),$$

then the following is a *derived* causal rule about G :

$$\forall s.(R(s) \wedge \neg K(s) \supset H(P, Result(G, s))). \quad (28)$$

For any $G \in \mathcal{G}$, let T_G be the set of domain constraint (25), causal rules about G in T , and derived causal rules about G . Then T_G is a causal theory of the action G in the sense of [Lin and Shoham 1991], and the procedure in that paper can be used to provide a monotonic completion of T_G .

A nonmonotonic completion

We first rewrite (26) as

$$\begin{aligned} \forall sg.(\forall a.(In(a, G) \supset In(a, g)) \supset \\ (R(s) \wedge \neg Canceled(G, g, s) \supset \\ H(P, Result(g, s))))). \end{aligned} \quad (29)$$

Then the derived causal rule (28) is obtained from (29) by applying predicate completion on $Canceled$:

$$\forall s.(Canceled(G_1, G_2, s) \equiv K(s)),$$

where $K(s)$ is as in (27). Let W be the conjunction of the domain constraint (25), the axioms in Def , and the cancellation axioms in T . Then under certain conditions [Reiter 1982], minimizing $Canceled$ in W with H allowed to vary, that is,

$$Circum(W; Canceled; H), \quad (30)$$

will achieve this predicate completion. Let T_2 be the conjunction of (30), the causal rules in T , (15), and some unique name assumptions. Then our nonmonotonic completion is the circumscription of ab in T_2 according to the policy in [Lin and Shoham 1991].

If (30) captures the predicate completion for $Canceled$, then T_2 would be “equivalent” to the union of T_G for all $G \in \mathcal{G}$. The results in [Lin and Shoham 1991] can then be used to show that under certain conditions, the monotonic and nonmonotonic completions are equivalent, and are both epistemologically complete.

Conflicts

Let T be a causal theory of \mathcal{G} . We suppose that T is consistent. Let $G \in \mathcal{G}$. If T_G is inconsistent, then G contains *unresolved* conflicting subactions. For example, if we have the following causal rules about the primitive actions $Close$ and $Open$, which closes and opens the door, respectively:

$$\begin{aligned} \forall s.H(Opened, Result(Open, s)), \\ \forall s.H(Closed, Result(Close, s)), \end{aligned}$$

the following domain constraint:

$$\forall s.\neg(H(Opened, s) \wedge H(Closed, s)),$$

and no cancellation axioms, then we’ll have

$$\begin{aligned} \forall s.H(Opened, Result(\{Open, Close\}, s)), \\ \forall s.H(Closed, Result(\{Open, Close\}, s)). \end{aligned}$$

It is easy to see that these two axioms are contradictory under the domain constraint.

In some cases G may contain unresolved conflicting subactions even if T_G is consistent. For example, suppose the agent must have a key in order to open the door:

$$\forall s.H(Key, s) \supset H(Opened, Result(Open, s)).$$

Then we'll have:

$\forall s. H(key, s) \supset H(Opened, Result(\{Open, Close\}, s)).$

It is easy to see that T_G , where $G = \{Open, Close\}$, is consistent. But it is clear that the potential conflict between *Open* and *Close* is still there. We shall say that G contains *unresolved potentially conflicting subactions* if for some situation S , there exists a state SS of S such that $T \cup SS$ is consistent, but $T_G \cup SS$ is inconsistent. Notice that the requirement that $T \cup SS$ be consistent means that the state SS satisfies the domain constraint.

Potential conflicts among actions can be resolved using cancellation axioms. For example, if we think that *Close* will always prevail, then we can write:

$\forall g. (In(Close, g) \wedge In(Open, g) \supset Canceled(Open, g, s)).$

If we think that *Open* and *Close* will cancel each other's effect, then we can write:

$\forall g. (In(Close, g) \wedge In(Open, g) \supset$
 $Canceled(Open, g, s) \wedge Canceled(Close, g, s)).$

In contrast to our approach, concurrency is modeled as so-called "interleaving concurrency" in [Gelfond *etc.* 1991]. In other words, in their system, concurrently performing two actions whose individual effects are contradictory amounts to performing the two actions sequentially, in one of the two orders. This is also the approach taken in [Pednault 1987].

Conclusions

We have proposed a formalization of concurrent actions in the situation calculus, and showed that the "provably-correct" approach proposed in [Lin and Shoham 1991] to formalizing the effects of actions can be extended to the new framework. Compared to other work on concurrent actions, our work is unique in that it is based on a formal adequacy criterion, and applies in a rigorous fashion to a well-defined class of causal theories.

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