

# CATMS: An ATMS Which Avoids Label Explosions

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## Abstract

Assumption-based truth maintenance systems have developed into powerful and popular means for considering multiple contexts simultaneously during problem solving. Unfortunately, increasing problem complexity can lead to explosive growth of node labels. In this paper, we present a new ATMS algorithm (CATMS) which avoids the problem of label explosions, while preserving most of the query-time efficiencies resulting from label compilations. CATMS generalizes the standard ATMS subsumption relation, allowing it to compress an entire label into a single assumption. This *compression* of labels is balanced by an *expansion* of environments to include any implied assumptions. The result is a new dimension of flexibility, allowing CATMS to trade-off the query-time efficiency of uncompressed labels against the costs of computing them. To demonstrate the significant computational gains of CATMS over de Kleer's ATMS, we compare the performance of the ATMS-based QPE [9] problem-solver using each.

## 1 Introduction

Assumption-based truth maintenance systems [2] have proven useful for reasoning about multiple contexts, especially in domains such as qualitative physics [9,6] and signal processing [14,12]. Each context indicates a different set of assumptions (i.e. choices) made during problem solving. An ATMS reasons with multiple contexts simultaneously, with larger contexts inheriting inferences made in smaller ones, so that the cost of reasoning is amortized over the set of contexts. The power of an ATMS comes from its compiling of inferences into *labels*, each representing the alternative contexts in which a particular proposition is true. Although such compilations can potentially lead to enormous efficiency gains at query time, those efficiencies may be unrealized due to an ATMS problem called *label explosion*. This problem arises during compilation when

labels grow exponentially large. The potential for label explosion is a consequence of the cross-product nature of label computations.

In this paper, we present a new ATMS algorithm (CATMS) which can avoid the problem of label explosions, while preserving most of the query-time efficiencies resulting from label compilations. CATMS generalizes the standard ATMS notion of *subsumption*, allowing it to compress an entire label into a single assumption. This *compression* of labels is balanced by an *expansion* of environments to include implied assumptions. The result is an added flexibility to trade-off the query-time efficiency of uncompressed labels against the costs of computing them. In short, CATMS provides an efficient hybrid of the compiled approach taken by a traditional ATMS and the interpreted approach noted in [15].

Section 2 briefly provides the ATMS background relevant to this paper. Section 3 explains the key idea underlying the CATMS algorithm: our theory of label compression. Section 4 presents the basic CATMS algorithm and Section 5 describes how it can detect and avoid label explosions. Section 6 demonstrates the significant computational gains allowed by CATMS over de Kleer's ATMS, by comparing their performance for the ATMS-based QPE [9] problem-solver. Finally, Section 7 discusses the implications of CATMS to future ATMS technology.

## 2 ATMS Background

Briefly, an ATMS can be characterized as follows. Each proposition considered by the problem solver is assigned an ATMS *node*. As the problem solver derives relations among propositions, it declares them to the ATMS as *clauses* among nodes. A particular ATMS implementation may support Horn clauses, or general CNF clauses. The problem solver declares a subset of the nodes to be *assumptions*—propositions which are assumed to be true. Each set of assumptions indicates a particular context and is represented as an ATMS

*environment.* Throughout the paper, we let  $E$ ,  $E_i$ ,  $E_j$  refer to environments. We denote the assumptions comprising  $E$  by  $Asns(E)$ . A node is considered *true* in  $E$  if it can be propositionally deduced from  $Asns(E)$  together with the clauses  $\mathcal{C}$ . A node is considered *false* in  $E$  if its *negation node* is true in  $E$ . The ATMS *contradiction node* denotes a logical contradiction. It is inferred whenever a node is both true and false in some  $E$ , in which case  $E$  is called *nogood* (i.e. inconsistent). The primary responsibility of an ATMS is to process an incremental stream of nodes ( $\mathcal{N}$ ), assumptions ( $\mathcal{A}$ ), and clauses ( $\mathcal{C}$ ) to efficiently answer queries about the status (i.e. true, false, or unknown) of some node in a given environment, or about the consistency of some environment.

An important concept in any ATMS is that of subsumption among environments:

**Definition 2.1 (Environment Subsumption)**  
 $E_i$  subsumes  $E_j \equiv Asns(E_i) \subseteq Asns(E_j)$ .

Because an ATMS operates monotonically, all nodes true in  $E_i$  are also true in the subsumed  $E_j$ , as long as  $E_j$  is consistent. Thus, one can avoid making redundant inferences by inheriting inferences from subsuming environments. However, the typical problem solver rarely requires an explicit list of all nodes true in some environment. So, instead of caching implied nodes with each environment, an ATMS caches for each node the minimal set of implying environments, called its *label*.

**Definition 2.2 (Node Label)** Let  $Label(N)$  denote the set of environments for node  $N$  satisfying:

1. **Consistency:** No  $E_i \in Label(N)$  is a *nogood*;
2. **Soundness:**  $N$  is true in each  $E_i \in Label(N)$ ;
3. **Completeness:** If  $N$  is true in  $E_j$ , then some  $E_i \in Label(N)$  subsumes  $E_j$ ;
4. **Minimality:** For  $E_i, E_j \in Label(N)$  and  $E_i \neq E_j$ ,  $E_i$  does not subsume  $E_j$ .

By compiling node labels, an ATMS can efficiently determine whether a node  $N$  is true in a given  $E$  by checking whether some  $E_i$  in  $Label(N)$  subsumes  $E$ .

As assumptions and clauses are given to an ATMS, labels are incrementally maintained through a technique called *boolean constraint propagation* (BCP) [4], which ensures that each clause is locally satisfied. This process involves computing *cross-products* of labels and unioning the results with the existing label. Due to the nature of cross products, ATMS labels have the potential to grow exponentially, resulting in label explosions.

Conceptually, an ATMS maintains a special label for the contradiction node, where consistency is not enforced. This label represents the set of minimal nogoods. Thus, an ATMS can determine whether a given  $E$  is nogood by checking whether  $E$  is subsumed by some  $E_i$  in the label of the contradiction node. To

maintain label consistency, an ATMS must further provide a means for removing minimal nogoods, and any environments subsumed by them, from all other labels.

Let  $\mathcal{E}$  denote the union of the labels of all nodes, plus any environments mentioned in queries posed by the problem solver.  $\mathcal{E}$  indicates the set of all environments which must be explicitly represented by an ATMS in order to reason over all possible contexts. Typically,  $\mathcal{E}$  will be much smaller than the worst-case (the power set of all assumptions), due to the consistency and minimality properties of labels.

### 3 Theory of Label Compression

CATMS generalizes the standard ATMS algorithm by generalizing the notion of environment subsumption. This impacts the minimality and completeness requirements for labels, as explained below.

#### 3.1 Assumption Closures

In CATMS, an important property of each environment is its *assumption closure*—the set of assumptions which logically follow from its base assumptions:

**Definition 3.1 (Assumption Closure)**  
 $Closure(E) \equiv \{A_i \in \mathcal{A} \mid Asns(E) \cup \mathcal{C} \models A_i\}$ .

An obvious property of these closures is that they include the base assumptions:

**Property 3.1 (Base Assumption Inclusion)**  
 $Asns(E) \subseteq Closure(E)$ .

It is possible for two environments with different base assumptions to have identical closures:

**Definition 3.2 (Equivalent Environments)**  
 $E_i$  is equivalent to  $E_j \equiv Closure(E_i) = Closure(E_j)$ .

Recall that in a standard ATMS, two environments are compared for subsumption by comparing the (base) assumption sets comprising them. In CATMS, subsumption is based on assumption closures:

**Definition 3.3 (Closure Subsumption)**  
 $E_i$  c-subsumes  $E_j \equiv Closure(E_i) \subseteq Closure(E_j)$ .

Closure subsumption is a generalization of standard ATMS subsumption, due to transitivity and the nature of closures:

**Property 3.2 (Subsumption Generalized)**  
 If  $E_i$  subsumes  $E_j$  then  $E_i$  c-subsumes  $E_j$ ; or, if  $Asns(E_i) \subseteq Asns(E_j)$  then  $Closure(E_i) \subseteq Closure(E_j)$ .

Properties 3.1 and 3.2 suggest a test for closure subsumption that requires a single closure computation:

**Property 3.3 (Closure Subsumption Test)**  
 $E_i$  c-subsumes  $E_j$  iff  $Asns(E_i) \subseteq Closure(E_j)$ .

## 3.2 Using Assumption Closures

Applying CATMS's notion of closure subsumption to the ATMS minimality property for labels can result in CATMS labels being smaller than the corresponding labels in a standard ATMS. For example, this new notion of minimality prevents a label from containing two equivalent environments. By itself, this would violate the completeness requirement for labels; however, label completeness is reestablished by using closure subsumption for queries as well. The proof is given in [8]. Soundness and consistency of labels are also retained, given that environments are removed but not added by this transformation.

A CATMS label is always a subset of the corresponding standard ATMS label. The *compression* of CATMS labels accounts for the "c" in CATMS. Label compression can only occur when there are implied assumptions for environments—that is, when some assumption node is the recipient of a justification.<sup>1</sup> de Kleer has argued [2] that allowing assumptions to be implied makes little intuitive sense. However in practice assumption nodes are commonly implied. This occurs in part because it is often difficult for the problem solver to know in advance which nodes it will eventually assume (as noted in [9]). Furthermore, allowing implied assumptions enables our technique for avoiding label explosions (see Section 5).

The cost of CATMS's compressed labels comes at query time. To answer a query of whether a node  $N$  is true in a given  $E$ , CATMS must first *expand*  $E$  to the current  $Closure(E)$ , and then check whether  $E$  is c-subsumed by any environment in  $Label(N)$ . Because labels in CATMS are potentially much smaller, the cost of a query in CATMS may actually be less than in a standard ATMS.

CATMS labels can be significantly smaller than standard ATMS labels, including the label of the contradiction node (which defines the minimal nogoods). Since maintaining consistency is a major expense in an ATMS, CATMS's reduced number of minimal nogoods is an especially significant advantage. As with node status queries, nogood queries require first expanding the query environment to compute its current assumption closure. That closure set is then used to check if any minimal nogood c-subsumes the query environment. This need to expand is the price that CATMS must pay for having smaller labels and fewer minimal nogoods to check against—it usually turns out to be quite a bargain.

<sup>1</sup>We use the terms "justification" and "clause" somewhat interchangeably; CATMS supports both Horn and CNF clauses.

## 4 The Basic CATMS Algorithm

This section describes the fundamental aspects of the CATMS implementation. For a more detailed discussion, see [8].

### 4.1 Label Propagation

CATMS computes compressed labels using the standard ATMS label propagation algorithm, but using CATMS's generalized notion of subsumption based on closures. The most dramatic difference occurs when an assumption node is reached during label updating. In CATMS, propagation terminates at that assumption. The CATMS label of an assumption node  $A$  always consists of a single environment  $E_A$  containing only  $A$  itself. Any other environment that a standard ATMS would include in  $A$ 's label must necessarily contain  $A$  in its assumption closure, and thus be c-subsumed by  $E_A$ . All label propagations through non-assumption nodes proceed as in a standard ATMS, but using the CATMS version of subsumption based on closures.

The ability of assumptions to block label propagations can be understood as follows. In a standard ATMS, label environments are propagated "forward" from the assumed propositions. In CATMS, assumed propositions are, in effect, propagated "backward" to the minimal environments in which they are true and indirectly to the c-subsumed environments, via assumption closures. This avoids the need to propagate through assumptions, thereby reducing the potential for label explosion.

The ability in CATMS to propagate selected nodes (i.e., assumptions) backward to environments provides a continuum between the two extremes of caching nodes with environments and caching environments with nodes. In the extreme case where all propositions are assumed, CATMS performs no label updates, and every label will contain exactly one environment. In that case, CATMS becomes a mechanism for caching the status of each node with particular environments of interest. In the case where no assumptions are justified, the CATMS labels are identical to those of a standard ATMS.

### 4.2 Computing Assumption Closures

As noted in Section 3, a CATMS environment must be expanded (i.e. have its current assumption closure computed) before testing whether it is c-subsumed by some other environment. Conceptually, an environment  $E$  could be expanded by simply running over all of the justifications. One would start with a set  $S$  of nodes, representing the union of  $Asns(E)$  and the set of always-true propositions, and would extend  $S$  by the consequent nodes of justifications whose antecedents

are all members of  $S$ . Such expansion would terminate within time linear in the total size of clauses in  $C$ .  $Closure(E)$  would be the set of assumption nodes in  $S$ .

However, the justification structure is inefficient for computing assumption closures because it typically contains a large percentage of nodes which are not assumptions. In practice, we employ that straight-forward *expansion by justifications* only to verify CATMS's performance. A more efficient structure results from collapsing out the non-assumption nodes, leaving only assumptions and equivalent justifications. The actual implemented structure, which we call the *expansion lattice*, is conceptually analogous to this. The expansion lattice contains only those environments which minimally imply one or more assumptions.<sup>2</sup> These environments are connected such that a path exists from each single-assumption environment up to every *superset* environment in the expansion lattice, where "superset" is based on standard ATMS subsumption—not c-subsumption. This is the only instance in CATMS where standard subsumption is used instead of c-subsumption.

Consider an assumption node  $A$  with incoming justifications. As label propagation reaches  $A$ , each environment  $E_i$  that a standard ATMS would add to  $A$ 's label is marked as minimally implying  $A$ .  $E_i$  is then added to CATMS's *expansion lattice*, if it is not already there.

When a new environment  $E$  is added to  $\mathcal{E}$ , CATMS expands  $E$  by searching upward from the single-assumption environments corresponding to  $Asns(E)$ . The set  $S$  starts equal to  $Asns(E)$ . When an environment  $E_i$  satisfying  $Asns(E_i) \subseteq S$  is initially encountered, CATMS adds the minimally implied assumptions cached with  $E_i$  to  $S$ . Each such assumption actually added to  $S$  provides a new seed from which to search. This search terminates, with  $Closure(E) = S$ , as soon as all active search paths are exhausted without picking up any new implied assumptions.

### 4.3 Maintaining Assumption Closures

In a standard ATMS, an environment is a static collection of assumptions. In CATMS, an environment's assumption closure can grow as new assumptions or clauses are added.

When label propagation reaches an assumed node  $A$ , any  $E$  that a standard ATMS would add to  $Label(A)$  must now include  $A$  in  $Closure(E)$ . The same is true for all environments c-subsumed by  $E$ . The problem is how to index environments so that those c-subsumed

<sup>2</sup>However, Section 4.4.2 describes another use for this expansion lattice.

environments can be found and updated.<sup>3</sup> One obvious, but inefficient, approach would be to search through  $\mathcal{E}$  for all environments c-subsumed by  $E$ . A slight improvement results by indexing environments by *size* (i.e. size of their assumption closures), as environments smaller than  $E$  cannot be c-subsumed by it. This scheme is complicated by the need to re-index environments whenever their assumption closures grow.

Further improvement results from maintaining pointers from environments to c-subsumed environments. This involves maintaining a *dynamic lattice* of all relevant environments, having the following properties:

1. A path of pointers exists from  $E_i$  to  $E_j$  if and only if  $E_i$  c-subsumes  $E_j$ .
2. No two environments are both directly and indirectly connected.

This lattice is dynamic in that the required pointers up to  $E$  change as  $Closure(E)$  grows.

The dynamic lattice provides a mechanism for continuously maintaining assumption closures of all  $E_i$  in  $\mathcal{E}$ . Each  $E_i$  newly added to  $\mathcal{E}$  is first expanded and then inserted into the lattice. Once inserted,  $Closure(E_i)$  is updated by propagating newly implied assumptions up through the lattice. When these propagations cause  $Closure(E_i)$  to grow, then  $E_i$  must be re-expanded and possibly repositioned higher in the lattice. Collisions occur as environments become equivalent. Only one representative of an equivalence class is kept in the lattice, with pointers to the rest of the class.

The maintenance of the dynamic lattice is quite complex and costly. While the dynamic lattice was the basis for initial implementations of CATMS, it has been disabled in the current implementation, in favor of the lazy approach described in Section 4.5.

### 4.4 Maintaining Consistency

New minimal nogoods arise as environments are added to the label of the contradiction node. Consistency maintenance involves marking new minimal nogoods, finding and marking any new non-minimal nogoods in  $\mathcal{E}$ , removing (marked) nogoods from labels, and checking additions to  $\mathcal{E}$  against the minimal nogoods. CATMS's use of closure subsumption complicates the standard methods for finding new non-minimal nogoods and for checking new environments, as shown below.

<sup>3</sup>This problem is identical to finding subsumed environments of a new minimal nogood.

#### 4.4.1 Finding Non-Minimal Nogoods

The traditional ATMS approach for finding subsumed environments of a new minimal nogood is to cache environments in a table indexed by their size, as the nogood can only subsume environments larger than it. The use of such a table is more complicated in CATMS because the size of an environment grows as its assumption closure grows. Thus, CATMS must move environments up in the table as their assumption closures grow.

If the dynamic lattice is being used to maintain assumption closures, it provides an efficient structure for propagating inconsistency. The new non-minimal nogoods in  $\mathcal{E}$  which are c-subsumed by a new nogood are exactly those environments found above that nogood in the lattice. Non-minimal nogoods may be removed from the dynamic lattice once they are found.

#### 4.4.2 Checking Consistency

As new environments are added to  $\mathcal{E}$ , they must be initially checked for consistency. We have explored three different approaches for performing this check in CATMS. The tradeoffs of these approaches are still being explored.

**Dynamic Lattice** By keeping minimal nogoods in the dynamic lattice, detecting whether a new environment is a non-minimal nogood is easy: a non-minimal nogood will always be placed directly above some minimal nogood in the lattice. This is clearly the best choice as long as the dynamic lattice is being maintained.

**Nogood Table** The standard ATMS approach is to maintain a table of minimal nogoods. An environment is detected as nogood if a c-subsuming nogood can be found in the table. As for the environment table mentioned above, nogoods in the nogood table are indexed by size, as nogoods bigger than the query environment cannot c-subsume it. Thus, when the assumption closure of a nogood grows, CATMS should move that nogood up in the table. Failure to do so will not lead to inconsistencies, as all smaller nogoods will be checked; however, it can cause CATMS to make unnecessary checks of those nogoods.

**Bad Assumptions in the Expansion Lattice** Another approach does not cache minimal nogoods directly; it instead maintains the set of *bad assumptions* for each environment. Upon finding a new minimal nogood  $E_n$ , a consistent subset environment  $E_s$  is found by removing (any) one assumption  $A$  from  $E_n$ .  $A$  is added to  $E_s$ 's list of *minimally bad assumptions*, and  $E_s$  is added to the expansion lattice (unless already there). The bad assumptions are then picked up during expansion, in a manner analogous to computing

assumption closures. An environment is recognized as nogood whenever some assumption is in both its assumption closure and in its set of bad assumptions. We have found this technique to be particularly useful when using the lazy update approach described next.

#### 4.5 Lazy Updating of Closures

Because of the high cost of continuously maintaining assumption closures via the dynamic lattice, the current implementation of CATMS employs an alternative which updates assumption closures upon demand. This involves temporarily relaxing the minimality and consistency requirements for labels. Non-minimal or inconsistent environments are allowed to remain in a label until that label is examined during a label propagation or a query.

When assumption closures are not continuously maintained, it is especially impractical to attempt to fully maintain consistency. That would require updating the assumption closures of all environments in  $\mathcal{E}$  each time a new minimal nogood is found, to see if any are c-subsumed by that nogood and are thus new non-minimal nogoods. Therefore, CATMS takes a lazy approach for detecting nogoods, whereby non-minimal nogoods are not identified until their consistency is required during a label propagation or a query. Consistency checking is performed on such environments using either of the last two techniques of Section 4.4.2. This lazy approach results in slightly larger CATMS labels, though typically still much smaller than the corresponding labels of a standard ATMS.

CATMS also employs some time-stamp efficiencies which allow it to avoid a significant percentage of lazy updates and consistency checks. For example, it avoids updating  $Closure(E)$  when no new assumptions have been minimally implied since the last time  $E$  was updated. Similarly, it avoids consistency checks for  $E$  whenever no new minimal nogood has been found since the last consistency check for  $E$ .

### 5 Avoiding Label Explosions

CATMS can avoid the problem of label explosion by reducing the label of any node  $N$  to size-one, by assuming  $N$ . The resulting  $Label(N)$  contains just one environment representing the single assumption  $N$ . All of the environments previously in  $Label(N)$  are added to the expansion lattice as minimally implying the assumed  $N$ , as explained in Section 4.2. For example, if a large  $Label(N)$  is about to participate in a costly cross-product operation, then assuming  $N$  can make the operation tractible. We call this technique *auto-assuming* a node, and distinguish such assumptions from ones made by the problem solver. Even though

minimality and consistency can make the final result of a cross-product significantly smaller than the worst-case, tractability may demand avoiding the cost of performing the cross-product.

The complexity of CATMS’s expansion lattice grows at worst quadratic in the total (precompressed) size of assumption node labels. Thus, auto-assuming to avoid exponential explosions in any label eliminates the potential for exponential explosions inherent in a standard ATMS. The trade-off, of course, is that fewer inferences are cached into the labels, requiring some of them to be made repeatedly each time a new environment is expanded. CATMS reduces the problem of label explosion to one of deciding the best time for auto-assuming versus computing a standard ATMS cross-product.

The worst-case complexity of assuming all nodes in CATMS is linear in the total size of the clauses  $C$ , due to the limited complexity of expansion by justifications. Therefore, as label growth exceeds some linear function of problem complexity, the performance of CATMS using auto-assumptions becomes increasingly superior to that of a standard ATMS.

We are experimenting with an assortment of criteria for when to auto-assume, based on the particular  $N$ ,  $A$ ,  $C$  and  $\mathcal{E}$  of a given problem. For now, we have found a simple constant threshold function that avoids performing cross-products of size greater than about 50 to be sufficient for performance superior to a standard ATMS for many problems. More sophisticated criteria could be based on some limited look-ahead into the justification structure defined by  $C$ , to avoid making overly-conservative auto-assumptions.

## 6 Examples

Because our CATMS implementation provides all the functionality of a standard ATMS, problem solvers can use it in place of de Kleer’s ATMS program. To evaluate the utility of CATMS’s use of compressed labels, we have run a variety of benchmarks comparing CATMS versus ATMS on real, complex problems. In particular, we have extensively tested CATMS with the Qualitative Process Engine (QPE) [9], which was originally designed to use ATMS. The results vary with the specific QPE domain and scenario being modelled. For small examples the differences are minor, but as the size and complexity of the examples increases, the advantages of CATMS become clear.

Table 1 compares the results of running QPE on two different scenarios: the familiar two container liquid flow and a linkage example from the domain of mechanisms. In fact, this latter example provided the original inspiration for developing CATMS. Although CATMS’s superior performance in these examples is sufficient to

	QPE Example			
	Two Containers		Mechanical Linkage	
	CATMS	ATMS	CATMS	ATMS
Nodes	1049	1050	1585	1586
Assumptions	18	16	26	24
Environments	135	188	810	4270
in labels	58	50	239	1130
nogoods	52	85	198	1514
minimal	36	66	90	402
Avg. label size	1.15	1.73	1.46	9.91
QPE time (secs)	33.5	36.4	244.4	479.9

Table 1: Example QPE results using CATMS versus ATMS

suggest the leverage it can provide, these results are somewhat misleading because CATMS does not yet employ many of the standard ATMS efficiency hacks that ATMS does. We expect that using de Kleer’s extremely fast (but complex) bit-vector, hash table, and label weave operations could help significantly.

These examples demonstrate the following (typical) results: 1) average CATMS label sizes are much smaller, 2) CATMS considers fewer unique environments and nogoods, and 3) CATMS may take slightly longer at query time but the speedup at justification time leads to overall better performance.

## 7 Conclusions

CATMS provides an efficient hybrid of the compiled approach taken by a traditional ATMS and the interpreted approach noted in [15]. It does so by generalizing the standard ATMS notion of subsumption to allow label propagation to stop at justified assumptions. To use the resulting compressed labels for queries, CATMS provides a means for computing the closure of assumptions implied by a query environment. Furthermore, CATMS uses its ability to maintain compressed labels to prevent the label explosion problem inherent in a fully compiled ATMS approach. CATMS avoids label explosions by auto-assuming a node if its label is too large to allow a tractable cross-product during label propagation. Thus, CATMS transforms the problem of label explosion to one of deciding when a node should be auto-assumed.

The label compression of CATMS is similar to the partitioned approach of PATMS [1], in that both allow a single assumption to take the place of an entire label. However, we believe that CATMS provides a more general solution to the problem of label explosions.

The actual CATMS implementation allows many more trade-offs and efficiencies than have been discussed here. For example, label propagation need not stop at all justified assumptions—such blockings can be limited to just auto-assumptions. Furthermore, new environments created by the union of two updated environments are expanded much more efficiently than

suggested by Section 4.2. We describe such trade-offs and efficiency issues in greater detail in [8].

The following areas seem especially worthy of future exploration: 1) better criteria for auto-assuming (perhaps based on limited look-ahead through the justification structure); 2) allowing assumptions to selectively block some environments while propagating others; 3) when using the dynamic lattice (based on c-subsumption) is worth its cost to maintain (using an explicit lattice based on standard ATMS subsumption is discussed in [3,2]); and 4) the impact of CATMS on standard ATMS techniques for focusing [10], backtracking [7], parallelizing [5], and others [13,11].

Finally, some problem solvers (such as GDE [6]) actually require, for their own use, the standard ATMS labels. To support such reasoning, CATMS would have to either *explode* a compressed label into its standard ATMS form, or provide a means of protecting particular labels from ever becoming compressed.<sup>4</sup> Of course, for such tasks, as well as when the number of queries is exponential or ATMS interpretation construction is performed, CATMS cannot prevent overall exponential complexity. What it can do, however, is allow enough label compilation so that any tractable reasoning over multiple contexts can be performed efficiently.

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<sup>4</sup>However, one of the authors is investigating the potential of CATMS to allow hierarchical reasoning within a GDE-like diagnostic system.

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