QUERY ANSWERING IN CIRCUMSCRIPTIVE AND CLOSED-WORLD THEORIES

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ABSTRACT.

Among various approaches to handling incomplete and negative information in knowledge representation systems based on predicate logic, McCarthy's circumscription appears to be the most powerful. In this paper we describe a **decidable algorithm to answer queries in circumscriptive theories.** The algorithm is based on a modified version of ordered linear resolution, which constitutes a **sound and complete procedure** to determine whether there exists a **minimal model satisfying a given formula.**

The Closed-World Assumption and its generalizations, GCWA and ECWA, can be considered as a special form of circumscription. Consequently, our method also applies to answering queries in theories using the Closed-World Assumption or its generalizations.

For the sake of clarity, we restrict our attention to theories consisting of ground clauses. Our algorithm, however, has a natural extension to theories consisting of arbitrary clauses.

1. Introduction

We describe a decidable algorithm to answer queries in indefinite theories with the proper treatment of incomplete information and, in particular, with the correct representation of negative information. The need for such algorithms has been recently stressed in the literature (cf. [GMN], [R2], [Mi]). Our algorithm is based on McCarthy's theory of circumscription (see [M],[M2],[L],[L2],[L3]), which appears to be the most powerful among various approaches to handling incomplete and negative information in knowledge representation systems based on predicate logic.

For the sake of clarity, in this paper we restrict our attention to theories consisting of ground clauses. Under natural conditions, explained in Section 5, our algorithm has a straighforward **decidable extension to theories consisting of arbitrary clauses.**

Suppose that T is a first order theory in clausal form and F is a sentence. We develop a **MInimal model Linear Ordered resolution** (MILO-resolution) which constitutes a <u>sound and complete</u> method to determine whether there exists a minimal model M of T satisfying the formula F. Since a circumscriptive theory CIRC(T) implies a formula H if and only if there are no minimal models M of T satisfying the negation of H, MILO-resolution gives rise to an algorithm for answering queries in circumscriptive theories.

Our method also applies to answering queries in theories using Reiter's Closed-World Assumption (CWA; see [R]) or its generalizations. It has been shown in [L2], that under the assumptions of unique names, domain closure and finitely many terms, CWA (applicable only to definite theories) is equivalent to circumscription. A generalization GCWA of the CWA for indefinite theories has been proposed by Minker [Mi] (see also [GP]). In [GPP], an extension, ECWA, of GCWA for non-unit clauses has been described and proven (under the same assumptions) to be equivalent to circumscription. Since the above mentioned assumptions are routinely made when applying the CWA, it can be argued that CWA and its generalizations constitute a special case of circumscription.

Finally, we wish to point out that our algorithm will naturally suffer from all the inherent inefficiencies present in a general theorem prover. In fact, being more complex, it will be even more inefficient. Therefore, we see its main importance as an **analytical tool** to study theorem proving methods in general closed-world theories, which - when restricted to a suitable domain - becomes a **sound and complete inference engine.** It is fairly clear, that if efficient implementation of a closed-world inference engine is the main objective, then strong **syntactical restrictions** have to be imposed on the theory involved. The so called **stratifiable databases** (see [ABW] and [P]) provide a case in point.

2. Parallel Circumscription

From now on we assume that T is a first order theory consisting of finitely many ground clauses over the language L. We also assume that the Unique Names Assumption is satisfied for L , i.e. that $t_1 \neq t_2$ for any two different terms t_1 and t_2 of L.

Parallel circumscription was introduced by J. McCarthy [M],[M2]. Suppose thet $P = \{P_1,\ldots,P_n\}$ is a list of some predicate symbols from T that we intend to minimize and $Q = \{Q_1,\ldots,Q_m\}$ is the list of the remaining predicates, called parameters. The process of circumscribing (or minimizing) predicates P in T transforms T into a stronger second order theory CIRC(T;P) as defined below.

Definition 2.1. The **circumscription of P in T** is the following sentence CIRC(T;P) : $T(P) \wedge \forall P' [(T(P') \wedge P' \rightarrow P) \rightarrow (P' = P)]$, where $P' \rightarrow P$ stands for $\forall x (P'(x) \rightarrow P(x))$.∎

This formula states that predicates from P have a minimal possible extension under the condition T(P) (cf.[L],[L2],[L3]).

Remark. For the sake of simplicity, in this paper we do not consider **variable predicates Z** (see [M2]). At the cost of becoming more complex, the procedure can be generalized to handle variable predicates.□

To clarify the notion of circumscription we reformulate it in model-theoretic terms.

Definition 2.2. For any two models M and N of T we write **$M \leq N$ mod P** if M and N differ only in how they interpret the predicates in P, and if the extension of every predicate in M is a subset of its extension in N.∎

This relation is a partial-order and hence we can talk about minimal models M w.r.t. $\leq$ in the class S of all models of T. Such models are called **P-minimal models of T.** The following result is fundamental:

Theorem 2.3. [L] A structure M is a model of CIRC(T;P) iff M is a P-minimal model of T. In other words, for any formula F we have CIRC(T;P) $\models$ F iff M $\models$ F, for every P-minimal model M of T.∎

Our algorithm will be based on the following important characterization of circumscription obtained in [GPP] and stating, in effect, that circumscription is equivalent to the so called Extended Closed-World Assumption:

Theorem 2.4. [GPP] Suppose that F is a ground formula. Then CIRC(T;P) $\models$ F if and only if there are **no** clauses C such that:

(i) C does not contain literals from P^-;

(ii) $T \vdash \neg F \vee C$, but $T \nvdash C$.∎

Theorems 2.3 and 2.4 yield the following:

Corollary 2.5. [GPP] Suppose that K is a ground formula. There exists a P-minimal model M of T satisfying K if and only if there exists a clause C such that:

(i) C does not contain literals from P^-;

(ii) $T \vdash K \vee C$, but $T \nvdash C$.∎

The purpose of the MILO-resolution defined in the next section is to determine the existence of a clause C and thus the existence of a P-minimal model.

3. Minimal Model Resolution

In this section we describe a **MInimal model Linear Ordered resolution (MILO-resolution)** which constitutes a sound and complete method to determine whether there exists a P-minimal model M of a theory T satisfying a given formula F. Since a circumscriptive theory CIRC(T;P) implies a formula H if and only if there are no P-minimal models M of T satisfying the negation of H, MILO-resolution leads to an algorithm for answering queries in circumscriptive theories. MILO-resolution is a modification of the ordered linear resolution (OL-resolution; see [CL]).

We denote by P^- the set of all negative literals, whose predicate symbols are in P and we consider every clause as an ordered list of literals $\{l_1,\ldots,l_m\}$. By an **extended clause** we mean an ordered list of literals, some of which may be framed. A framed literal k is denoted by [k]. Framed literals are merely used for recording those literals that have been resolved upon; they do not participate in the resolution. An extended clause is a **tautology** if it contains a pair of unframed complementary literals. An extended clause C **subsumes** an extended clause D if the set of unframed literals of C is contained in D. Now we are ready to define **MILO-deduction.** For readers unfamiliar with the OL-deduction, we have indicated in **bold** case those parts of the definition that have to be removed to obtain standard OL-deduction.

Definition 3.1. Given a theory T and a clause C_0, a **MILO-deduction** of a clause C_n from T + C_0 is any sequence of extended clauses $C_0,C_1,\ldots,C_n$ in which C_{i+1} is generated from C_i according to the following rules:

(i) First, an extended clause D_{i+1} is constructed, which is the ordered resolvent of $C_i=\{l_1,\ldots,l_m\}$ and some clause $B=\{k_1,\ldots,k_s\}$ from T upon the first literal l_j in C_i **that belongs to P^-,** i.e. $D_{i+1}=$ $l_1,\ldots,l_{j-1},k_1,\ldots,k_{u-1},k_{u+1},\ldots,k_s,[l_j],l_{j+1},\ldots,l_m$ where $k_u=\neg l_j$ (framed literal $[l_j]$ is used to record the performed operation);

(ii) The clause C_{i+1} is obtained from D_{i+1} by performing the following reductions in the order specified:

(a) deleting any unframed literals k in D_{i+1} for which there exists a framed literal $[\neg k]$ in D_{i+1};

(b) merging any identical literals in D_{i+1} to the right;

(c) removing any framed literals in D_{i+1} that are not preceded by unframed literals **from** P^-.

(iii) The clause C_{i+1} cannot be a tautology and it cannot be subsumed by any of the previous clauses.■

As indicated above, MILO-deduction differs from the standard ordered linear deduction (OL-deduction; see [CL]) only by: (1) restricting the resolution to literals from P^- and (2) removing all those framed literals in D_{i+1} that are not preceded by unframed literals from P^-, rather than just removing those framed literals which are not preceded by <u>any</u> unframed literals.

Definition 3.2. A MILO-deduction of a clause C_n from $T + C_0$ is called a **MILO-derivation** of C_n from $T + C_0$, if

(i) C_n does not contain any literals from P^-;

(ii) $T \nvdash C_n$ (i.e. $\neg C_n$ is satisfiable).

The process of finding a MILO-derivation is called a **MILO-resolution.**■

Let us explain the meaning of this definition. First of all, if C_n does not contain any literals from P^- then, according to Definition 3.1, no further deduction can be performed from it, and therefore C_n is a <u>terminal</u> clause. Secondly, it is not difficult to show, that if K is a conjunction of literals and if a clause C_n is MILO-derivable from $\neg K$, then $T \vdash K \vee C_n$, but $T \nvdash C_n$, which in view of Corollary 2.5. shows that <u>there exists a P-minimal model of T satisfying K</u>. This establishes the easy part (soundness) of the following fundamental result:

Theorem 3.3. (Soundness and completeness of the MILO-resolution) Suppose that K is a conjunction of literals. There exists a P-minimal model of T such that $M \models K$ iff there exists a MILO-derivation from $T + \neg K$.■

Example 3.4. Suppose that T consists of the following clauses:

(1) $s(C) \vee \neg s(B)$
(2) $s(A) \vee s(B) \vee \neg s(C)$
(3) $s(A) \vee s(B) \vee s(C)$

and suppose that $P = \{s\}$. The following deduction is a MILO-derivation from $T + \neg(s(B) \wedge s(C))$ (literals resolved upon are underlined and side clauses are given in parentheses):

```
¬s(B)v¬s(C)
      | (2)
s(A)v¬s(C)v[¬s(B)]v¬s(C)
      | (reduction)
s(A)v¬s(C)
      | (1)
s(A)v¬s(B)v[¬s(C)]
      | (3)
s(A)vs(C)v[¬s(B)]v[¬s(C)]
      | (reduction)
   s(A)
```

because, it is easy to verify, using e.g. standard OL-resolution, that $T \nvdash s(A)$. Therefore, Theorem 3.3 implies that there exists a P-minimal model of T in Example 3.4 satisfying $s(B) \wedge s(C)$.■

Although the notions of MILO-deduction and OL-deduction are similar, the proof of Theorem 3.3 is considerably more involved than the proof of the soundness and completeness of the standard OL-resolution. This is due to the special treatment of literals from P^-.

Suppose now that F is any formula. We can obviously assume that F is represented in normal disjunctive form, i.e. $F = K_1 \vee \ldots \vee K_m$, where K_i's are conjunctions of literals.

Corollary 3.5. For a formula F the following conditions are equivalent:

(i) there exists a P-minimal model M of T such that $M \models F$;

(ii) there exists a P-minimal model M of T such that $M \models K_i$, for some i;

(iii) there exists a MILO-derivation from $T + \neg K_i$, for some i.■

From the description of the MILO-resolution, it is clear that its role is to <u>reduce the original problem</u> of the existence of P-minimal models M of T satisfying a given formula F <u>to the problem of establishing whether a given clause C_n is derivable from T</u>. The last problem can be handled by a standard theorem prover. Obviously, if the theory T is not decidable, we will not be always able to establish that C_n is not derivable from T. This dependence on the decidability of T is not surprising: after all our query concerns the existence of specific models of T.

4. Query answering in circumscriptive and closed-world theories

Suppose that F is any formula. We can obviously assume that F is represented in normal conjunctive form, i.e. $F = G_1 \wedge \ldots \wedge G_m$, where G_i's are clauses. From Theorem 2.3 and Corollary 3.5 we easily obtain:

Theorem 4.1. For a formula F the following conditions are equivalent:

(i) $CIRC(T;P) \nvDash F$;

(ii) there is a P-minimal model for $\neg F$;

(iii) there is a P-minimal model for $\neg G_i$, for some i ;

(vi) there is a MILO-derivation from $T + G_i$, for some i .■

Corollary 4.2. For a formula F the following conditions are equivalent:

(i) $CIRC(T;P) \models F$;

(ii) there is no P-minimal model for $\neg F$;

(iii) for every i there is no P-minimal model for $\neg G_i$;

(vi) for every i there is no MILO-derivation from $T + G_i$.■

Example 4.3. According to Theorem 4.1, CIRC(T;P) does not imply ¬s(B) ∨ ¬s(C), where T is the theory described in Example 3.4.■

The following corollary is a special case of results established in [EMR] and [GPP] and shows that as long as F does not contain any literals from P^-, F is implied by CIRC(T;P) iff it is derivable from T, which further explains the reduction process described in the previous section.

Corollary 4.4. ([EMR],[GPP]) Suppose that F is a formula which does not contain any negative occurrences of predicates from P. Then:

$$T \vdash F \text{ iff } CIRC(T;P) \models F .$$ ■

Corollary 4.2 leads to the following **decidable algorithm for query answering in circumscriptive theories** based on MILO-resolution.

Theorem 4.5. (Decidable Query Answering Algorithm). The following procedure constitutes a decidable algorithm for determining whether a given formula F is implied by a circumscriptive theory CIRC(T;P):

<u>Step 1.</u> Represent F in normal conjunctive form, i.e. let $F = G_1 \wedge \ldots \wedge G_m$, where G_i's are clauses.

<u>Step2.</u> For all j=1,...,m use the depth-first search on the MILO-resolution tree with the top clause G_j to find all MILO-deductions terminating with clauses H that do not contain literals from P^-.

<u>Step 3.</u> If no such terminal clauses H are found for any j=1,...,m, then $CIRC(T;P) \models F$.

<u>Step 4.</u> Else, for any terminal clause H found use any <u>decidable</u> standard theorem prover to determine whether $T \vdash H$.

<u>Step 5.</u> If there is an H such that $T \nvdash H$, then $CIRC(T;P) \not\models F$, else $CIRC(T;P) \models F$.■

The decidability of the above algorithm follows from the fact that, due to the <u>subsumption check</u> in the definition of MILO-deduction, the <u>search tree for MILO-resolution is finite</u>. The following example illustrates the above algorithm. In order to show that the algorithm is not limited to ground clauses, we apply it to clauses that contain variables.

Example 4.6.(cf.[BS]) Suppose that our theory T is given by the following clauses:

(1) learns(x, Latin)∨learns(x,Greek) <- senior(x)
(2) learns(x, French)∨learns(x,Spanish) <- junior(x)
(3) senior(x)∨junior(x)
(4) senior(Ann)
(5) learns(Ann,Latin).

Suppose that P={learns,senior,junior} and that we want to find out whether CIRC(T;P) ⊨ ¬learns(Ann,Greek). As shown below (using obvious abbreviations), all MILO-deductions from T + ¬learns(Ann,Greek) terminate with clauses implied by T (because T⊢learns(Ann,Latin)), thus showing that CIRC(T;P) ⊨ ¬learns(Ann,Greek).

```
        ¬l(A,G)
           | (1)
 l(A,L)∨¬s(A)∨[¬l(A,G)]
           | (4)              \ (3)
           |                   \
        l(A,L)            l(A,L)∨junior(A)
```

Similarly, we can show that: CIRC(T;P) ⊨ ¬learns(Ann,French)∧¬learns(Ann,Spanish).

On the other hand, if P={learns,senior} then, as shown below, there exists a MILO-derivation from T + ¬learns(Ann,French), thus CIRC(T;P) ⊭ ¬learns(Ann,French). Similarly, CIRC(T;P) ⊭ ¬learns(Ann,Spanish).

```
        ¬l(A,F)
           | (2)
 l(A,S)∨¬j(A)∨[¬l(A,F)]
           | (reduction)
 l(A,S)∨¬j(A)
```

It is easy to verify that T⊬l(A,S)∨¬j(A).■

Remark. Under the assumptions mentioned in the introduction, which are routinely made when CWA is applied, the Extended Closed-World Assumption [GPP] is exactly equivalent to circumscription, i.e. for any formula F we have: ECWA(T;P)⊢F iff CIRC(T;P)⊨F. In particular, for any unit clause F, GCWA(T;P)⊢F iff CIRC(T;P)⊨F, where GCWA stands for the Generalized Closed-World Assumption of J. Minker [Mi]. Moreover, for Horn clauses, all the four approaches - CWA,GCWA,ECWA and circumscription - coincide. This shows that our methods apply to answering queries in closed-world theories.■

5. Concluding remarks

For the sake of simplicity, we have presented our results under the assumption that all clauses are ground, i.e. in the propositional case. This assumption is not necessary. Without significant changes, **our results can be generalized to the following case:**

(1) the theory T consists of any, not necessarily ground, clauses;
(2) the query F in Section 4 is universal;
(3) the Unique Names Axiom is assumed.

In particular, there is no need to replace the theory T by the set of ground instances of its clauses, because the algorithm described in Section 4 (with natural modifications) works properly with variables. Moreover, if the language L does not contain function symbols, then the algorithm remains **decidable.**

In [BS], Bossu and Siegel describe an entirely different theorem proving procedure to answer queries in circumscriptive theories. Their procedure, however, is restricted to the so called 'groundable clauses' and applies only to the case when all predicates are minimized. Moreover, it seems that in many instances it can be grossly inefficient as it always deals with the entire set of clauses, not just with those that pertain to a particular query.

Other methods of answering queries in closed-world theories can be found in [GM], [YH] and [GP]. Grant and Minker's algorithm [GM] is restricted to ground positive clauses. Yahya and Henshen's algorithm also assumes that all clauses are ground and seems too inefficient to be implemented in practice. Gelfond and Przymusinska's approach [GP] is sound but often far from completeness.

Finally, Clark's QEP-procedure [C], essentially equivalent to the one implemented in PROLOG, correctly evaluates queries under CWA only for Horn clauses.

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Full description of the MILO-resolution for non-ground clauses including proofs and variable predicates will appear elsewhere.

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