

## Modeling Interventions Using Belief Causal Networks

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### Abstract

Causality plays an important role in our comprehension of the world. It amounts to determine what truly causes what and what it matters. Interventions allow the identification of elements in a sequence of events that are related in a causal way. In this paper, we introduce belief causation and we propose a method for handling interventions in graphical model under an uncertain environment where the uncertainty is represented by belief masses, so-called belief causal networks. More specifically, we propose a generalization of the “DO” operator and explain the needed changes on the structure of the graph to model a belief causal network on which interventions are proceeded.

### Introduction

Though at first glance, it seems obvious what causation is, there are difficulties to define which event truly causes another. In fact, it should be well distinguished from the statistical correlation where the occurrence of both events is observed at the same time, but an action on one of them will not affect the other event (Sprites, Glymour, and Scheines 2001), (Pearl 2000) (e.g. smoking causes lung cancer while it is statistically correlated with alcoholism). Researchers in artificial intelligence (AI) use the concept of causality in several applications as for the diagnosis of the potential causes from observed effects, to induce causal laws from observations as well as for causal ascription (e.g. see (Benferhat et al. 2008), (Bonnefon et al. 2008) for an overview description). In fact, it is important to provide the systems of inference or decision-making with explanations capacity for an operator or human user.

Deterministic approaches consider that all events can be predicted with certainty. However, these approaches seem to be incompatible with human reasoning and its perception for its environment (Hume 1910) since an event may be present in the absence of its causes or absent in its presence.

For this purpose, Bayesian causal models treated in (Pearl 2000) as well as in (Sprites, Glymour, and Scheines 2001) are used where cause-effect relationships are modeled using probabilistic tools. A distinction between observations and interventions is made. In fact, while an observation is a new

information about the value of a variable in a static world, an intervention, (Pearl 2000), (Halpern and Pearl 2005), is the effect of an *external* action that forces a variable to have a specific value in a dynamic world. This distinction is somewhat similar to the one between belief revision (Gänderfors 1992) and update (Katsuno and Mendelzon 1991) used for modeling belief change.

The main issue of Bayesian causal networks in real world applications, is the necessity to gather enough data to determine all required a priori knowledge including conditional ones. An alternative causal model under a possibilistic framework was also proposed showing its efficiency especially when cases require pure qualitative and ordinal handling (Benferhat and Smaoui 2007).

Belief function theory (Shafer 1976) is a general framework for reasoning with uncertainty. It has connections to other frameworks such as probability, possibility and imprecise probability theories. Belief function networks are important tools to represent and reason under uncertainty. Networks with conditional dependencies were explored with (Shenoy 1993) in valuation based systems (VBS), called valuation networks. In (Xu and Smets 1994), (Ben Yaghlene, Smets, and Mellouli 2003) the network has the same structure as a Bayesian network, since it is a directed acyclic graph. However, the manner in which the conditional beliefs are defined is different from that one in which the conditional probabilities are traditionally defined in Bayesian networks: each edge in the graph represents a conditional relation between the two nodes it connects. Since those networks use the fusion principle, when focal elements are singletons, the belief function network does not collapse into a Bayesian one. In (Simon, Weber, and Evsukoff 2008), conditional dependencies are defined given all the parents like for Bayesian networks (Darwiche 2009).

After an observation, the system changes from a state to another. A belief network allows the prediction of this evolution. Those observations happen by themselves without any manipulation on the system.

Since external actions affect differently the system, the reasoning process requires different modeling tools. It is important to note that no models handling interventions are provided in the belief function framework. Existing works in belief function framework only consider observations.

This paper presents a causal model under a belief function

framework namely belief causal networks which is an alternative and an extension to Bayesian causal networks, that offer interesting tools to handle interventions. The main advantage of belief causal networks is that they solve the problem of ill-known or ill-defined prior probability required in Bayesian causal networks to compute a posteriori distributions. In such cases using arbitrary information under a probabilistic framework is dangerous because it may lead to the application of inappropriate important decisions.

The rest of the paper is organized as follows: In Section 2, we introduce the belief function theory and belief function networks. In Section 3, we propose a definition of causation under a belief function framework. Then, we expose our model in Section 4 based on belief causal networks and explain the changes on the graphs toward handling interventions. Section 5 concludes the paper.

## Belief Function Theory

### Basics of Belief Function Theory

Belief function theory (Shafer 1976) deals with imperfect data. The set of elementary events is referred to the frame of discernment and is denoted by  $\Theta$ . These events are exhaustive and mutually exclusive. Beliefs are expressed on propositions belonging to the powerset of  $\Theta$  denoted  $2^\Theta$ .

The mapping  $m: 2^\Theta \rightarrow [0,1]$  is the basic belief assignment (bba) such that  $\sum_{A \subseteq \Theta} m(A) = 1$ .  $m(A)$  is a basic belief mass (bbm) assigned to  $A$ . It represents the part of belief exactly committed to the event  $A$  of  $\Theta$ . A bba is normalized, if  $m(\emptyset)=0$ . Two bba's provided by two independent sources  $m_1$  and  $m_2$  may be combined to give one resulting mass  $m_{12}$ . The Dempster rule of combination is used, when both sources are reliable.

$$m_1 \oplus m_2(A) = K \cdot \sum_{B \cap C = A} m_1(B) \cdot m_2(C), \forall B, C \subseteq \Theta \quad (1)$$

where  $K^{-1} = 1 - m_1 \oplus m_2(\emptyset)$  is the normalization factor. Conditioning allows to change the knowledge we had, update masses originally defined on  $A$ , following the disposal of a new more precise information saying that the proposition  $B$  is certain, i.e.  $m(B)=1$ . The mass initially allocated to  $A$  will be transferred to  $A \cap B$ . In the case, where  $A \cap B = \emptyset$ , several methods exist for transferring the remaining evidence (Smets 1991).  $m[B](A)$  denotes the degree of belief of  $A$  in the context where  $B$  holds with  $A, B \subseteq \Theta$ . The Dempster rule of conditioning is computed by:

$$m[B](A) = \begin{cases} K \cdot \sum_{C \subseteq \overline{B}} m(A \cup C) & \text{if } A \subseteq B, A \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where  $K^{-1} = 1 - m[B](\emptyset)$ .

### Operations on the Product Space

Let  $U = \{X, Y, Z, \dots\}$  be a set of variables, where each variable has its frame of discernment. Let  $X$  and  $Y$  be two disjoint subsets of  $U$ . Their frames are the product space of the frames of the variables they include. The joint relation  $\Theta_X \times \Theta_Y$  is denoted  $X \times Y$  for short.

**Marginalization:** Given a bba defined on the product space

$X \times Y$ , a bba defined on one of the subset of the product space is obtained by dropping the extra coordinates. It is denoted by:

$$m^{X \downarrow Y}(B) = \sum_{C \downarrow Y = B} m^{X \times Y}(C) \quad (3)$$

where  $C \downarrow Y = B$  is called the projection of  $C$  on  $Y$ ;  $C \cap (B \times X) \neq \emptyset$ .

**Example 1** Given the bba defined on  $A \times B$   $m(\{a1, b1\}, \{a1, b2\}) = 0.7$ ,  $m(\{a2, b1\}, \{a2, b2\}) = 0.1$ ,  $m(\{a1, b1\}) = 0.2$ . Marginalizing  $m$  on  $B$  will lead to the following distribution:  $m(\{a1\}) = 0.7 + 0.2 = 0.9$ ,  $m(\{a2\}) = 0.1$

**Vacuous extension:** Given a bba defined on  $X$ , its vacuous extension on  $X \times Y$  denoted  $m^{X \uparrow X \times Y}$  is given by:

$$m^{X \uparrow X \times Y}(A) = \begin{cases} m^X(A') & \text{if } A = A' \times Y, A' \subseteq X \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

**Example 2** Given the following bba defined on  $A$  as:

$m(\{a1\}) = 0.5$ ,  $m(\{a2\}) = 0.2$ ,  $m(\{a1, a2\}) = 0.3$ . Its corresponding vacuous extension to the product space is given by taking into consideration all the value of  $B$  for a given value of  $A$ .  $m(\{a1, b1\}, \{a1, b2\}) = 0.5$ ,  $m(\{a2, b1\}, \{a2, b2\}) = 0.2$ ,  $m(\{a, B\}) = 0.3$ .

**Ballooning extension:** Let  $m^{X'}$  be a bba defined on a frame of discernment  $X'$ . We would like to build a bba on a larger frame  $X \supseteq X'$ . The least committed bba (Smets 1998) on  $X$  is given by the so called "ballooning". It is defined as:

$$m^{X' \uparrow X}(A) = \begin{cases} m^{X'}(A') & \text{if } A' \subseteq X', A = A' \cup \overline{X'} \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

**Example 3** Let  $X' = \{x1, x2\}$  such that  $m^{X'}(\{x1\}) = 0.6$  and  $m^{X'}(\{x1, x2\}) = 0.4$ . Let  $X = \{x1, x2, x3, x4\}$ , the ballooning extension to  $X$  gives the following bba:  $m^X(\{x1, x3, x4\}) = 0.6$  and  $m^X(\{x1, x2, x3, x4\}) = 0.4$ .

Let  $m^X[y](x)$  defined on  $X$  for  $y \in Y$ , its ballooning extension is defined on  $X \times Y$  in such a way that its conditioning on  $X$  is  $m^X$ ,

$$m_{y_i}^{X \uparrow X \times Y}(\theta) = \begin{cases} m^X[y_i](x) & \text{if } \theta = (x, y_i) \cup (X, \overline{y_i}) \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

**Example 4** Given a bba defined on  $A$ , its definition on  $A \times B$  in such way that its conditioning on  $B$  gives  $m(A)$  is obtained by the application of the ballooning extension. Let us consider the following conditional basic belief mass:  $m[b_1](a_1) = 0.7$ . Its corresponding basic belief mass on  $A \times B$  is obtained by taking into consideration  $(\{a_1, b_1\})$  and all the instances of  $A$  for the complement of  $b_1$  (here  $b_2$ ). Thus the mass initially allocated to  $a1$  will be allocated to  $m(\{a_1, b_1\}, \{a_1, b_2\}, \{a_2, b_2\})$  given that  $B=b1$  is certain.

### Belief Function Networks

Belief function networks are important tools to model and reason with uncertainty. Since focal elements are subsets instead of singletons, partial and total ignorance are well handled. A belief network is a graphical model, denoted  $G$  defined on two levels:

- Qualitative level: represented by a directed acyclic graph (DAG),  $G=(V,E)$  where  $V$  is the set of variables and  $E$  the set of edges encoding the dependencies among variables. A variable  $V_j$  is called a parent of a variable  $V_i$  if there is an edge from  $V_j$  to  $V_i$ . Thus the set  $V_j$  parents of  $V_i$  is denoted  $Pa(V_i)$ .
- Quantitative level: represented by the set of bba's associated to each node in the graph. For each root node  $V_i$  (i.e. node without parent nodes) having a frame of discernment  $\Theta$ , an a priori mass  $m(V_i)$  is defined on the powerset  $2^\Theta$ . For other nodes, a conditional bba's  $m[Pa(V_i)](V_i)$  is specified for each value of  $V_i$  knowing the value  $Pa(V_i)$ .

With belief function networks, it is possible to deal with ill-known or ill-defined a priori knowledge including conditional ones to compute a posteriori distribution and therefore solving the problem of choosing an appropriate a priori. The definition of the joint distribution under a belief function framework is different from the construction made in Bayesian networks. In fact, it is obtained by combining the joint distribution of each node. This is done in three steps as following:

1. For a conditional variable  $V_i$  defined given its parents  $Pa(V_i)$ :
  - 1.1 For each instance of the parents denoted  $Pa_j(V_i)$  compute the ballooning extension of  $m_i^V[Pa_j(V_i)]$  for the deconditionalization process:  $m^{V_i}[Pa_j(V_i)]^{\uparrow V_i \times Pa(V_i)}$ .
  - 1.2 Combine the ballooning extensions using Dempster's rule of combination:  $\oplus_j m^{V_i}[Pa_j(V_i)]^{\uparrow V_i \times Pa(V_i)}$ .
2. Extend each node (root node and child node) to the universe of all variables in the network by applying the vacuous extension:  $(\oplus m^{V_i}[Pa_j(V_i)]^{\uparrow V_i \times Pa(V_i)})^{\uparrow V_1, \dots, V_n}$ .
3. Combine local joint distributions using Dempster's rule of combination and thus get the following chain rule:
 
$$m^{V_1, \dots, V_n} = \oplus_{i=1, \dots, n} (\oplus m^{V_i}[Pa_j(V_i)]^{\uparrow V_i \times Pa(V_i)})^{\uparrow V_1, \dots, V_n} \quad (7)$$

### Belief Causation: Observations vs Interventions

In this paper, we define causal relations under a normalized belief function framework. Unlike deterministic approaches where causes are necessary to the occurrence of their effects, a belief causal link defines a higher belief of effects when a cause takes place and accordingly if a cause does not arise then the belief of the effect will decrease. External actions on the system disturb the relationships between variables and thus should have a different impact on the other events. It seems obvious that the application of belief conditioning is appropriate when an event occurs spontaneously (observation) and will lead to erroneous results when something forces the event to happen (intervention). For example, one should reason differently when he observes that the harvest is not good and when he knows that someone intervenes by lighting a fire and so the entire crop will be lost. In the following, we present the difference between an intervention and an observation in a belief function framework.

### Observation

Observation is seeing. It can provide some information about the statistical relations amongst events. When we have passively observed an event, we can reason backwards diagnostically to infer the causes of this event, or we can reason forward and predict future effects. For example, if you travel to Russia and you notice that the harvest is not good, you will change your beliefs about the rainfall (even if it usually rains enough  $m(\text{rain}=\{\text{yes}\}) > m(\text{rain}=\{\text{yes}, \text{no}\}) > m(\text{rain}=\{\text{no}\})$ ). The bba on the variable "Rain" is revised according to the observation and thus a new bba  $m_1$  is obtained:

$$m_1(\text{rain}=\{\text{no}\}) > m_1(\text{rain}=\{\text{yes}, \text{no}\}) > m_1(\text{rain}=\{\text{yes}\}).$$

### Intervention

Intervention is the act of manipulating. Unlike observation, it is the effect of an external action to the system that forces a variable to have a specific value in a dynamic world. It means that the natural behavior of an object is voluntary changed without being sure of the outcome.

Interventions allow the identification of elements in a sequence of events that are related in a causal way and thus make possible to explain especially the negative influence of the occurrence of an event. Assume, that you know that someone has lit a fire. It is obvious that this action has an impact on the harvest and it forces it to take the value "low". Your initial beliefs about the rainfall remain unchanged.

$$\begin{aligned} m_2(\text{rain} = \{\text{yes}\}) &= m(\text{rain} = \{\text{yes}\}) > \\ m_2(\text{rain} = \{\text{yes}, \text{no}\}) &= m(\text{rain} = \{\text{yes}, \text{no}\}) > \\ m_2(\text{rain} = \{\text{no}\}) &= m(\text{rain} = \{\text{no}\}) \end{aligned}$$

Interventional beliefs allow the reasoning in a causal way by the mean of the "DO" operator, originally introduced by (Goldszmidt and Pearl 1992) for the ordinal conditional functions of Spohn (Spohn 1988) and proposed after that in (Pearl 2000) under a probabilistic framework.

An intervention on a variable  $V_i$  pushes it to take the value  $v_i$  is denoted  $do(V_i = v_i)$  or  $do(v_i)$  without modifying our beliefs over direct causes of  $v_i$ . While a conditional belief representing the effect of an observation is computed by  $m(.|v_i)$ , we propose to adapt the DO operator to the belief function framework and therefore the interventional belief modeling the effect of an external action will be given by  $m(.|do(v_i))$ .

### Belief Causal Networks for Handling Intervention

Causality and interventional actions can be intuitively and formally described with graphs (Pearl 2000), (Benferhat and Smaoui 2007). It was shown that it provides interesting tools to predict the effects of external actions on the system. A belief network represents an efficient way to model dependency between variables and to predict the effect of observations on the joint distribution of the variables. In order to predict the effects of external actions on the system, the construction of the belief causal network must be different from belief network.

## Belief Causal Network

A belief causal network ( $B_f\text{CN}$ ) is a DAG in which nodes represent variables and arcs describe cause-effect relations. The causal process follows the direction of the edges. Thus, an event is a cause of its child node and an effect of its parent node. For each root node an a priori bba is defined. For each child node, a conditional bba given the value of its parents are defined. Like for belief function networks, variables in  $B_f\text{CN}$  are defined on  $2^\Theta$ .

**Example 5** The graph in the left side of Figure 1 is a belief causal network.

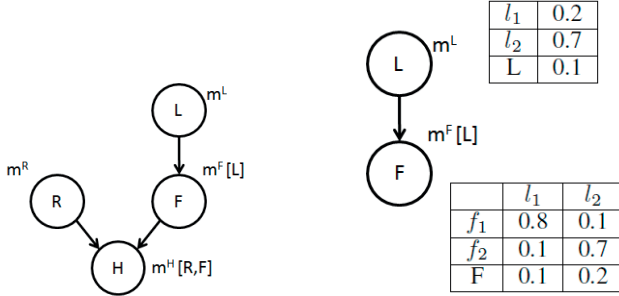


Figure 1: Belief causal network

Nodes are described as follows:  $L$  means that the type of the land reacts with fertilizers ( $l_1$ :yes,  $l_2$  no),  $F$  means that fertilizers has an effect ( $f_1$ :yes,  $f_2$ :no),  $R$  means the rain falls ( $r_1$ :yes,  $r_2$ :no),  $H$  means that the harvest is good ( $h_1$ :yes,  $h_2$ :no). For sake of simplicity, we focus our attention on the belief causal network shown in the right side of Figure 1.

The joint distribution is computed using Equation 7. The first step consists in computing the ballooning extension of each conditional. For example the bba  $m^F[l_1](f_1)$ , will be transferred to  $\{f_1, l_1\} \cup \{F, \bar{l}_1\}$ , i.e. to  $\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_2, f_2\}\}$  as shown in Table 1.

Table 1: Ballooning extension of conditional bba's  $m^F[l_i]^{\uparrow F \times L}$

$l_1$	$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_2, f_2\}\}$	0.4
	$\{\{l_1, f_2\}, \{l_2, f_1\}, \{l_2, f_2\}\}$	0.5
	$\{L \times F\}$	0.1
$l_2$	$\{\{l_2, f_1\}, \{l_1, f_1\}, \{l_1, f_2\}\}$	0.6
	$\{\{l_2, f_2\}, \{l_1, f_1\}, \{l_1, f_2\}\}$	0.1
	$\{L \times F\}$	0.3

Its corresponding joint distribution obtained by the combination of the ballooning extensions of each conditional in shown in Table 2. For example the combination of  $m^F[l_1](f_1)$  with  $m^F[l_2](f_1)$ , is obtained by the combination of their ballooning extension as exposed in Table 1:

$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_2, f_2\}\} = 0.8 \cap \{\{l_2, f_1\}, \{l_1, f_1\}, \{l_1, f_2\}\} = 0.1$ . It is equal to  $\{\{l_1, f_1\}, \{l_2, f_1\}\} = 0.08$ .

Table 2: Joint form of the conditional bba's  $M^F[L]$

$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_2, f_2\}\} = 0.8$	$\{\{l_1, f_1\}, \{l_2, f_1\}\} = 0.08$	$\{\{l_1, f_2\}, \{l_2, f_1\}\} = 0.01$	$\{L \times F\} = 0.1$
$\{\{l_2, f_1\}, \{l_1, f_1\}, \{l_1, f_2\}\} = 0.1$	$\{\{l_1, f_1\}, \{l_2, f_1\}\} = 0.08$	$\{\{l_1, f_2\}, \{l_2, f_1\}\} = 0.01$	$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_1, f_2\}\} = 0.01$
$\{\{l_2, f_2\}, \{l_1, f_1\}, \{l_1, f_2\}\} = 0.7$	$\{\{l_1, f_1\}, \{l_2, f_2\}\} = 0.56$	$\{\{l_1, f_2\}, \{l_2, f_2\}\} = 0.07$	$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_1, f_2\}\} = 0.07$
$\{L \times F\} = 0.2$	$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_2, f_1\}\} = 0.16$	$\{\{l_1, f_2\}, \{l_2, f_2\}, \{l_2, f_1\}\} = 0.02$	$\{L \times F\} = 0.02$

As explained before, at the second step all variables will be extended to the product space. In this example,  $L$  should be vacuously extended to  $F \times L$  (see Table 3).

Table 3: Vacuous Extension:  $m^{L \uparrow F \times L}$

$\{l_1, F\} = 0.2$
$\{l_2, F\} = 0.7$
$\{L, F\} = 0.1$

Finally, as exposed in Table 4, the global joint distribution is computed by combining the bba's obtained by vacuous extension of  $L$  with the bba's presented in Table 2. The resulting joint bba's is summarized in Table 5.

Table 4: Joint distribution (elements represent the intersection of subsets)

	$\{\{l_1, f_1\}, \{l_1, f_2\}\} = 0.2$	$\{\{l_2, f_1\}, \{l_2, f_2\}\} = 0.7$	$\{L, F\} = 0.1$
$\{\{l_1, f_1\}, \{l_2, f_1\}\} = 0.08$	$\{l_1, f_1\} = 0.016$	$\{l_2, f_1\} = 0.056$	$\{\{l_1, f_1\}, \{l_2, f_1\}\} = 0.008$
$\{\{l_1, f_2\}, \{l_2, f_1\}\} = 0.01$	$\{l_1, f_2\} = 0.002$	$\{l_2, f_1\} = 0.007$	$\{\{l_1, f_2\}, \{l_2, f_1\}\} = 0.001$
$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_1, f_2\}\} = 0.01$	$\{l, F\} = 0.002$	$\{l_2, f_1\} = 0.007$	$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_1, f_2\}\} = 0.001$
$\{\{l_1, f_1\}, \{l_2, f_2\}\} = 0.56$	$\{l_1, f_1\} = 0.112$	$\{l_2, f_2\} = 0.392$	$\{\{l_1, f_1\}, \{l_2, f_2\}\} = 0.056$
$\{\{l_1, f_2\}, \{l_2, f_2\}\} = 0.07$	$\{l_1, f_2\} = 0.014$	$\{l_2, f_2\} = 0.049$	$\{\{l_1, f_2\}, \{l_2, f_2\}\} = 0.007$
$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_1, f_2\}\} = 0.07$	$\{l_1, F\} = 0.014$	$\{l_2, f_2\} = 0.049$	$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_1, f_2\}\} = 0.007$
$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_2, f_1\}\} = 0.16$	$\{l_1, f_1\} = 0.032$	$\{l_2, F\} = 0.112$	$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_2, f_1\}\} = 0.016$
$\{\{l_1, f_2\}, \{l_2, f_2\}, \{l_2, f_1\}\} = 0.02$	$\{l_1, f_2\} = 0.004$	$\{l_2, F\} = 0.014$	$\{\{l_1, f_2\}, \{l_2, f_2\}, \{l_2, f_1\}\} = 0.002$
$\{L \times F\} = 0.02$	$\{l_1, F\} = 0.004$	$\{l_2, F\} = 0.014$	$\{L \times F\} = 0.002$



Table 5: Global joint distribution  $m^{LF}$

$\{l_1, f_1\}$	0.16
$\{l_1, f_2\}$	0.02
$\{l_1, F\}$	0.02
$\{l_2, f_1\}$	0.07
$\{l_2, f_2\}$	0.49
$\{l_2, F\}$	0.14
$\{\{l_1, f_1\}, \{l_2, f_1\}\}$	0.008
$\{\{l_1, f_2\}, \{l_2, f_1\}\}$	0.001
$\{\{l_1, f_1\}, \{l_2, f_1\}, \{l_1, f_2\}\}$	0.001
$\{\{l_1, f_1\}, \{l_2, f_2\}\}$	0.056
$\{\{l_1, f_2\}, \{l_2, f_2\}\}$	0.007
$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_1, f_2\}\}$	0.007
$\{\{l_1, f_1\}, \{l_2, f_2\}, \{l_2, f_1\}\}$	0.016
$\{\{l_1, f_2\}, \{l_2, f_2\}, \{l_2, f_1\}\}$	0.002
$\{L \times F\}$	0.002

### Intervention by Graph Mutilation

An external action will alter the system. Those effects should be studied especially if they have a negative influence on the remaining events. They have to be predicted. A manipulation on a variable makes its direct causes (parents) not more responsible of its state. Thus, all the edges directed to the node concerned by the action will be deleted. No changes affect other nodes. Pearl considers it as a surgery (a mutilation) by which all the other causes than the one of the intervention will be excluded.

The graph mutilated is denoted  $G_m$  and its associated belief distribution is denoted  $m_{Gmut}$ . Its corresponding joint distribution is given by  $m_{Gmut}(\cdot|v_i) = m(\cdot|do(v_i))$ . Thus, in order to compute the effect of a manipulation on  $V_i$  ( $do(v_i)$ ), the joint distribution  $m$  is transformed to  $m(\cdot|do(v_i))$ . After a manipulation on  $V_i$ , its corresponding bba becomes:

$$m^{V_i}(v) = \begin{cases} 1 & \text{if } v = v_i \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

The impact of this intervention on the system is obtained by the computation of the global joint distribution given that an external action is made on the variable  $V_i$  setting its value to  $v_i$ :

$$m(\cdot|do(v_i)) = \begin{cases} \oplus_{j \neq i} (m^{V_j}[Pa_{V_j}]^{\uparrow V_j \times Pa_{V_j}})^{\uparrow V_1, \dots, V_n} & \text{if } V_i = v_i \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

**Example 6** *Lighting a fire in a meadow will affect the harvest. It is considered as an external intervention on the variable  $H$  that forces it to take the value low. On the network shown in Figure 1, an action on the variable  $H$ , that obliged it to take the value  $H=h1$ , will lead to the disconnection of  $H$  from its original causes, here Rainfall  $R$  and the use of Fertilizers  $F$ .*

Figure 2 illustrates the new graph, the mutilated one. By this way, parents of the manipulated variable become independent from it and thus the belief about their occurrence remains unchanged.

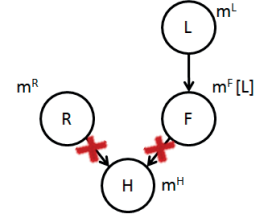


Figure 2: Graph mutilation

After a mutilation on the graph presented in Figure 1, the joint bba's is then computed using (10b) instead of (10a).

$$m^{L,F} = m^L \oplus m^F[L] \quad (10a)$$

$$m_{Gmut}^{L,F} = m^L \oplus m^F \quad (10b)$$

with:

$$m^F(f) = \begin{cases} 1 & \text{if } f = f_1 \\ 0 & \text{otherwise} \end{cases}$$

**Proposition 1** *Let  $V_i$  be a manipulated variable set to a value  $v_i$  and let  $Pa_{V_i}$  its directed causes.*

*If  $U$  is a set of variable distinct from  $V_i \cup Pa_{V_i}$  then after a marginalization on  $Pa_{V_i}$  of Equation 9, we get a counterpart and a generalization of Pearl proposition (Pearl 2000) for the belief function theory framework:*

$$m^U(|do(v_i)) = (m^U(\cdot|v_i, Pa_{V_i}) \oplus m(Pa_{V_i}))^{\downarrow Pa_{V_i}} \quad (11)$$

*It amounts to make a conditioning on the mutilated graph.*

### Intervention by Graph Augmentation

Another interpretation of interventions on a causal network, is to add a parent node called "DO" to the node on which an intervention is made (Pearl 1993). This node is considered as an extra node in the system.

The augmented graph with those nodes is denoted  $G_{aug}$ . Thus, the set of parents  $Pa(V_i)$  becomes  $Pa' = Pa(V_i) \cup DO$ . This alternative to model interventions is also studied in this paper under a belief function framework.

This variable can take the values in  $DO(V_i=v_i)$ ,  $do_{nothing}$ . Nothing means that there are no actions on the variable  $V_i$ , it represents the state of the system when no interventions are made.  $DO(V_i=v_i)$  means that the variable  $V_i$  is forced to take the value  $v_i$ .

Notice that in this paper, we only consider simple form of intervention where only one instance is forced to be true. Hence conditioning on the "DO" variable only concerns singletons. The new distribution is then:

$$m[Pa(V_i), DO(v_i)](v_i) = \begin{cases} 1 & \text{if } V_i = do(v_i) \\ 0 & \text{if } V_i \neq do(v_i) \\ m[Pa(V_i)](v_i) & \text{if } DO = do_{nothing} \end{cases} \quad (12)$$

It remains to specify what is the bba assigned to the added node (i.e. DO). As said previously we only consider simple form of interventions, two cases are considered:

- If there is no intervention then bba's of the DO node is defined by:

$$m^{DO} = \begin{cases} 1 & \text{if } DO = do_{nothing} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

In this case, we can show that:

**Proposition 2** An augmented belief causal graph where the *DO* node is set to the value nothing encodes the same joint distribution that an original belief causal graph.

$$m^{V_i=1,\dots,n} = m_{Gaug}(.|DO = do_{nothing}) \quad (14)$$

- If there is an intervention that pushes the variable  $V_i$  to take the value  $v_i$ , then the DO bba's is defined by:

$$m^{DO} = \begin{cases} 1 & \text{if } DO = do(v_i) \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

In this situation, we have:

**Proposition 3** As for probability and possibility theory, dealing with interventions using the mutilation of the graph or its augmentation gives the same results.

$$\begin{aligned} m^{V_i=1,\dots,n}(.|do(v_i)) &= \\ m_{Gmut}(.|V_i = v_i) &= \\ m_{Gaug}(.|DO = do_{nothing}) & \end{aligned} \quad (16)$$

We note that even though there is a difference in the construction of the global joint distribution between the initial, the mutilated and the augmented graph the result remains the same.

**Example 7** Let us continue with the network in Figure 1. Its corresponding augmented graph is shown in Figure 3. The parents of the manipulated variable  $H$  ( $H=h1$ ), are not only its direct causes ( $R, F$ ) but also the added variable “*DO*”. A new distribution  $m_{Gaug}$  have to be taken into consideration, it is defined on  $L \times F \times R \times H \times DO$ .

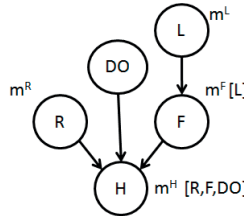


Figure 3: Graph Augmentation

The local distribution on the node  $H$  is obtained using Equation 12. For example:  $m[r1,f1,DO_{nothing}](h1) = m[r1,f1](h1)$ ;  $m[r1,f2,DO_{h1}](h1) = 1$ ;  $m[r2,f1,DO_{h2}](h1) = 0$ .

## Conclusion

This paper provides a graphical model to deal with interventions under a normalized belief framework. We have first presented a definition of a belief causation and a belief causal network. We have shown that in order to correctly represent causal relations and reason in a causal way, the structure of the network has to be modified and the conditioning on observation should be distinguished from a conditioning on an external action. A generalization of the do operator under the belief function framework was therefore proposed, mutilated and augmented belief graphs were presented even if the joint distribution is not defined as for probability distribution for instance.

As future works, we plan to define properties of belief

causality and to profit from the allocation of a non-zero belief mass on the empty set, this allows an extension of the causality ascription model proposed in (Bonnefon et al. 2008) to the belief function framework.

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