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Exceptions in Ontologies: Deducing Properties from Topological Axioms

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Abstract

This paper is a contribution to formal ontology study. We propose a new model of knowledge representation by combining ontologies and topology. In order to represent atypical entities in the ontologies, we introduce topological operators of interior, exterior, border and closure. These operators allow us to describe whether an entity, belonging to a class, is typical or not. We define a system of relations of inclusion and membership by adapting the topological operators. We propose to formalize the topological relations of inclusion and membership by using the mathematical properties of topological operators. However, there are properties of combining operators of interior, exterior, border and closure allowing the definition of an algebra (Kuratowski, 1958). We propose to use these mathematical properties as a set of axioms. This set of axioms allows us to establish the properties of topological relations of inclusion and membership.

1. The problematic

Let us illustrate this problematic using the following example. Some individual entities are attached to classes whereas they do not check all the properties of the class. To illustrate this phenomenon, let us consider the ontological network above (see Fig. 1).

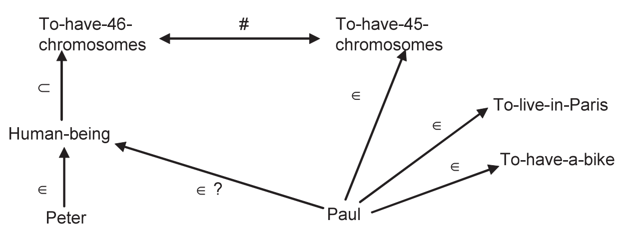

Figure 1: The element [Paul] does not satisfy all the properties of the class [Human-being].

This network corresponds to the seven following declarative statements:

(1) A human being has 46 chromosomes;
(2) Peter is a human being;
(3) Paul is a human being;
(4) Paul has 45 chromosomes;
(5) Paul lives in Paris;
(6) Paul has a bike;
(7) One thing can not have at the same time 46 chromosomes and 45 chromosomes.

Because [Paul] is a [Human-being], he inherits all the typical properties of [Human-being], in particular [To-have-46-chromosomes]. A paradox is introduced by the statement (7) because "A human-being has 46 chromosomes" is a general fact but not a universal fact. The statement (1) means "In general, human beings have 46 chromosomes but there are some exceptions to this rule".

The same phenomenon is observed with distributive classes. Some subclasses are attached, more or less, to a general class because some of theirs elements may not check all the properties of this general class.

In Artificial Intelligence, the solution for this kind of problem is default reasoning: an individual A belonging to a concept F inherits concepts subsuming F except contrary indications. This technique of default reasoning led for example Reiter (Reiter 1980) to propose non-monotonic logics.

2. Topological Relations

We postulate that networks of concepts and semantic relationships between concepts can be represented on a plan. Thus, instances are points of the plan, while classes are demarcated areas of the plan, which consist of : (a) an interior (the typical elements belonging to the class), (b) an exterior (the elements that are not in the class), (c) a border (atypical elements that do not check all the properties of the class, i.e atypical elements that are neither within nor outside the class).

The properties of the four topological operators of interior (noted i), exterior (noted e), border (noted b) and closure (noted f) are used as a set of axioms. We have, for instance, for the interior of a set F, noted iF:

1. F open $\Leftrightarrow$ iF = F (by definition)

2. $iiF = iF$ (idempotence)
3. $iF \subset F$
4. **a)** $i(F \cap G) = iF \cap iG$ and **b)** $iF \cup iG \subset i(F \cup G)$
5. $F \subset G \Rightarrow iF \subset iG$ (monotony)

We have, as well, properties for eF, bF et fF. We define, therefore, the next six topological relations in which X represents any given point and {X} represents a singleton (i.e. the smallest neighbourhood containing X). F,G, H represent any parts of D that are not singletons. Those are sets that represent the extensions of concepts:

1. Membership at the interior of a class (noted $\in_i$): we define ($X \in_i F$) if and only if X inherits all the properties of F: $X \in_i F$ iff $\{X\} \subset iF$;

2. Membership at the exterior of a class (noted $\in_e$): we define ($X \in_e F$) if and only if X cannot belong neither the interior nor the border of F (and in the same way recursively for the subclasses of F): $X \in_e F$ iff $\{X\} \subset eF$;

3. Membership at the border of a class (noted $\in_b$): we define ($X \in_b F$) if and only if X is an atypical individual entity of F: $X \in_b F$ iff $\{X\} \subset bF$;

4. Inclusion at the interior of a class (noted $\subset_i$): we define ($F \subset_i G$) if and only if F is a typical subclass of G: $F \subset_i G$ iff $F \subset iG$;

5. Inclusion at the exterior of a class (noted $\subset_e$): we define ($F \subset_e G$) iff F cannot be a subclass neither at the interior nor at the border of G : $F \subset_e G$ iff $F \subset eG$;

6. Inclusion at the border of a class (noted $\subset_b$): we define ($F \subset_b G$) if and only if F is an atypical subclass of the class G: $F \subset_b G$ iff $F \subset bG$.

Using properties of i, e, b and f operators, we can deduce inference rules of the relations of inclusion and membership. We identified thirteen rules. In particular, we have:

- A4: $(X \in_i G) \wedge (G \subset_i C) \Rightarrow (X \in_i H)$
- A5: $(X \in_i G) \wedge (G \subset_b H) \Rightarrow (X \in_b H)$
- f1: $(G \subset_e F) \wedge (H \subset_i G) \Rightarrow (H \subset_e F)$.
- f2: $(A \subset_e B) \wedge (C \in_i A) \Rightarrow (C \in_e B)$

Figure 2 represents an interpretation of figure 1 using our topological relations. An individual entity is represented by a point while a class C is represented in the form of a topological ball projected on a plan, with its interior, its border an its exterior. In particular, we notice that [Paul] is an atypical element of the class [Human-being], while [Peter] is a typical element of the class [Human-being]. Dotted arrows represent some possible deductions thanks to the rules of combination we defined. For example (see Fig. 2):

By using the rule A4, we can deduce that [Peter] is a typical element of the class [To-have-46-chromosomes]. With the rule f1, we deduce that the class [Human-being] is exterior of the class [To-have-45-chromosomes]. Similarly, with the rule f2, we deduce that the element [Paul] is exterior of the class [To-have-46-chromosomes], etc. Thus, using our topological relations, it becomes possible not only to remove the paradox, but also to make clear the semantic of the relations of inclusion and membership. In the long text, it is expected to show other examples where paradoxes are lifted, especially between atypical subclasses of a given class.

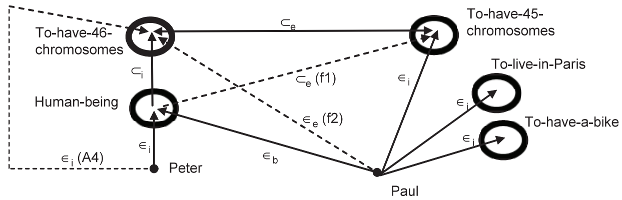

Figure 2: Interpretation of Figure 1 using our topological relations and some possible deductions

3. Conclusion

In this paper, topological concepts of interior, border, closure and exterior are used to specify whether an individual entity, belonging to a class, is typical or not. By adapting these operators, a system of relations is defined. We proposed to formalize the topological relations of inclusion and membership by using the mathematical properties of topological operators. We used the properties of combination of these operators that allow the definition of an algebra. We proposed, as well, to use these mathematical properties as a set of axioms. This set of axioms allows us to establish the properties of the relations of topological inclusion and membership. This model has been implemented in ANSPROLOG* (Baral 2003). This language is used to describe directly the facts (i.e. the initial network) and inference rules in the same formalism as used in PROLOG. It has the ability to represent normative statements, exceptions and default statements, and is able to reason with them.

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