The Promise of LP to Boost CSP Techniques for Combinatorial Probl ems*

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Abstract

In recent years we have seen the development of successful methods for solving optimization problems by integrating techniques from Constraint Programming (CP) and Operations research (OR). Such hybrid approaches draw on the individual strengths of these different paradigms: OR heavily relies on mathematical programming formulations such as integer and linear programming, while CP uses constrained-based search and inference methods. This is particularly true in domains where we have a combination of linear constraints, well-suited for linear programming (LP) formulations, and discrete constraints, suited for constraint satisfaction problem (CSP) formulations.

Nevertheless, *in a purely combinatorial setting*, so far it has been suprisingly difficult to integrate LP-based and CSP-based techniques. For example, despite a significant amount of work on using LP relaxations to solve Boolean satisfiability (SAT) problems, practical state-of-the-art solvers do not incorporate LP relaxation techniques. From a practical point of view, the challenge is how to integrate such techniques into practical solvers. The basic idea is to use the information from LP relaxations to guide the combinatorial search process. A key issue is whether the LP relaxation provides useful additional information that is not already uncovered by constraint propagation and inference techniques. Of course, also the cost of solving the LP relaxation should not outweigh the benefits in terms of reduction in search cost.

We present a *complete* randomized backtrack search method that tightly couples CSP propagation techniques with randomized LP rounding. Our approach draws on recent results on some of the best approximation algorithms with theoretical guarantees based on LP relaxations and randomized rounding techniques, as well on results that uncovered the extreme variance or "unpredictability" in the running time of complete search procedures, often explained by the phenomenon of heavy-tailed cost distributions.

We use as a benchmark domain the Quasigroup or Latin Square Completion problem. Each instance consists of an N by N matrix with N^2 cells. A complete quasigroup consists of a coloring of each cell with one of N colors in such a way that that there is no repeated color in any row or column. Given a partial coloring of the the N by N cells, determining whether there is a valid completion into a full quasigroup is an NP-complete problem.We can finely tune the complexity of the completion task by varying the fraction of the uncolored cells. The underlying stucture of this benchmark is similar to that found in a series of real-world applications, such as time-tabling, experimental design, and fiber optics routing problems.

We present a randomized 0.632-approximation algorithm for the Quasigroup or Latin Square Completion problem. We also describe promising experimental findings in support of the effectiveness of our complete randomized hybrid CSP/LP backtrack search method on the Quasigroup Completion Problem. A detailed empirical and theoretical evaluation is currently under way. Our results show how we can use the information provided by the randomized LP rounding as a powerful heuristic to guide the backtrack algorithm — the LP relaxation with rounding strategy provides global information about the values to assign to CSP variables. Interestingly, and contrarily to other domains, the role of the LP relaxation in detecting infeasibility is not as important as a search heuristic. In fact, while LP infeasibility detection can be useful at the top of the search tree, at medium/lower levels, CSP propagation techniques are more effective at pruning the search space with considerably less computational effort. Our approach also uses restart strategies in order to combat the long tails that characterize combinatorial search. By using restart strategies we take advantage of *any significant probability mass early on in the distribution* reducing the variance in runtime and the probability of failure of the search procedure, resulting in a more robust overall search method.

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