Diagrammatical Aspects of Qualitative Representations of Space

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Abstract

In this paper we demonstrate the usefulness of the diagrammatical aspects of a qualitative representation of positions in 2-D space [6]. Qualitative representations make only as many distinctions as necessary to identify objects, events, situations, etc. in a given context (identification task) as opposed to those needed to fully reconstruct a situation (reconstruction task). While the distinctions made are expressed propositionally in form of relations, we use data structures that analogically reflect the structure of the relational domain on a higher level of abstraction. This representation allows to perform operations such as a change in point of view or the composition of relations efficiently.

As a result of the extended "imagery" debate in cognitive science [8; 13] approaches to the representation of spatial knowledge tend to fall into one of the categories "propositional" or "pictorial". There are several problems with these two extreme positions:

The propositional approaches focus primarily on formal properties of the representation such as soundness and completeness [14]. While doing so, however, they are forced to explicitly[1] express the rich structural properties of space using propositions. Thus, basic structural properties of physical space such as:

- homogenity and continuity of physical space
- objects have only positive extension
- two objects cannot fill the same space at the same time
- each object exists only once (identity)
- an object can only move to neighboring positions (adjacency)

must be stated explicitly leading to an enormous computational overhead even for moderately realistic applications.

On the other hand, current pictorial representations [4; 7], while preserving many of the spatial properties mentioned above, model some aspects of space on too low a level. For example, by defining a fixed grid of a given granularity (each field of which can be either filled or not) to represent occupancy, an arbitrary discretization of the domain is introduced. This may lead to wrong results, because binary decisions are forced at an inadequate stage of processing.

However, the desirable properties of both propositional and pictorial representations can be combined. This has been recognized by several authors [12; 2], who nevertheless propose hybrid models that combine them by interfacing them as *separate* representations. The qualitative representation of space we propose here *integrates* them by exploiting the generalization capabilities of abstract relational representations while inherently reflecting domain constraints such as the neighborhood of the positional relations.

This is possible, because pictorial representations are just a special case of the more general class of analogical representations [15], i.e. those, where important properties of the represented domain are intrinsically represented by the inherent properties of the representing domain. The structural similarity between the represented and the representing domains doesn't need to be an isomorphism and can be given at an abstract level.

1 Qualitative representation of positions in 2-D space

We focus on 2-D projections of 3-D scenes. In order to represent the relative position of two objects in 2-D space qualitatively we define a small set of spatial relations from the two relevant dimensions projection and orientation. By projection we mean how the boundaries of the two objects relate, i.e. their topological relations. By orientation we mean where the objects are placed relative to one another. The relative position is given by a projection/orientation relation pair.

A complete set of topological relations can be derived from the combinatorial variations of the point set intersection of boundaries and interiors of the involved objects by imposing the constraints of physical space on them [3]. The projection relations are **disjoint** (d), **tangent** (t), **overlaps** (o), **contains-at-border** (c@b),

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[1]The sense in which "explicit" and "implicit" are used in this paper is different from the one in the "Call for Participation" for this workshop. While the latter states for example that "The power of diagrammatic representations stems from the property that they allow the explicit representation and direct retrieval of information that can be represented only implicitly in other types of representations and then has to be computed, sometimes at great cost, to make it explicit for use.", our use refers to the way the represented and the representing domains are related. Thus, while the structural properties of space must be explicitly stated in a propositional representation, they are implicitly given by the corresponding properties of the pictorial representations.

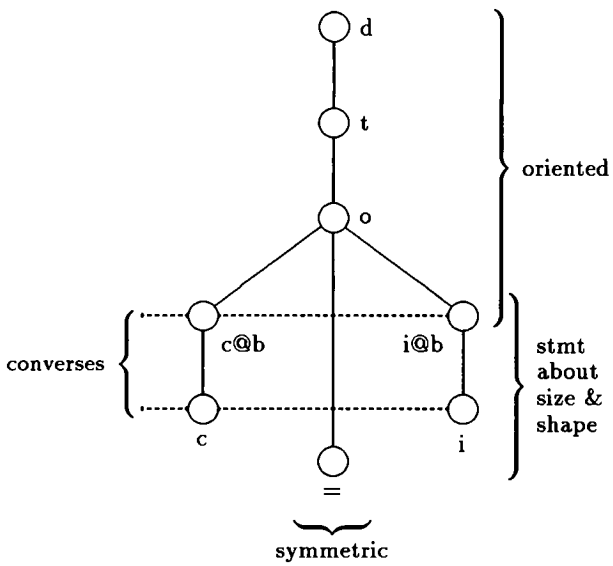

Figure 1: Structure of the projection dimension

included-at-border (**i@b**), **contains** (**c**), **included** (**i**), and **equal** (**=**). Figure 1 shows the structure of the projection domain and some of the properties of the relations involved. The relations {**d**, **t**, **o**, **=**} are symmetric. The containment relations {**c@b**, **c**, **i@b**, **i**} are pairwise converses, e.g. A [**c**] B iff B [**i**] A. The relations {**d**, **t**, **o**, **c@b**, **i@b**} are oriented, i.e. if two objects are related by one of these projections it is possible to establish their relative orientations as well. This is not the case for {**c**, **i**, **=**}, which preclude orientation. Finally the containment and equality relations make a statement about relative size and shape, e.g. only a a larger object can contain a smaller one, and only objects of the same size and shape can be equal.

The structure of the relational domain results from the structure of physical space. Two relations are directly linked by an arc in the figure (neighbors) if the corresponding physical situations can result from each other without an intervening situation that could be described by a third relation of the same dimension. Thus, **disjoint** and **tangent** are neighboring relations, because two disjoint objects can come together and touch each other (tangency) without an intervening situation (distance ist *not* a topological concept and cannot be determined qualitatively, i.e. by comparing just the two objects involved). **Overlaps** and **contains** are not neighbors, because, if one object can contain another one (in the projection) and they overlap, there is a situation in the change from **overlaps** to **contains** in which one object **contains-at-border** the other one. Neighboring relations behave similarly, which allows us to define hierarchically organized levels of granularity by considering groups of neighboring relations as a single "coarse" relation.

The orientation dimension results from the transfer of distinguished reference axes from an observer to the reference object. The orientation relations are (at their finest level) **front** (**f**), **back** (**b**), **left** (**l**), **right** (**r**), **left-back** (**lb**), **right-back** (**rb**), **left-front** (**lf**), and **right-front** (**rf**). As figure 2 shows, they form a uniform circular neighboring structure. Relative orientations must be given w.r.t. a *reference frame*, which can be *intrinsic* (orientation given by some inherent property of the reference object), *extrinsic* (orientation imposed by external factors), or *deictic* (orientation imposed by point of view). When reasoning about orientations, the reference frame is *implicitly* assumed to be the intrinsic orientation of the parent object (i.e. the one containing the objects involved), unless *explicitly* stated otherwise. Converting among reference frames can be done diagrammatically as will be shown below[2].

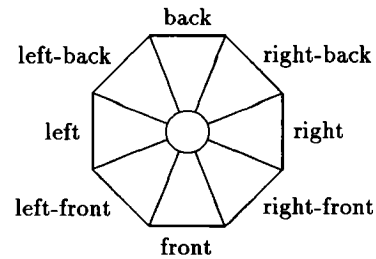

Figure 2: Structure of the orientation dimension

These "structured relational domains" are related to the concept of "quantity spaces" introduced by Hayes [5]. A quantity space is a small, discrete set of values a physical variable may take and is usually totally ordered. Our structured domains are also small, discrete sets of spatial relations between two objects, but have a much richer structure than just total order.

2 Reasoning with abstract maps: diagrammatical aspects

While the long term storage of spatial information is done propositionally, the spatial reasoning process (envisionment) is done diagrammatically through the use of an interim internal representation that inherently reflects the properties of the represented domain. These "abstract maps" contain for each object in a scene a data structure called "rpon" (for relative projection and orientation node), which has the same neighborhood structure of the projection and orientation relations. Figure 3 shows a simplified visualization in which the containment and equality projection relations have been merged in a single node. As in figure 1 arcs between nodes denote neighboring projection/orientation pairs. A spatial relation between two objects is modelled by creating a bidirectional pointer from the projection slot of the given (implicit) orientation of the reference object to the corresponding slot of the "inverse relation" in the primary object.

A change in point of view, which affects relations with an explicit deictic type of reference frame, can be easily accomplished diagrammatically by "rotating" the labels of the orientation with respect to the intrinsic one. In

[2]Determining relative orientations is more involved than what is relevant to show the diagrammatical aspects of this representation. Among the things which are only mentioned here are acceptance areas (depending on relative size and shape [11]) and the availability of various levels of granularity.

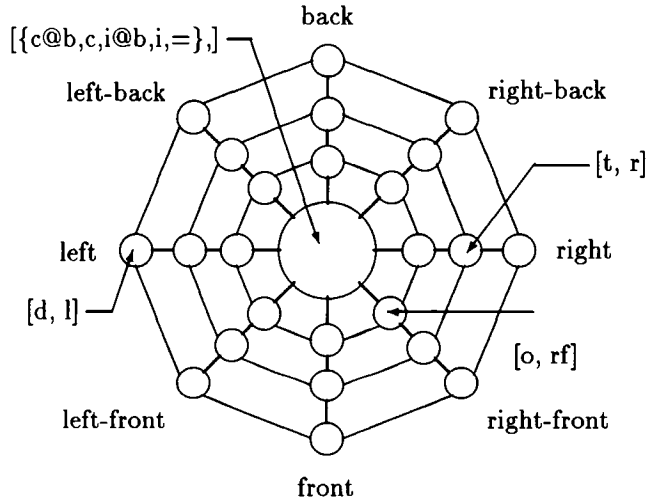

Figure 3: Relative Projection and Orientation Node

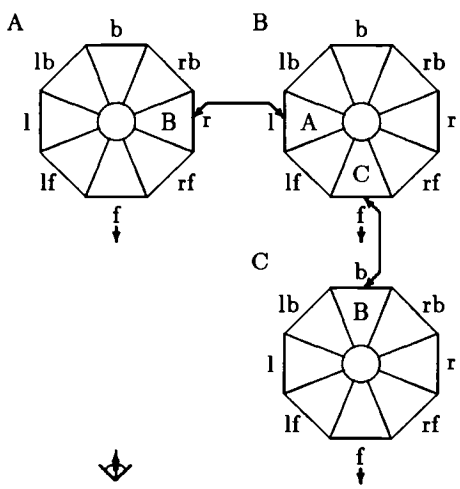

Figure 4: Original point of view

the example shown, A is to the left of B (A [d,l] B) as seen from the original point of view (Fig. 4). If the point of view changes to be the one depicted in Fig. 5, A will rather be considered to be left-back of B as indicated by the rotated labels. Note that all relations of a given object to others in the scene are updated simultaneously while keeping projections consistent. So, while B is to the back of C in Fig. 4, their relationship is correctly relabeled in Fig. 5 to be "B is right-back of C". Furthermore, this can be done "in parallel" for all objects in a scene.

An essential operation in every relational representation is the composition of relations: Given the relation between A and B, on the one hand, and between B and C, on the other, we want to know the relation between A and C. For simplicity we consider here only the special case of composition of orientations for disjoint objects, which can be nicely computed diagrammatically as follows: Given A [,x] B and B [,y] C, the composition A [,t] C contains all orientations that are "in-between" and including x and y (see Fig. 2) on the "shortest path". The composition is symmetric, i.e. t(x, y)=t(y, x). The following examples

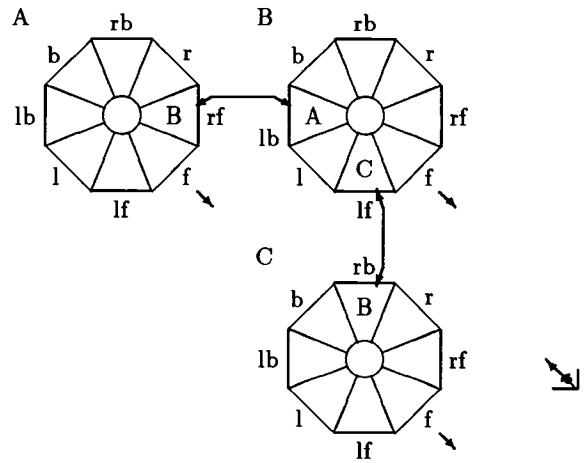

Figure 5: Rotation

illustrate this rule:

1. If $x = y$ then $t = x = y$, since there are no orientations in-between ("distance" between orientations = 0).
2. If A is to the **back** of B and B is to the **left** of C, A can be to the **back**, **left-back** or **left** of C as the three square configurations on the right side of figure 6 show.
3. For x = **lb** and y = **r** we obtain t ={**lb,b,rb,r**} (and *not* {**lb,l,lf,f,rf,r**} which are the orientations between x and y on the longer path "the other way around"). Here, 0 < distance < 4.
4. If x and y are opposites (e.g. **b/f**, **l/r**, **lb/rf**) then t = {?}, that is *any* of the orientations apply, since both paths between opposites have the same length (distance = 4).

While the table-lookup method is certainly more efficient for single compositions, it requires $n * m$ lookups and a set union operation for the composition of underdetermined relations consisting of disjunctions of n and m primitive relations each. Given that all resulting compositions as well as most typical complex relations are connected (neighbors), the diagrammatical method does all the required operations by virtue of its built-in structure.

Spatial reasoning, both changing the point of view and computing the composition of relations, can be done at coarser or finer levels depending on the kind of information available. In particular, if only coarse information is available, the reasoning process is less involved than if more details are known. This is a very intuitive aspect of our approach, which distinguishes it from other frameworks (e.g. using value ranges or confidence intervals) in which less information means more computation.

3 Conclusion

There is another sense in which qualitative representations relate to diagrammatical ones. In the narrow sense of sketch-like representations [4; 9] diagrammatical representations are qualitative in nature, since they

- make only as many commitments as necessary

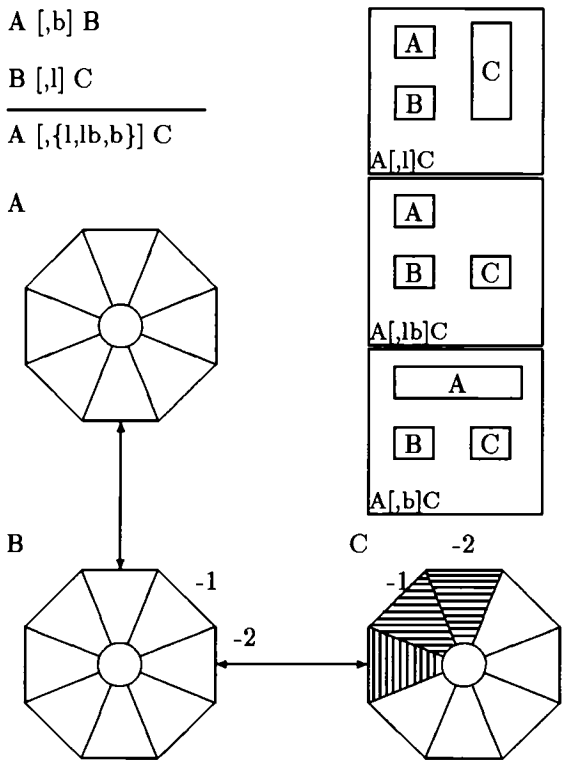

Figure 6: Propagation of tokens in a rpon-graph

- are underdetermined (i.e. stand for a whole class of possible instances) and context dependent
- allow reasoning at various levels of granularity (coarse vs. fine reasoning).

Thus in this paper we have argued, as Palmer [10] does, against the "typical claims [which are] that analog representations are quantitative while propositional representations are qualitative" both by showing the diagrammatical aspects of qualitative representations, and by hinting at the qualitative aspects of diagrammatical representations.

References

[1] N. Block, editor. *Imagery*. MIT Press, Cambridge, 1981.

[2] B. Chandrasekaran and N. Narayanan. Towards a theory of commonsense visual reasoning. In K. Nori and C. Madhavan, editors, *Foundations of Software Technology and Theroretical Computer Science, Tenth Conference*, Lecture Notes in Computer Science, pages 388–409, Bengalore, India, 1990. Springer-Verlag.

[3] M. Egenhofer. A formal definition of binary topological relationships. In W. Litwin and H.-J. Schek, editors, *Third International Conference on Foundations of Data Organization and Algorithms*, volume 367 of *Lecture Notes in Computer Science*, pages 457–472, Berlin, June 1989. Springer-Verlag.

[4] B. V. Funt. Problem solving with diagrammatic representations. *Artificial Intelligence*, 13(3):201–230, 1980.

[5] P. J. Hayes. The naive physics manifesto. In D. Michie, editor, *Expert Systems in the Electronic Age*, pages 242–270. Edinburgh University Press, Edinburgh, Scotland, 1979.

[6] D. Hernández. Relative representation of spatial knowledge: The 2-D case. In D. Mark and A. Frank, editors, *Cognitive and Linguistic Aspects of Geographic Space*, NATO Advanced Studies Institute. Kluwer, Dordrecht, 1991.

[7] M. Khenkhar. DEPIC-2D: Eine Komponente zur depiktionalen Repräsentation und Verarbeitung räumlichen Wissens. In D. Metzing, editor, *GWAI-89*, pages 318–322, Berlin, 1989. Springer.

[8] S. Kosslyn, S. Pinker, G. Smith, and S. Shwartz. On the demystification of mental imagery. In Block [1], pages 131–150.

[9] A. K. Mackworth. On reading sketch maps. In *Proc. of the 5th International Joint Conference on Artificial Intelligence*, 1977. Cambridge, USA.

[10] S. E. Palmer. The nature of perceptual representation: An examination of the analog, propositional controversy. In R. Schank and B. Nash-Webber, editors, *TINLAP-1*, pages 165–173, 1975.

[11] D. Peuquet and Z. Ci-Xiang. An algorithm to determine the directional relationship between arbitrarily shaped polygons in the plane. *Pattern Recognition*, 20(1):65–74, 1987.

[12] S. Pribbenow. Interaktion von propositionalen und bildhaften Repräsentationen. In C. Freksa and C. Habel, editors, *Repräsentation und Verarbeitung räumlichen Wissens*. Springer, Berlin, 1990.

[13] Z. Pylyshyn. The imagery debate: Analog media vs tacit knowledge. In Block [1], pages 151–206.

[14] R. Reiter and A. K. Mackworth. A logical framework for depiction and image interpretation. *Artificial Intelligence*, 41:125–155, 1989.

[15] A. Sloman. Interactions between philosophy and AI — The role of intuition and non-logical reasoning in intelligence. In *Proceedings 2nd IJCAI*, pages 270–278, London, 1971. Reprinted in AI (2) 1971.