

Qualitative Models in Conceptual Design: A Case Study

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Abstract¹

Design is an important but little understood intelligent activity. *Conceptual design* is the transformation of functional specifications to an initial concept of an artifact that achieves them. Human designers rely heavily on the use of sketches, which can be thought of as qualitative models of a device. An appealing model of conceptual design is that of a mapping from *qualitative* functional specifications to a corresponding *qualitative* object model.

As a case study, I have investigated this model for the conceptual design of part shapes in elementary mechanisms, such as ratchets or gears. I present a qualitative modelling formalism for mechanical function, the *place vocabulary*, and for shape, the *metric diagram*. I show that qualitative functional attributes can not be mapped into corresponding qualitative attributes of a device that achieves them, and consequently that qualitative function can not be computed based solely on a qualitative object model. Only a significantly weaker functional model, *kinematic topology*, can be derived based on qualitative object models alone.

This result means that at least in mechanical design, sketches do not represent a *single* qualitative model, but must be interpreted as a *set* of possible precise models. Each step in the design process then refers to a particular precise model in this set. This novel interpretation of the use of sketches suggests alternatives to the popular model of conceptual design as a symbolic mapping of functional into object attributes.

1 Introduction

Designing an artifact is a complex intellectual process of much interest to AI researchers. Most research in intelligent CAD systems has focussed on *detail* design, the adaption of an initial concept to precise specifications. Little is known about the process of *conceptual* design, the transition between functional specification and concept of an artifact that achieves it. As the concept of the artifact is only vaguely defined, conceptual design heavily involves qualitative reasoning and representations.

Elementary Mechanism Design I am investigating the problem of conceptual design for a specific and particularly in-

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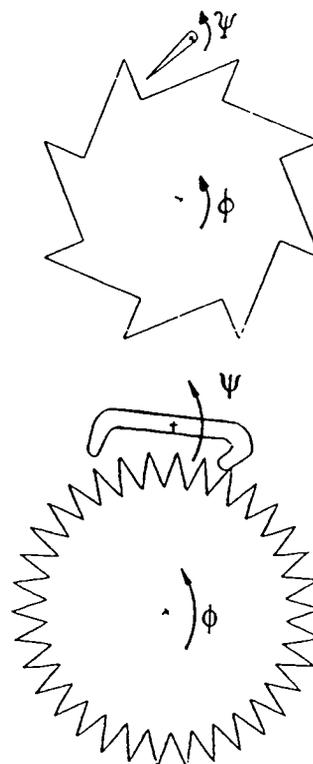


Figure 1: Examples of higher kinematic pairs: a ratchet and a clock escapement.

triguing sub-problem: the analysis and design of *higher kinematic pairs*, often called *elementary mechanisms*. A higher kinematic pair consists of two parts which achieve a kinematic function by interaction of their shapes. This class contains most of the "interesting" kinematic interactions, such as ratchets, escapements or gearwheels, as shown in the examples of Figure 1. Important properties of kinematic pairs are that each object has one degree of freedom (rotation or translation) only, and that their interaction can be modelled in two dimensions. The domain of kinematic pairs is ideal for studying design methodologies and representations because it is very rich in possible designs, but is also contained enough so that it can be treated in isolation without the need for extensive assumptions.

Models of the Conceptual Design Process Design is a mapping of functional specifications to an actual physical

device. A function of a mechanism is its behavior in a particular environment: a ratchet blocks a particular rotation, a Peaucellier device transforms circular motion of one point into straight line motion of another. A functional *specification* is a condition on a function of the device. A numerically precise specification of all functions of a device is often overly restrictive, and it would be pure coincidence if there actually existed a mechanism which satisfies them. In practice, the functional specifications are intentionally *vague*: they admit a whole range of numerical values for functional properties. In order to exploit the possibilities admitted by this vagueness, it must be represented in the functional specification, using a *qualitative* functional model. This functional model is provided by formalisms such as *place vocabularies* ([4, 9]), a qualitative functional modelling language for elementary mechanisms.

What is a qualitative representation? As there is no generally agreed on definition of what makes a model “qualitative,” it is necessary to define the term for the purposes of this paper. The word “qualitative” is derived from “quality”, which is a synonym of “property”. Consequently, a qualitative model is composed of properties of the modelled domain, represented as predicates in first-order logic. More precisely, a qualitative model is

a model of a system in first-order predicate logic where symbols correspond to independent entities of the modelled system, and predicates correspond to connections between or properties of the symbols.

This definition is consistent with all the major approaches to qualitative modelling, in particular it entails compositional and local models (as defined in [2]). Note, however, that models are usually represented using higher-level constructs, but could be translated into predicate calculus.

2 Qualitative Representation of Function

The model of conceptual design as a mapping from function to design object requires first of all a language for qualitatively specifying function. For many domains, the issue of what such a language should express is an open problem. For the limited domain of elementary mechanisms, however, a clear set of requirements can be stated. In the following, note that I use the word *function* to mean something distinct from the *purpose* of the device.

For expressing qualitative behavior or restrictions on qualitative behavior in a qualitative functional model, a general modeling language for qualitative mechanical behavior is necessary. A good qualitative model of behavior is the *envisionment* ([3]) of the device. However, the envisionment itself is not appropriate for modeling function, as it cannot express the functional connection between the environmental inputs and the resulting behavior.

In general, a complex device has an internal state which defines the applicable functional connections between inputs and outputs. A formalism for representing mechanical function must represent the different states and possible transi-

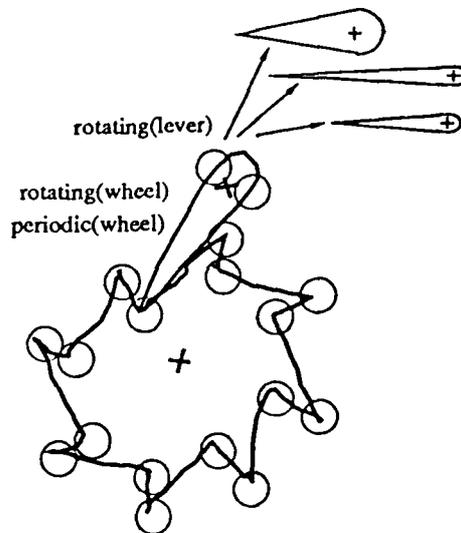


Figure 2: *Example of a sketch of a ratchet. Note that the sketch correctly models the discontinuities of the shapes (marked by circles) and the types of boundaries between them. However, the metric dimensions are much less precisely defined, leaving open the different interpretations shown. This characteristic is shared with the metric diagram.*

tions between them. A functional representation that fulfills these criteria is the *place vocabulary* ([4, 9]). Each place² is characterized by a particular kind of object contact and the set of applicable qualitative inference rules between the dynamic parameters of the device. The places are arranged in a graph, whose edges define the possible transitions and are labelled with the conditions on qualitative motion under which the transitions can occur.

Places vocabularies are described in more detail in [9], and more details on their application to functional specifications can be found in the unabridged version of this paper ([10]).

3 Modelling the Design Object

When discussing concepts, people like to refer to *sketches*, which appear to be qualitative representations of some form. In fact, designers often insist on using an extra wide pen in order to make purposely rough sketches of their initial ideas. In this section, I develop a qualitative modelling formalism for shape which is designed to capture the information represented in a sketch.

In the domain of elementary mechanisms, a sketch is a rough drawing of the shapes of a mechanism which achieves a desired function. An example of such a sketch is shown in Figure 2. The sketch shows the example of a ratchet, a device which blocks clockwise rotation of the wheel (state shown in the sketch), but allows counterclockwise rotation. The most striking fact about the sketch is that the wheel as it is shown in the sketch can not even be turned a full rotation. Consequently, the device as shown in the drawing does not even achieve the function of a ratchet that the sketch is intended

²While “state” would be a better term, we use “place” for historical reasons, and to avoid confusion with kinematic states.

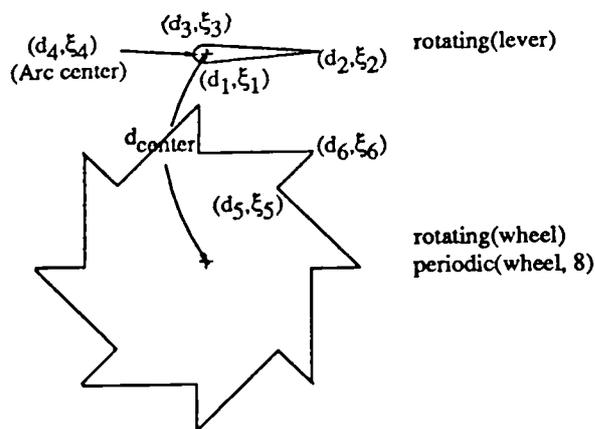


Figure 3: The metric diagram for a ratchet mechanism. The object dimensions are defined as symbolic coordinates, which can then be represented by qualitative values. The coordinates of discontinuities are local to the particular object and defined in polar coordinates.

to demonstrate! Clearly, understanding of a sketch must involve an *interpretation* as a desired function, rather than just a simulation of the device as shown. A theory of how such an interpretation might be constructed is described later in this paper.

If a sketch can not be understood as a precise shape representation, what is the information it conveys? Note that a sketch correctly shows the discontinuities of the shape, as indicated by the circles in Figure 2, as well as the type of edges between discontinuities. The inaccuracies in metric dimensions leave open a range of possible interpretations, some of which are also shown in Figure 2. The information contained in the sketch consists of:

- a graph of discontinuities and edges between them, and
- very approximate metric dimensions of the shape, and
- annotations, such as freedom of motion or periodicity of a shape.

This is the information modelled in the *metric diagram* of a shape, shown in the example in Figure 3. The structure of the shape is modelled as a graph of vertices and edges between them, indicated by the drawing of the shape in Figure 3. In my implementation, edges can be either straight lines or circular arcs.

Metric dimension parameters are associated with each of the elements of this structure. They are the positions of the discontinuities in polar coordinates, the positions of the centers of circular arc edges, and the distances of the objects in the frame of the mechanism. They can be represented in quantity spaces, systems of fixed intervals, precise numbers, or any other representation suitable to express the knowledge available about them. Annotations to the metric diagram express the freedom of motion of the parts, and the periodicity of the ratchet wheel.

The metric diagram is a qualitative representation: the graph modelling the connections can be expressed in predicate logic, and parameter values can be represented by a fixed set

of qualitative values. I now show that there exists no simpler shape representation which is both sufficient to predict the different possible object contacts - an important precondition for any kinematic analysis - and is also qualitative according to the definition in the introduction. The claim follows from a proof that all of the elements of the metric diagram are required in any qualitative shape representation powerful enough for qualitative kinematic analysis.

According to the definition of *qualitative*, each individual in a qualitative representation must correspond to an entity in the real world. Each part of a mechanism must therefore be modelled by distinct and independent individuals. The elements of the model of a single part shape can be justified as follows. Each *discontinuities* and *edges* define the possible object contacts, and consequently must be explicit elements of the model. Changing the *position* of a discontinuity can make an object contact possible or impossible, and consequently each must be represented individually. Knowledge of the *distance between objects* is required to predict object contact, and must be a separate quantity because of the independence of object models. Finally, *annotations* are either shorthand for more complicated expressions (periodicity), or express information required for kinematic analysis (freedom of motion). I conclude that the metric diagram is a required part of any shape representation which is to be related to a model of mechanism kinematics.

4 Mapping between Function and Artifact

The purpose of modelling in design is to allow reasoning about the relationship between a designed object and its function. Ideally, this would be accomplished by a direct mapping between attributes of the functional model and attributes of the geometric object model. Using the place vocabulary as a functional model, the following are examples of functional attributes which are of interest in design:

1. the feasibility of a particular object contact (place).
2. the inference rules which hold in a particular place.
3. the conditions for possible transitions between a pair of places.

Each of these attributes can be mapped into a condition on the model of the designed object, as shown in [7, 11]. Figure 4 shows example positions of a ratchet which serve to illustrate the three types of attributes. The three examples illustrate a progression of complexity: for the first, the mapping can be based on a purely qualitative model, the second requires rather precise approximations of the metric dimensions, and the third example can in practice only be validated on a numerically precise model.

4.1 Mapping to a qualitative object model

For an intelligent CAD system based on a qualitative object model, it is important to be able to map the qualitative functional attributes into equivalent attributes of the object model. In this section, we show the attributes in a metric

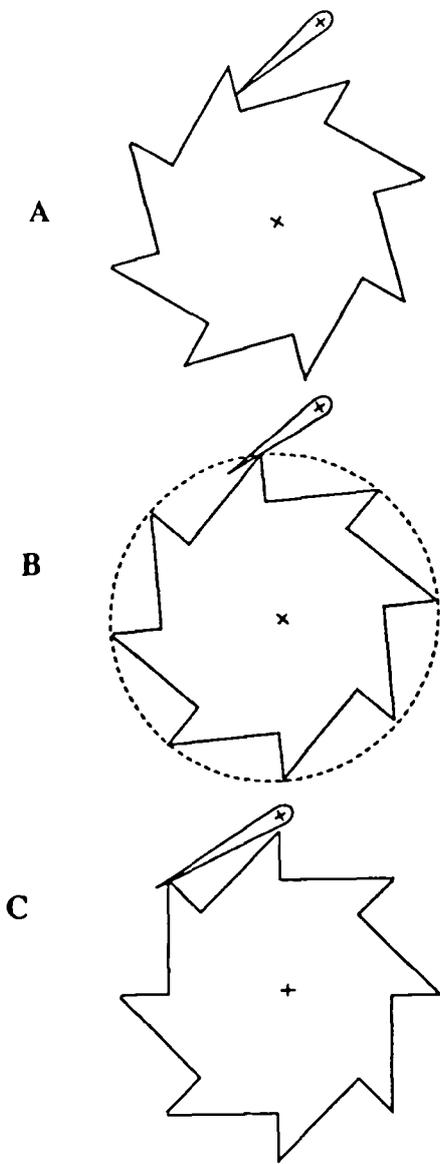


Figure 4: Three different states of a ratchet which illustrate the three example attributes. Deciding whether the contact shown in A is possible amounts to linear distance comparisons. Determining the inference rule which holds in B requires evaluating a complex algebraic expression on dimensions. Showing that the transition between contacts shown in C is actually possible requires iterative numerical analysis.

diagram which correspond to the functional attributes shown in the three examples of Figure 4.

Feasibility of an object contact: The functional attribute of the existence of a state where two particular object parts touch maps into an attribute on the relative dimensions of the objects. In particular, for two rotating objects such as in the ratchet example, two object parts with distance d_1 and d_2 from the respective centers can touch whenever

$$d_1 + d_2 \geq d_{center} \text{ and} \\ |d_1 - d_2| \leq d_{center}$$

where d_{center} is the distance between the centers of rotation of the two objects.

Situation A in Figure 4 is representative of the touch between the tip of the lever and the side of the wheel's tooth. The place corresponding to this situation exists whenever the tip of the lever can touch *some* point on the edge, so that (using the notation of Figure 3 given earlier) the attribute of the metric diagram becomes:

$$\min(d_5, d_6) + d_2 \geq d_{center} \text{ and} \\ |\max(d_5, d_6) - d_2| \leq d_{center}$$

The mapping of this functional predicate into the object model is thus a linear distance comparison. If the distance between the centers of rotation, d_{center} , is kept constant in the design problem, it can be expressed as an attribute of the *relative* values of the object dimensions. As these could be expressed qualitatively in a quantity space ([12]), a purely qualitative metric diagram is sufficient as an object model which allows one to express this attribute. However, if the distance between centers of rotation becomes variable, the quantity space representation becomes insufficient. Note also that the chosen metric diagram is optimal for this case: it only involves a single parameter for each object.

Inference rules for a given place: In the situation B in Figure 4, it might be of interest to know in which direction the pawl will turn when the wheel is turned counterclockwise. This depends on the direction of the edge of the lever with respect to the dashed circle which describes the incremental freedom of motion of the wheel. If the edge "points" to the inside, as shown in the figure, the lever will turn clockwise, otherwise, it will turn counterclockwise. The attribute can be generalized from a single contact point to all configurations with the given contact, but for one contact, there may be up to three regions with different inference rules. The functional attributes are not single inference rules, but the different combinations of regions with different inference rules which can exist for the type of contact.

The condition is equivalent to the sign of a linear expression of the sines and cosines of the rotation angles in the configuration where the points of interest touch. However, these angles in turn depend on a nonlinear combination of object dimensions ([5]). For the example B shown in Figure 4, the inference rule to be applied in the place containing the configuration where the tip of the lever touches a tip of the wheel depends on the sign of (using the notation of Figure 3:

$$\sqrt{d_2^2 + d_3^2 - 2d_2d_3\cos(\xi_2 - \xi_3)} (d_2^2 + d_6^2 - d_{center}^2) - 2d_2d_3d_6$$

This attribute is dependent on nonlinear combinations of distinct parameters of the metric diagram, which furthermore belong to independent objects. Since qualitative attributes can not depend on combinations of independent objects, it is not possible to store its value as part of a qualitative object model, but it must be composed from individual qualitative representations. Only symbolic algebra is sufficient to do this, and consequently symbolic algebra is also required to express the attributes corresponding to rules for force and motion propagation in the metric diagram. As any qualitative shape representation must contain the metric diagram, and the power to express such attributes makes the metric

diagram non-qualitative, *there is no qualitative shape representation that would allow one to express this attribute.*

Conditions for place transition: Situation C in Figure 4 shows a configuration where the lever and wheel touch in two different points. This instantaneous situation represents a transition between two places, called a *subsumption*. The attribute which states the possibility of this direct transition, i.e. the existence of this subsumption configuration, is an important element of the functional model of the device.

If we attempt to map this attribute to the object model, represented by a metric diagram, it amounts to the existence of a configuration which simultaneously satisfies two nonlinear constraint equations. Using symbolic algebra, it is possible to derive an equivalent condition as the existence of a root of a six-degree polynomial. By applying algebraic decision methods ([1]), it is possible to reduce this to a complex combination of algebraic predicates which express the condition, but these are highly nonlinear in the parameters of the metric diagram - so complex that it is impossible to print them readably on a single page. Besides the fact that the nonlinear combination of parameters of independent objects violates the condition of compositionality, the expression of the subsumption condition is much too complicated to be effectively used for reasoning.

The example of subsumptions points to even deeper problems with the mapping between qualitative models of function and objects. It is due to the fact that qualitative representations are *local*: all relations between individual symbols are defined and reasoned about individually. Consequently, in a qualitative analysis of kinematics, each object contact is reasoned about individually. This fails to take into account *interference* between object contacts: a particular state may in reality be impossible because it would create an overlap of other, not directly related parts of the mechanism. Such interference can be reliably inferred only from the presence of subsumption configurations - but these attributes in turn can not be formalized in a qualitative object model. I conclude from these arguments that qualitative object models are almost useless for making even qualitative predictions about function.

4.2 Mapping to a precise object representation

Even though most functional attributes can not be mapped directly into attributes on a *qualitative* object representation, they do define attributes of a *precise* object representation which can be reasoned about. Given a numerically precise model of the designed objects, its place vocabulary as a representation of the qualitative function can be computed using the methods described in [9, 5]). Each of the attributes of the place vocabulary can be labelled with the conditions on the object representation which are necessary to maintain its existence ([7, 11]). For the existence of places or inference rules associated with them, these are the algebraic conditions on the object dimensions, as shown in the examples given earlier.

For reasoning about the existence of a subsumption, it is now sufficient to express the condition for maintaining the

particular way in which the subsumption is achieved by the object shapes, not a condition for the existence of the subsumption in general. For this reason, it turns out to be possible to formulate the conditions for maintaining or achieving a particular subsumption in closed form ([11]). Even subsumptions can be reasoned about if a precise object model is used.

The many advantages offered by precise object representations leave open the question of why designers prefer rough sketches to precise drawings at the stage of conceptual design. I discuss possible explanations in the next section.

5 Interpretation of Sketches

The results of the preceding sections leave open the question why human designers often insist on using sketches. There are two possible interpretations of this phenomenon: either the sketch represents a single *qualitative* model, or the sketch is a representation of a family of *precise* models.

5.1 The sketch as a single qualitative model

As has been shown by the preceding discussion, interpreting a sketch as a single qualitative model can not be powerful enough to infer qualitative kinematic behavior. However, a qualitative metric diagram - equivalent to a sketch - is sufficient to predict the *kinematic topology* ([8]) of the device. Kinematic topology expresses the topology of the device's *configuration space*, the space spanned by the position parameters of the mechanism's parts. For many devices, the topology of its configuration space already says a lot about its function. For example, in a pair of gearwheels, the fact that the two gears can only move in coordination can already be deduced from the fact that the configuration space consists of several doubly-connected regions which "wrap around" both dimensions of configuration space ([8]). On the other hand, topology is too weak for a qualitative simulation of the meshing of the gear's teeth, or the blocking behavior in a ratchet.

However, even kinematic topology depends crucially on the existence or absence of global subsumptions, which establish additional connections between topological primitives and can have a profound effect on configuration space topology. In spite of this problem, kinematic topology and the associated primitive-based representation of object shape are useful for conceptual design. Because of the high degree of abstraction, the amount of ambiguity which results when subsumption conditions can not be evaluated is manageably low. For example, for a pair of gearwheels described in the primitive decomposition, there are only five different topologies to be considered ([8]). With only approximate metric information, such as that provided by a sketch, the analysis of kinematic topology already allows us to predict that the gears either mesh or jam - a prediction which rules out many other forms of behavior and provides a focus for subsequent detail design. An analysis at this level also explains how people can pick out the desired function out of the many functions permitted by the inaccuracies of a sketch. Furthermore, as shown in [8]), kinematic topology can be computed for any shape which can be decomposed into segments of convex and concave curvature. To my knowledge, it is the only form of kinematic

analysis which does not require a precise representation of object shapes.

5.2 The sketch as a family of precise models

An important characteristic of a sketch is that its precise dimensions often do not represent a correctly functioning device. The sketch requires an *interpretation* as a device with different precise dimensions in order to support the desired explanations. More precisely, the sketch defines a metric diagram in which the parameter values are underdetermined. This vagueness means that a single sketch can be interpreted as any of a family of possible precise models. In conceptual design, the sketch thus allows the designer to make frequent mental changes to his design without having to change the drawing - an important economy when such changes are frequent.

The most likely explanation of the designer's use of sketches is a combination of the two possibilities. On the one hand, the sketch itself defines a restricted domain in which a precise solution is searched. This is based on a single interpretation, for example based on kinematic topology. On the other hand, it allows the designer to re-use the same drawing throughout the frequent changes inherent in conceptual design. The arguments in this paper have shown that the interpretation of sketches as precise models is inevitable for design, and consequently that the popular model of design being based on mapping functional attributes to a single qualitative model represented in a sketch is wrong.

6 Conclusions

I have shown how qualitative models of kinematic function and of object shapes can be constructed for the limited domain of this case study. However, it has proven impossible to generate useful direct correspondences between the qualitative functional models and the qualitative shape models. Even though this result is limited to a narrow domain, it shows that the hypothesized model of conceptual design can at least not be generally applicable.

This result suggests an alternative model of conceptual design: the iterative refinement of a *precise* model of the design object ([6]). In this approach, shape modifications are obtained by reasoning about the limits of validity of the functional attributes of a current design. This avoids the difficulties with qualitative object models, while maintaining the use of a qualitative functional model which can easily be related to specifications. A precise object model is also used in the work of Joskowicz and Addanki ([13]), who present an incomplete algorithm for mapping exact functional specifications into corresponding exact object shapes.

Why, then, do human designers like to use sketches? One reason is that a representation like kinematic topology is useful for controlling search: a design whose kinematic topology does not correspond to the desired one should not be pursued any further. But another, more important reason may be that while the sketch defines the correct metric diagram, its precise dimensions can be freely reinterpreted according to the interests of the analysis. The sketch thus allows the designer to make frequent mental changes to his design without

having to change the drawing - an important economy when changes are frequent. Because of the difficulties with kinematic predictions discussed earlier, this latter reason seems much more plausible than the use of a sketch as a qualitative shape representation. It is also likely that this model of the designer's use of sketches also holds for other domains, such as architecture.

References

- [1] Ben-Or, D. Kozen, J. Reif: "The Complexity of Elementary Algebra and Geometry," *Journal of Computer and System Sciences* 32, 1986, pp. 251-264
- [2] Daniel Bobrow: "Qualitative Reasoning about Physical Systems: An Introduction," *Artificial Intelligence* 24, 1984
- [3] J. de Kleer: "Multiple Representations of Knowledge in a Mechanics Problem Solver," *Proceedings the 5th IJCAI*, Cambridge, 1977
- [4] Boi Faltings: "Qualitative Kinematics in Mechanisms," *Proceedings of the 10th IJCAI*, Milan, 1987
- [5] Boi Faltings: "The Place Vocabulary Theory of Qualitative Kinematics in Mechanisms," University of Illinois Technical Report UIUCDCS-R-87-1360, July 1987
- [6] Boi Faltings: "Qualitative Kinematics and Intelligent CAD," *Proceedings of the 2nd IFIP WG 5.2 workshop on ICAD*, 1988, North Holland
- [7] Boi Faltings: "A Symbolic Approach to Qualitative Kinematics," *Proceedings of the Third International Conference on Fifth Generation Computer Systems*, Tokyo, November 1988
- [8] Boi Faltings, Emmanuel Baechler and Jeff Primus: "Reasoning about Kinematic Topology," *Proceedings of the 11th IJCAI*, Detroit, August 1989
- [9] Boi Faltings: "Qualitative Kinematics in Mechanisms," *Artificial Intelligence* 44(1), June 1990
- [10] Boi Faltings: "Qualitative Models in Conceptual Design: A Case Study," *1st International Conference on Artificial Intelligence in Design*, Butterworth, 1991
- [11] Boi Faltings: "A Symbolic Approach to Qualitative Kinematics," to appear in *Artificial Intelligence*, 1992
- [12] Ken Forbus: "Qualitative Process Theory," *Artificial Intelligence* 24, 1984
- [13] Leo Joskowicz, S. Addanki: "From Kinematics to Shape: An Approach to Innovative Design," *Proceedings of the AAAI*, St. Paul, August 1988