Uncertainty Management in Shock Models Applied to Prognostic Problems

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Abstract

A particular aspect of uncertainty management in degradation-based reliability models is investigated. The presence of unobservable failure mechanisms in the reliability model is considered to be a source of variability greatly affecting the model predictive accuracy. A methodology is proposed to reduce the uncertainty effects imposed by the degradation data obtained in the presence of unobservable failure mechanisms. The key idea of the methodology is to transform the available degradation data so that variability of the critical degradation indicator values would be minimal. A numerical example utilizing degradation data is presented. The practical benefits of using the transformed degradation data are outlined.

Introduction

Degradation-based reliability analysis has been widely adopted as a useful reliability management tool for applications where few or no failures are available (Meeker and Escobar, 1998).

Degradation models are based on probabilistic treatment of a collection of degradation paths formed by the dominant failure mechanism that degrades the component reliability. Degradation paths evolve in the space of a degradation measure (indicator) that quantifies the unit's ability to operate in accordance with its specifications. In literature, sometimes several degradation indicators are considered to quantify the component reliability. To simplify the analysis, the degradation indicators are usually grouped together to form the health indicator (status), which is usually scalar rather than a vector quantity.

The degradation indicator evolves toward the critical degradation level corresponding to the moment when the component is no longer able to perform its designated functions. Thus, the time instant when the degradation indicator overshoots the critical level is said to be a failure event.

A collection of degradation paths forms a probability distribution function for the failure events. For any monment in time, reliability can be evaluated as the probability of the degradation indicator being less than the critical threshold (Lu and Meeker, 1993).

This paper is concerned with uncertainty issues related to degradation-based reliability models that exhibit a certain pattern in the degradation data available for model evaluation. The essence of the issues to be investigated is illustrated in the following qualitative example.

Illustrative Example

Consider an electronic power supply deteriorating due to two failure mechanisms that are described as follows. An internal defect reinforced by various external stress factors such as temperature and vibration, initiates a crack in the printable circuit board (PCB) so that the crack propagation tends to affect a vitally important electronic component residing on the PCB near the propagating crack. The crack propagates mostly due to temperature gradients suffered by the power supply. The temperature gradients cause certain spots on the PCB to undergo mechanical stresses, which, in turn, cause the crack growth. Eventually the propagated crack deteriorates the electronic element's functioning so that the power supply is no longer capable to provide its output voltage within the specified range. This failure will be attributed to Failure Mechanism 1 (FM_1).

The second failure mechanism is related to corrosion processes mostly affecting the solder joints populating the PCB. Any severely corroded solder joint may cause the power supply to fail (Vichare and Pecht, 2006). This failure type will be attributed to Failure Mechanism 2 (FM_2).

The outlined failure mechanisms are assumed to be different with respect their observability. In the case of FM_1 the mechanical stress reinforcing the crack growth manifests itself as an occurrence of random spikes in the output voltage. By measuring the voltage spikes frequency and magnitude, one is able to assess the damage acquired by the PCB due to FM_1. The assumption made here is that the spike magnitudes and frequencies are correlated with the crack length. The longer the crack length, the more severe the effect upon the electronic component output voltage. Also there is assumed to be a critical crack length, exceeding which the crack causes the power supply to cease its proper functioning, since the output voltage is no longer stable.

In the case of FM_2,one is not able to perceive online any information related to the level of degradation (corrosion) in the solder joints. The degree at which the corrosion has deteriorated the power supply reliability can be revealed only after a thorough offline inspection, which is usually impractical to perform on a not-yet-failed item.

The absence of perceivable information in regards to FM_2 can hinder the reliability modeling. Figure 1 shows the degradation paths observed in reliability testing of real-world electronic power supplies (Hines and Usynin, 2006). Each of the depicted degradation paths has ended with a failure event.

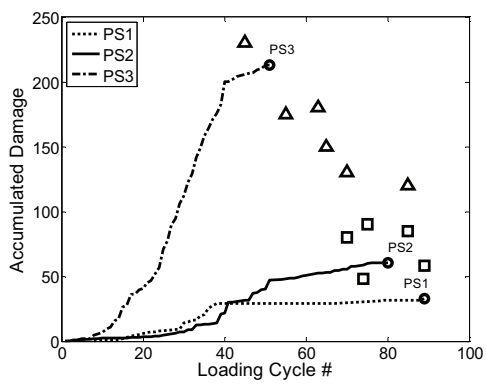

Figure 1. The degradation paths observed from electronic power supplies. The triangle and square marks represent imagined power supplies failure points. The square-marked failure points are grouped near PS1 and PS2. The triangle-marked failure points are distributed between PS3 and PS2

To develop a reliability prediction model, one needs to define a critical degradation threshold, exceeding which, the component is said to fail. If the precise value of the critical degradation threshold is unknown in the initial development phase, it has to be estimated from available degradation and failure observations. However, as shown in Figure 1 the degradation paths can exhibit a great deal of variability in the critical degradation values corresponding to the observed failure moments (Data Points PS1, PS2, and PS3).

As can be seen in Figure 1, the reason for the large variability in the critical threshold is the fact that Power Supply 3 has degraded in a manner significantly different from those observed at PS1 and PS2. The deviated degradation path can be easily declared to be an outlier since a) its appearance differs from the majority of the tested items (even though the majority is of two items: PS1, PS2), b) the presence of such an anomalous degradation pattern can complicate the reliability model development in the sense that the model would have to account for this unusual degradation path, probably at the expense of the model predictive accuracy.

However the small number of degradation paths obtained in this experiment does not allow one to decide if the PS3 degradation path can be disregarded in the reliability model development because of being an outlier.

If additional degradation paths, which terminated with a failure, were available, and these extra failure observations were situated near the PS1 and PS2 failure points (as is shown in Figure 1 by the black square marks), the PS3 degradation path would clearly be classified as an outlier. On the other hand, if the extra observations were evenly distributed between the PS3 failure point and those corresponding to PS1, PS2 (as shown in Figure 1 by black triangle marks), neglecting the PS3 data would result in an unreasonable loss of information.

If the PS3 data are not to be disregarded, the observed deviation in the PS3 degradation pattern can be explained by the presence of failure mechanism FM_2 associated with corrosion in the solder joints. The degradation measure (accumulated damage) shown in Figure 1 reflects only the damage the power supplies have acquired due to FM_1. The damage due to FM_2 cannot be accounted for in the observed degradation paths since FM_2 is unobservable. Although the FM_2 cannot be observed, the damage imposed by this hidden failure mode may eventually cause the power supply to fail. From this perspective, the failure mode FM_2 manifests its presence only through the failure events.

The power supplies (real and imagined), whose failure data points are situated at the right-lower side of Figure 1 are likely to have acquired damage mostly due to FM_2, since the damage accumulated due to FM_1 is relatively low, and the lifetimes of these power supplies are relatively large. Corrosion processes are known to proceed in the calendar time domain so that the calendar age of the power supplies is expected to be related to FM_2, in the sense that the calendar age is positively correlated with the probability of failure due to FM_2. However, since FM_2 is unobservable, there is not any numerically expressed evidence of the degradation imposed by FM_2. Hence, in developing the power supply reliability model, one has to deal with degradation data that do not indicate degradation due to FM_2.

The degradation data depicted in Figure 1 show significant variability in the critical damage levels, at which the power supplies experienced failure. This variance will primarily affect the estimation of the critical damage level for the degradation-based reliability model.

In this example it is difficult to claim with certainty that the PS3 degradation path in Figure 1 is an outlier. In this situation, the important decision to make is whether to disregard the possible outlier and continue the reliability modeling only with the data that are well fitted to the assumed model, or adjust the reliability model so that the odd looking data will be a piece of information useful for the reliability prediction.

This paper proposes a methodology to deal with such situations where the empirical degradation threshold tends to be uncertain because of variability in degradation and

failure data as shown in Figure 1. The study is performed in the framework of shock models that have been used in the reliability analysis for decades (Esary and Marshall, 1973). However the proposed approach is quite general, and can be applied to any degradation-based technique, that utilizes the notion of critical degradation threshold.

The remainder of the paper is organized as follows. In the next section the problem statement will be formulated, the key ideas behind reliability shock models will be outlined, and the approach to address the problem will be proposed. Section 3 will illustrate the proposed methodology in a real-world data example. Section 4 will provide some conclusions and remarks regarding further development of the proposed idea.

Methodology

Consider a component subject to several degradation processes, or failure mechanisms, eventually leading to component hard failure. The degradation effects imposed by the failure mechanisms upon the component's reliability are assumed to be approximately equal in their magnitude so that it is difficult to distinguish a dominant failure mode. However, the failure mechanisms can be differentiated with respect their observability.

A failure mechanism is called observable if its effect can be measured directly or inferred through the use of various degradation indicators. For example, an opto-isolator is one of a few critical elements in a switch mode power supply (SMPS). The degradation progression for this component can be detected and tracked through the usage of resonance measurements and the value of current transfer ratio (CTR), which is correlated with the failure progression (Judkins, Hofmeister and Vohnout, 2007).

A failure mechanism is called unobservable if its effect upon the component's reliability cannot be measured or even detected because of sensor equipment limitations, or the impracticality of diagnostic and detection routines. While in operation, the component does not manifest the presence of such a failure mechanism in a perceivable manner. The presence and effects of unobservable failure mechanisms can be confirmed and investigated only in a post-mortem analysis. Unobservable failure mechanisms are assumed to make a significant contribution to the component's degradation and fault progression.

Let **F** denote a set of various failure mechanisms that simultaneously affect the component's reliability.

$$\mathbf{F} = \{F_1, F_2, F_3, \ldots F_n\} \tag{1}$$

Assume that there is a value of k such that a subset $\mathbf{F}_{obs} = \{F_{i_1}, F_{i_2}, F_{i_3}, \ldots F_{i_k}\}$ is a set of observable failure mechanisms, and $\mathbf{F}_{hid} = \{F_{i_{k+1}}, F_{i_{k+2}}, F_{i_{k+3}}, \ldots F_{i_n}\}$ is a subset of unobservable (hidden) failure mechanisms.

Each failure mechanism imposes a certain degradation effect upon the component's reliability. The degradation effect D_i imposed by a particular failure mechanism F_i can be thought of as a function that quantifies the amount of damage suffered by the component solely due to the failure mechanism.

$$D_i = f(T_i, F_i) \tag{2}$$

where T_i is the timescale associated with the failure mechanism F_i. Apparently, different failure modes can evolve in different timescales. For example, corrosion processes tend to degrade the component in the calendar age timescale, whereas temperature stress-related damage worsens the component's reliability in the operational age timescale.

In this study the overall degradation effect is assumed to be additive so that it is represented by the following expression

$$D_{total} = \sum_{i=1}^{n} D_i \tag{3}$$

It should be noted that in the case of multiplicative damage, one should perform the log-transformation to use the additive model given in Equation 3.

Since some of the D_i are unobservable, the damage effects can be grouped according to their observability.

$$D_{total} = \sum_{i=1}^{k} D_i + D_{hid} \tag{4}$$

where D_{hid} is the total degradation effect imposed by the unobservable failure mechanisms. For the sake of brevity, this term is called the hidden damage. The value of hidden damage is assumed to be completely unknown and unobservable.

The presence of the unknown term D_{hid} can be treated as a source of uncertainty in the reliability prediction problem. The influence of the unknown hidden damage cannot be neglected since it is assumed that there is no dominant failure mechanism, so that all of the degradation contributors (observable and hidden as well) play equally important roles in the component deterioration. The uncertainty effect of the hidden damage is illustrated in the following qualitative example.

Assume that the item exhibits two degradation modes F_1 and F_2. F_1 is observable, whereas F_2 is unobservable. The degradation due to the assumed failure mechanisms accumulates linearly in time. The item fails as soon as the total component's damage $D_{total} = D_1 + D_2$ exceeds a certain critical threshold D^*, whose value is unknown. Figure 2 shows the observed and true degradation paths, (OA' and OA, respectively) for a particular item, which has suffered some damage due to F_1 and F_2.

The solid line OA' depicts the observable damage D_1 accumulated in the item. The dotted line OA depicts the true (total) damage D_{total} accumulated in the item.

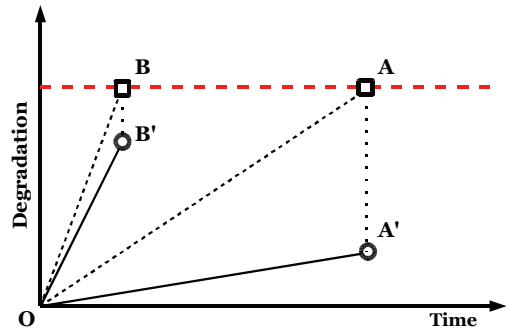

Figure 2. A schematic representation of two observable degradation paths OA' and OB' and their true (yet unobservable) counterparts OA and OB.

The point A on the critical threshold line is the apparent moment of failure. However, since the only observable damage is due to F_1, one observes the failure moment at the end point A', thus, concluding that the critical damage level is that corresponding to the point A', which seems to be significantly lower than the true critical threshold.

This underestimation can be explained by the fact that the item under consideration happens to suffer mostly from failure mode F_2. The ordinates of the points along the degradation path OA' are significantly lower than those of the points along the true degradation path OA.

Degradation paths OB and OB' correspond to an item that suffers mostly due to failure mode F_1. The major portion of the total damage accumulated in this item is observable. Hence the distance between the observed failure point B' and the true failure point B is not significant.

To conclude this qualitative example the following statement is made. The presence of unobservable failure modes introduces uncertainty in the critical degradation level estimation, since the damage levels observed in the moments of failure tend to vary because of variability in the ratio of observable to unobservable failure mechanisms affecting the components.

The problem posed in this study is to reduce the uncertainty effect resulting from the presence of unobservable failure mechanisms in the component degradation process. The uncertainty effect is going to be investigated in the framework of reliability shock models that will be outlined in the following subsection.

Shock Models

General cumulative shock models have been considered in the reliability analysis literature (Barlow and Proschan, 1975) and are defined as follows.

Let {N(t), t>0} be a point process with sequence of jump times T_1, T_2,… Each jump time T_i has a corresponding random variable C_i. The stochastic process $\{X(t), t \geq 0\}$ given by

$$X(t) = \sum_{i=1}^{N(t)} C_i \tag{5}$$

is called a cumulative shock model. In this setting, the value of C_i is the magnitude of the shock arrived in time T_i. Figure 3 illustrates a typical cumulative stochastic process X(t).

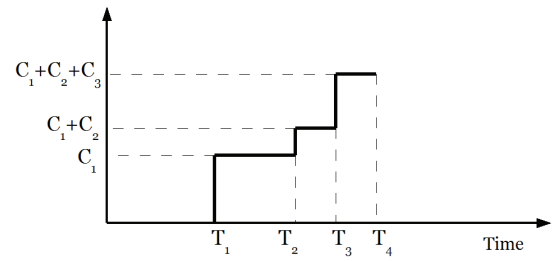

Figure 3. A typical degradation path in a cumulative shock model.

From the reliability analysis perspective, of primary interest is the random time L(x) when the accumulated shock magnitude X(t) exceeds a given critical threshold x for the first time.

$$L(x) = \inf \{ t, X(t) > x\} \tag{6}$$

It has been proven in (Gut, 1990) that the distribution of L(x) approaches the normal distribution with the following parameters, if the value of x is relatively large.

$$L(x) \sim N\left(\frac{\mu}{\nu}x, \frac{\gamma^2}{\nu^3}x\right) \tag{7}$$

where μ = E[T], ν = E[C], γ^2 = Var(μC – νT); E[.] denotes the mathematical expectation and Var(.) is the variance.

Essentially the normal distribution in Equation 7 is a limiting case for the TTF distribution implied by a cumulative shock model. The analytical expression for the limiting normal distribution is of practical importance since it provides information on the time-to-failure distribution for an arbitrary cumulative shock model. In reliability analysis the probability distribution given in Equation 7 has been known as the Birnbaum-Saunders distribution, which was originally developed to model the rupture time of metals exposed to fluctuating stress and tension (Birnbaum and Saunders, 1969).

As can be seen in Equation 7 the variance of the time-to-failure explicitly depends on the value of critical threshold x. Of particular interest to this study is the case where the critical threshold is to be defined probabilistically rather than in a strict manner. In this case there is a range of possible threshold values and probabilities of failure associated with them. These probabilities of failure are thought of as uncertainty in the critical threshold. A comparative study of threshold uncertainty effects for

several types of cumulative shock models is given in (Usynin, Hines, and Urmanov, 2008).

Concluding this brief description of cumulative shock models, we define the notion of warning setpoint and critical degradation zone. The level of cumulative damage at which the component has the probability of failure (POF), which is assumed to be critical, is called a warning setpoint. Having reached the warning setpoint the component immediately needs preventive maintenance. The notion of warning setpoint is primarily important for determining an optimal preventive maintenance policy.

The critical degradation zone includes the degradation levels starting with the warning setpoint and ending at the damage level where all items from the population are expected to fail. The defined notions of warning setpoint and critical degradation zone are going to be useful in the following subsection that will introduce the methodology.

Optimal Transformation of Degradation Measure

Consider a degradation-based reliability shock model M(T, D), which evolves in the timescale T, and D is defined to be a probability distribution of the critical degradation threshold. The parameters of D are to be estimated from available failure data.

Let $\mathbf{\Theta}$ denote the set of available failure data of the following form

$$\mathbf{\Theta} = \{ (t_1, d_1), (t_2, d_2), \dots (t_k, d_k) \} \quad (8)$$

where (t_i, d_i) denotes the time and damage level attained by the component i at the failure moment.

Assuming that the critical degradation threshold is to be described by the probabilistic model D, one estimates the threshold distribution parameters as the sample mean and variance according to the following formulae.

$$\bar{d} = \frac{1}{k}\sum_{i=1}^{k} d_i\,, \quad s^2 = \frac{1}{k-1}\sum_{i=1}^{k}\left(d_i - \bar{d}\right)^2 \quad (9)$$

It is intuitively understandable that a large value of variance in the critical degradation threshold can deteriorate the prediction provided by the model M. In particular, a large variance in the estimate of the critical threshold forces the practitioner to set the warning point too low, thus reducing the component's service life. This uncertainty effect cannot be smoothed or eliminated completely if the critical threshold variance is mostly due to fully random deviations in the failure moments. However, if the failure data pattern exhibit some systematic regularity, a certain transformation of the available data can be made to reduce the variability in the critical degradation threshold.

The presence of regularity in the failure data can emanate from various origins which are difficult to classify and they are usually highly case-dependent. This paper considers regularity due to hidden degradation effects imposed by unobservable failure mechanisms. If the component is subject to at least two different failure modes, and one of the failure modes is unobservable, the degradation data observed on the component may look like it is shown in Figure 4. The small circles in Figure 5 represent the time instants and damage levels attained by some components at their failure moments. The origin and particular features of these data will be discussed in detail in the numerical example section.

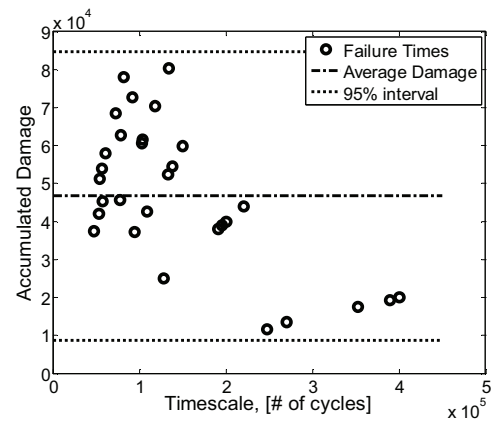

Figure 4. The real-world degradation data exhibiting a certain regularity.

The degradation data shown in Figure 4 exhibit a certain regular pattern in the failure moments. The items survived a large number of duty cycles tend to acquire a relatively small amount of damage compared to the items survived a short number of duty cycles. Hence the failure points form a data cloud inclined downward.

One possible explanation for this appearance can be that the items situated in the right-hand side of Figure 4 (long age survivors) have acquired critical degradation mostly due to the unobservable failure mode. Therefore, their degradation level d_i at the failure moment is relatively low since the observable degradation mode happened to be a minor contributor to the item failure. On the opposite side, the items that are short survivors (situated in the left-hand side of Figure 4) have acquired critical damage mostly due the observable failure mode.

Apparently such regularity in failure data introduces a great deal of variability into the degradation-based reliability model, effectively enlarging the critical degradation zone, which is depicted as a range of damage levels confined between the dotted lines.

The large critical degradation zone effectively lowers the warning setpoint, approximately depicted by the lower dotted line.

A data transformation to a new coordinate system is proposed to reduce the variability in the critical degradation zone. If failure data exhibit some linearity in the original coordinate system (T, D), a Principal Component Analysis (PCA)-based transformation can be applied to reveal the new coordinate system (T', D'), the

usage of which will be more beneficial in terms of the end result uncertainty.

PCA is a useful statistical technique for finding patterns in high-dimensional data. Essentially PCA is an applied-linear-algebra-based method which provides a simple, non-parametric approach for extracting relevant information from confusing data sets. The main question a PCA-based method addresses can be formulated as follows. Is it possible to find a linear combination of the original coordinate basis that best represents the given data (Shlens, 2005)?

Let **X** be the original data set, where each column is a multidimensional data point. If **X** is an $m \times n$ matrix, the number of available data points in **X** is n; the dimensionality of the data is m. In this setting the PCA transformation is given by

$$\mathbf{Y}^T = \mathbf{X}^T\mathbf{W} = \mathbf{V\Sigma} \tag{10}$$

where $\mathbf{V\Sigma W}^T$ is the singular value decomposition (SVD) of **X**, **Y** is the matrix of transformed data re-expressed in the new coordinate system.

Given the data set **Θ,** one performs the PCA transformation to come up with transformed failure data of the following form.

$$\mathbf{\Theta}' = \{ (t_i', d_i') \},\ i = 1,2,\ldots k \tag{11}$$

It is well known that PCA transforms data so that the data will have the largest variance along the first transformed coordinate, the second largest variance will be along the second transformed coordinate and so forth. Since the considered data set **Θ** is two-dimensional, the second transformed coordinate d' produces the smallest variance in the data. This obviously follows from the fact that the largest variance in the 2-dimensional data set **Θ'** is along the first transformed coordinate t'.

Therefore having performed PCA over the original degradation data **Θ** one has the degradation data set **Θ'** such that the transformed degradation measure d' has a minimal possible variability in the given data. Having the minimal variability in the degradation measure is highly beneficial for the reliability modeling. The practical benefits of the minimal variance will be illustrated in the numerical example section.

The performed PCA transformation can be discussed from the perspective of failure mode observability. According to the PCA definition (Equation 10) the transformed degradation measure is represented as

$$d' = w_{12}t + w_{22}d \tag{12}$$

where w_{ij} is an element of the 2×2 matrix **W**. As can be seen, the degradation measure d' is a linear combination of the observable degradation measure d and the time measure t. Obviously the term $w_{22}d$ accounts for the degradation that is explicitly present in the original data set **Θ**. The term $w_{12}t$ can be thought of as a linear approximation for the damage acquired due to unobservable failure mechanisms $\hat{D}_{hid}(t) = w_{12}t$.

Goodness of such a linear approximation depends on the regularity pattern the failure data exhibit in the original coordinate system (t, d). If the exhibited regularity is linear, the PCA transformation is expected to provide a good approximation to the unobservable degradation effects (Equation 13).

In the case of a non-linear pattern in degradation data, the proposed methodology can be generalized through the use of kernel principal component analysis (KPCA). However, this generalized non-linear approach is out of this paper's scope and will be considered in future research.

The next section presents an example where the proposed methodology is applied to real-world data. The example also discusses the practical benefits derived from the usage of transformed degradation data.

Numerical Example

This example considers the fatigue test data originally discussed in (Gertsbakh, 2000). A sample of 30 steel specimens was subjected to a series of loading tests until they failed because of fatigue. Each loading test consisted of 5000 fatigue cycles. The magnitude of fatigue loads was chosen such that a specimen underwent $5000\alpha_i$ low-load cycles and $5000(1-\alpha_i)$ high-load cycles in one loading test. Thus the value of α_i represents the ratio of low-load cycles to the total number of cycles applied to Specimen i within one load test.

The entire sample was divided into 6 groups G_k k=1,2,..6, each of which was characterized with a certain ratio of α_k.

Table 1 summarizes the steel specimens failure data. As can be seen the ratio α_k is varying from 0.05 to 0.95.

Table 1. The fatigue data adopted from (Gertsbakh, 2000). The numbers of cycles are given in thousands ($\times 10^3$)

i	α_i	Low-load	High-load	i	α_i	Low-load	High-load
1	0.95	256.8	13.5	16	0.40	32.0	45.7
2		235.8	11.6	17		48.0	70.4
3		370.15	19.25	18		42.0	61.5
4		335.1	17.5	19		42.0	60.6
5		380.3	20.0	20		54.0	80.4
6	0.80	153.0	38.0	21	0.20	10.0	37.5
7		176.2	44.0	22		16.0	62.7
8		160.3	40.0	23		12.0	45.3
9		156.0	39.0	24		19.0	72.6
10		103.0	25.0	25		11.0	42.0
11	0.60	84.0	54.4	26	0.05	3.0	53.9
12		81.0	52.3	27		3.75	68.55
13		90.0	59.9	28		4.25	77.95
14		57.0	37.3	29		3.32	57.95
15		66.0	42.7	30		2.75	51.25

In this study the loading regimes (high-load and low-load) are considered to be two distinct failure mechanisms observed in the tested specimens. Let HL and LL denote the failure mechanisms associated with high-load and low-load cycles respectively. In this experimentation the cycle frequency is assumed to be 1 load cycle a time unit so that the model timescale is expressed in numbers of cycles.

To apply a shock model-based approach, the loading cycles are assumed to deliver a certain amount of damage to the specimens. Although the original data do not provide any information in regards to how much damage a high- or low-load cycle delivers, for the sake of simplicity the damage delivered by a high-load cycle is assumed to be unit-size. Damage delivered by a low-load cycle is assumed to be unknown (unobservable).

The failure mechanism HL is assumed to be observable. In other words, the damage delivered by high-load cycles is measurable. The failure mechanism LL is assumed to be unobservable because of, for instance, certain limitations in sensor equipment incapable to pick up the degradation indicators of the low-load cycle damage.

The original data do not provide information as to how the fatigue damage in the specimens evolves in time. In the absence of any knowledge of the damage progression it is assumed that damage accumulates linearly. If this assumption turns out to be unrealistic, the proposed methodology will not suffer, primarily because this assumption of linearity is important only for assessing the efficiency of the particular reliability prediction model, as will be shown some later in this section.

From the assumptions, it follows that the original data can be represented as shown in Figure 5. The abscissa represents the total number of load cycles survived by the specimens; the ordinate represents the observable accumulated damage (degradation) which according to the assumption of the unit-size damage increments is the number of high-load cycles suffered by the specimens.

Since the LL damage is assumed unobservable, it is impossible to take into account the effect of the unobservable damage upon the reliability prediction. Computing the mean and variance of the critical degradation values (Equation 9) and assuming the critical probability failure to be 0.025, which corresponds to the 2σ offset in the case of Gaussian distribution, one can estimate the warning setpoint.

$$\textit{Warning Setpoint} = 8.7\times10^{3} \qquad (13)$$

The meaning of the warning setpoint is that any specimen exhibiting the damage level of 8.7×10^{3} has a POF of 0.025, which is assumed to be unsafe for continuing the specimen's operation. Thus, being at the warning setpoint is the indication that the specimen immediately requires preventive maintenance.

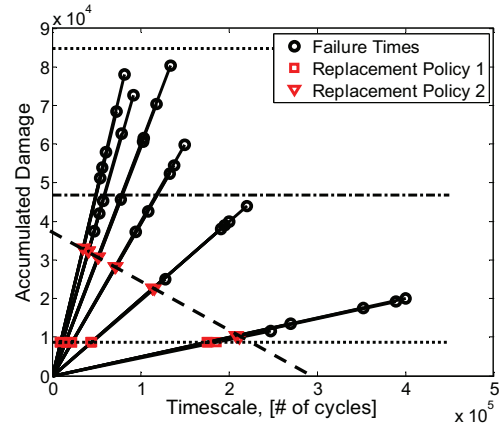

Figure 5. The fatigue data are depicted as degradation paths. The square marks depict the replacement times based on the original warning setpoint (Equ.13). The triangle marks depict the replacement times based on the transformed warning setpoint.

To assess the efficiency of performing preventive maintenance given a value of the warning setpoint, the average useful lifetime metric is introduced. The average useful lifetime metric is defined to be the mean value of a specimen's lifespan given the specimen is to be replaced as soon as its observed degradation reaches the predefined warning setpoint. Figure 5 shows the times of crossing the estimated critical warning setpoint as small squares lined up along the lower dotted line representing the warning setpoint. The times corresponding to the small squares would be the useful lifetimes of the specimens if they were taken out of service as soon as their observed degradation exceeded the warning setpoint given in Equation 13.

If a PCA-based orthogonal transformation is applied to the data, the transformed warning setpoint will appear as shown in Figure 5 by the dashed line inclined downward.

The times of performing preventive maintenance are depicted as triangle marks lined up along the transformed warning setpoint level. As can be seen the specimens' useful lifespan tends to prolong since the transformed warning setpoint takes into account the degradation acquired by the specimen and the time the specimen has been in operation as well.

To quantitatively compare the replacement policies suggested by the original and transformed data, Table 2 summarizes the efficiency metrics corresponding to the replacement policies. The efficiency metric (the average useful life) evaluated for the transformed-data-based policy turns out to be significantly better (longer) than that evaluated for the reliability model built on the original untransformed data.

As can be seen in Table 2, the PCA-based transformation significantly improves the average useful life of the steel specimens. If one follows the preventive maintenance policy based on the warning setpoint evaluated from the transformed data, the expected useful life of the components is almost twice as long as the useful

life derived from following the policy based on the original data.

Table 2. The average useful lifetime (the average number of useful cycles) provided by the preventive maintenance policies based on the original data and transformed data.

	Original Data	Transformed Data
Avg. Service Lifetime [# of cycles]	46.5×10^3	87.0×10^3

This significant improvement is explained primarily by the fact that the transformed data exhibit a minimal possible variance in the critical threshold. The minimal variation in the model failure threshold in turn provides the most accurate prediction of the TTF distribution, which provides the best achievable preventive maintenance strategy based on the notion of warning setpoint.

Conclusions

A particular aspect of uncertainty management in reliability prediction models has been investigated. The case where both observable and unobservable failure mechanisms affect component reliability has been considered. It has been revealed that the presence of unobservable degradation sources can manifests itself as a certain regular pattern in the failure data.

The regular pattern can introduce a great deal of variability into the estimated critical threshold involved into the reliability prediction model. A methodology has been proposed to reduce possible uncertainty effects of failure data regularity upon the reliability model. The key idea of the proposed methodology is to find a proper data transformation to produce a coordinate system, in which the failure data exhibit minimum variance in the critical degradation threshold. The expected benefit of using the transformed failure data lies in the fact that the transformation accounts for the regular pattern in the original failure data so that the effect of the regularity in the transformed data is reduced as much as possible.

The case where the regularity is of a linear nature has been illustrated by a numerical example involving real-world data. In the example, a PCA-based transformation was applied to the data. The benefit of using the PCA transformed data was assessed by calculating the efficiency metric, which was defined to be an average useful life. The reliability model built on the PCA transformed failure data has provided an efficiency metric value almost twice as good than that provided by the model using the original failure data.

The proposed methodology can be extended to cases in which the failure data regularity is non-linear. Kernel Principal Component Analysis (KPCA) seems to be well suited for transforming failure data that has a non-linear pattern in the failure points.

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