The Re-Representation Problem in a Logic-Based Framework for Analogy Making

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Introduction

The ability of analogy making is considered to be an essential part of intelligent behavior. By mapping concepts and knowledge about a well-known domain into a target domain, new hypotheses about that domain can be inferred. Such a transfer may result in a new conceptualization of the target domain and therefore can be seen as a source of creativity. Moreover, the discovery of common structures can initiate a generalization process and support the introduction of abstract concepts, which are not directly observable.

While the creation of an analogical mapping is well-examined, provided the corresponding representations of both domains are already chosen in a way that the structural compatibility is obvious, such situations seem to be somewhat artificial. In fact, the structural commonalities characterizing two analogous domains are usually not obvious in advance, but become visible as a result of the analogy making process. The conceptualization (i.e. the representation) must be modified and adapted to make implicit analogous structures explicit. It is argued that an essential part of establishing an analogy is a change of representation of one or both domains to allow for discovering the common structure (Indurkhya 1992).

In this paper, we sketch an approach to deal with the problem of re-representation in a logic-based framework for analogy making that applies anti-unification to compute the analogical relation.

Anti-Unification for Analogy Making

There exist different proposals to use anti-unification as a means to compute analogies (Hasker 1995; Burghardt 2005). The notion of anti-unification is based on the instantiation ordering of terms: given some first order language $\mathcal{L}$, a term t_2 is called an instance of another term t_1 (and vice versa t_1 is called an anti-instance of t_2), if there exists a substitution σ such that $t_1\sigma = t_2$. In this case, we write $t_1 \xrightarrow{\sigma} t_2$ or just $t_1 \rightarrow t_2$. The instantiation relation imposes a partial ordering on the set of all terms. Given a pair of terms s and t, an anti-unifier of s and t is a term g such that $s \leftarrow g \rightarrow t$. An anti-unifier g is called least general anti-unifier, if g is an instance of every other anti-unifier. It has been shown that a least general anti-unifier exists for every pair of terms, and that it is unique up to renaming of variables (Plotkin 1969). A natural idea is to extend the anti-unification of terms to the anti-unification of formulas, in order to find generalizations of facts and rules (Gust, Kühnberger & Schmid 2006).

We will explain the idea of how to use anti-unification for analogy making by considering the well-known Rutherford analogy. Table 1 shows an axiomatization of the two domains for which the analogy shall be established. Now anti-unification can be used to relate these two sets of formulas: for example, anti-unifying the two axioms α_1 and β_1 results in a generalized term and two substitutions σ and τ, as indicated by the following diagram:

$$\begin{tikzcd}
 & \begin{array}{c}\mathit{mass}(A) > \\ \mathit{mass}(B)\end{array} \arrow[dl, "{\sigma = \{A \mapsto \mathit{sun},\ B \mapsto \mathit{planet}\}}"'] \arrow[dr, "{\tau = \{A \mapsto \mathit{nucleus},\ B \mapsto \mathit{electron}\}}"] & \\
\begin{array}{c}\mathit{mass}(\mathit{sun}) > \\ \mathit{mass}(\mathit{planet})\end{array} & & \begin{array}{c}\mathit{mass}(\mathit{nucleus}) > \\ \mathit{mass}(\mathit{electron})\end{array}
\end{tikzcd}$$

By combining the two substitution, an analogical relation between the domains can be established: *sun* corresponds to *nucleus* and *planet* corresponds to *electron*.

Re-representation

The analogical relation computed depends obviously on the choice of formulas that are anti-unified. For example, anti-unifying the axioms α_3 and β_2 would result in a mapping of *sun* to *electron* and *planet* to *nucleus*, which does not express the intended analogy. In this situation, a new representation of the two domains is needed, to provides a base for the computation of the intended analogical relation. For example, from the background knowledge it is known that *distance* is a symmetric function, and so

$$\mathit{distance}(\mathit{nucleus}, \mathit{electron}, t) > 0$$

can be derived from $\{\phi_1, \beta_2\}$, which is a good candidate for anti-unification with α_3. Similarly, the formulas

$$\mathit{gravity}(\mathit{sun}, \mathit{planet}) > 0$$
$$\mathit{coulomb}(\mathit{nucleus}, \mathit{electron}) > 0$$

can be derived from $\{\phi_2, \alpha_1, \alpha_2, \alpha_4\}$ and $\{\beta_3, \beta_4, \beta_5\}$ respectively, which allows for an analogical mapping of *gravity* to *coulomb*.

Background Knowledge	
ϕ_1 :	$\forall x \forall y \forall t : \mathit{distance}(x, y, t) = \mathit{distance}(y, x, t)$
ϕ_2 :	$\forall x \forall y : x > y \land y > z \rightarrow x > z$

Solar System	Rutherford Atom
α_1 : $\mathit{mass}(\mathit{sun}) > \mathit{mass}(\mathit{planet})$	β_1 : $\mathit{mass}(\mathit{nucleus}) > \mathit{mass}(\mathit{electron})$
α_2 : $\mathit{mass}(\mathit{planet}) > 0$	β_2 : $\forall t : \mathit{distance}(\mathit{electron}, \mathit{nucleus}, t) > 0$
α_3 : $\forall t : \mathit{distance}(\mathit{sun}, \mathit{planet}, t) > 0$	β_3 : $\mathit{charge}(\mathit{nucleus}) > 0$
α_4 : $\forall x \forall y : \mathit{mass}(x) > 0 \land \mathit{mass}(y) > 0 \rightarrow \mathit{gravity}(x, y) > 0$	β_4 : $\mathit{charge}(\mathit{electron}) < 0$
α_5 : $\forall x \forall y : \mathit{gravity}(x, y) > 0 \rightarrow \mathit{attracts}(x, y)$	β_5 : $\forall x \forall y : \mathit{charge}(x) > 0 \land \mathit{charge}(y) < 0 \rightarrow \mathit{coulomb}(x, y) > 0$
$\vdots$	β_6 : $\forall x \forall y : \mathit{coulomb}(x, y) > 0 \rightarrow \mathit{attracts}(x, y)$

Table 1: A formalization of the Rutherford analogy (fragment)

In general, the task of re-representation consists of finding pairs of formulas from the domain theories, that posses a common structure and expose the analogical relation.[1] Here the choice of logic as a representation formalism exhibits its power, since beside the formulas explicitly given, there are also implicit formulas that can be inferred from that axioms. This provides a notion of re-representation in a quite natural way, where other approaches for analogy making have to introduce special and sometimes quite artificial means.

Anti-Unification of Theories

To allow for a formal treatment of these ideas, we will now extend the notion of anti-unification to sets of formulas. For a set of formulas F, we denote the theory of F, i.e. the set of all formulas that can be inferred from F, by $\mathit{Th}(F)$. We call a set of formulas F_1 an *anti-instance* of a set F_2, if there exists a substitution σ such that $\mathit{Th}(F_1\sigma) \subseteq \mathit{Th}(F_2)$.[2] Again we write $F_1 \xrightarrow{\sigma} F_2$ or just $F_1 \rightarrow F_2$. If $\mathit{Th}(F_1\sigma) = \mathit{Th}(F_2)$, F_1 is called a *full anti-instance* of F_2 and we write $F_1 \overset{\sigma}{\Rightarrow} F_2$ or just $F_1 \Rightarrow F_2$. Given two sets of formulas Ax_S and Ax_T, we call a triple $\langle G, \sigma, \tau \rangle$, consisting of a finite set of formulas G and substitutions σ and τ, an *anti-unifier* of Ax_S and Ax_T, iff $Ax_S \xleftarrow{\sigma} G \xrightarrow{\tau} Ax_T$. $\langle G, \sigma, \tau \rangle$ is *more specific* than $\langle G', \sigma', \tau' \rangle$, if G' is a full anti-instance of G in a way that is compatible with the domain substitutions as indicated by:

$$\begin{tikzcd} & G' \arrow[d, Rightarrow, "\theta"] \arrow[ddl, bend right, "\sigma'"'] \arrow[ddr, bend left, "\tau'"] & \\ & G \arrow[dl, "\sigma"'] \arrow[dr, "\tau"] & \\ Ax_S & & Ax_T \end{tikzcd} \qquad \begin{array}{l} \sigma \circ \theta = \sigma' \\ \tau \circ \theta = \tau' \end{array}$$

Using a more specific anti-unifier can help to prevent proliferation of variables in the generalization. Therefore, in the context of analogy making, we will apply least general anti-unifiers, since any more general anti-unifier only adds complexity without extending the analogical relation.

Obviously, least general anti-unifiers for sets of formulas always exist: the minimal example is the empty set $\emptyset$. But this is probably not a set we are looking for, since it results in the empty analogical relation. Therefore we introduce the concept of coverage: Given an anti-unifier $\langle G, \sigma, \tau \rangle$ for Ax_S and Ax_T, the subset $\mathit{Th}(G\sigma)$ of $\mathit{Th}(Ax_S)$ is said to be covered by G and for Ax_T accordingly. In general, a greater coverage is preferable, since it provides more support for the analogy, although there exist some caveats.

[1] While in the given example, re-representation is just needed to enhance the support for the analogy, there are cases in which no analogy can be computed at all, if the given representation for domains is not altered.

[2] We consider only admissible substitution, i.e. substitutions that do not introduce variables into the scope of a quantifier.

Conclusion and Future Work

This paper presents an approach to analogy making that uses anti-unification to compute an analogical relation between two domains that are formalized in first order logic. It is shown how logical inference provides a way to tackle the problem of re-representation. Future work concerns a theoretical assessment of the trade-off between the maximization of coverage and the minimization of substitution complexities as well as an algorithmic realization of the ideas proposed and a thorough practical evaluation.

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