Fuzzy Numbers for the Improvement of Causal Knowledge Representation in Fuzzy Cognitive Maps

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Abstract

We present an extension of the fuzzy cognitive map knowledge representation, based on fuzzy numbers, to improve the management of uncertainty related to linguistic expressions. In this regard, we also review the fuzzy causal algebra and outline the opportunities for applications.

Introduction

Fuzzy cognitive maps (FCM) are fuzzy directed graphs where nodes represent concepts; edges are labeled with a plus, '+', to denote causal increase and a minus, '-', to denote causal decrease. In FCMs causal relationships are assessed with linguistic terms.

FCMs are used to graphically model a system's behavior through its cause and effect relationships. However, current approaches to the management of FCMs, miss valuable information related to the uncertainty of linguistic estimations about causality. As an attempt to solve this problem, we present an extension of the FCM knowledge representation based on fuzzy numbers.

Causal Knowledge Representation with Fuzzy Numbers

Gradual association between a cause and an effect is expressed as a causal strength in the cause-effect relationship. This notion could be induced by partial or gradual occurrence of effects, or by the uncertainty of observations [Dubois and Prade, 1995]. Expressing the concept of causal strength in terms of fuzzy sets theory, we can say that having a fuzzy set $\tilde{C} = \{c_1, c_2, \dots c_n\}$ of causes for some effect e, the degree of membership of a given cause c_i in $\tilde{C}$, denoted as $u_e(c_i)$, is the degree of sufficiency for the occurrence of e given the causality imparted by c_i.

A fuzzy set of causes for an effect is the theoretical framework that supports the FCM causal knowledge representation. However, the estimation about the degree of causation is in many cases the result of subjective perception. To extend and improve the knowledge representation it would be useful to use fuzzy degrees of membership representing an uncertain estimation, possibly linguistic, of the degree of sufficiency. Consequently, the fuzzy set of causes will be a type-2 fuzzy set [Zadeh, 1975] where, having a set of causes C, causal relationships will be expressed by membership functions of the form:

$$A : C \rightarrow F([0,1]) \qquad (1)$$

Where F([0,1]) is the fuzzy power set of [0,1] [Klir and Yuan, 1995]. Applying this concept to the FCM knowledge representation, causal edges are represented by fuzzy numbers with membership functions associated to linguistic terms.

The Fuzzy Causal Algebra

Fuzzy causal algebra deals with the calculation of the indirect and total causal effect [Axelrod, 1976]. The indirect effect that some concept node Ci imparts to some concept node Cj is the causality that Ci imparts to Cj via the causal path that links both nodes. On the other hand, to assess the total causal effect we have to combine all causal paths leading to the node.

The operations for indirect and total causal effects are interpreted as fuzzy intersection and union respectively, defined on a partially ordered set of causal values [Peláez, 1994].

The problem is to find operators for the fuzzy intersection and union that resembles our intuition about causality. We will base our analysis on two arguments provided by [Zimmermann, 1981] that gain special relevance in the causal reasoning context.

Compensation. An operator is compensatory if a change in the resulting membership degree due to a change in one of the operands can be counteracted by a change in another operand.

Aggregative Behavior. An operation holds aggregative behavior if the resulting membership degree depends on the number of membership functions combined.

In the context of causal reasoning, both concepts describe our intuition about a causal system. For example, compensation is present in causal chaining, where the causality that *C*i imparts to *C*j declines as the causal path *C*i→*C*j gets more populated by intermediate causes. The aggregative behavior is present in causal confluence, where the causality imparted to an effect gets higher as the number of causes of the same sign increases.

We will use the standard algebraic product as indirect-effect operator and the generalized algebraic sum for total effect. Both operators are compliant with the arguments previously discussed [Zimmermann, 1987]. Hence, the indirect-effect operator will be:

$$\mu_{\cap} = \prod_{i=1}^{m} \mu_i \tag{2}$$

Where $\mu_{\cap}$ stands for the fuzzy intersection such that $\mu_{\cap} \in F([0,1])$, being $F([0,1])$ the fuzzy power set of $[0,1]$. μ_i is the marginal causality and *m* is the number of nodes in the causal path.

For the total effect operator, in order to offset positive and negative causes, the algebraic sum will consider the sign label representing positive or negative causality as an algebraic sign. Therefore, for causal combination we will have the following operation:

$$\mu_{\cup} = 1 - \prod_{i=1}^{m}(1-(-1)^{\alpha}\mu_i^{+}) \cdot \prod_{j=1}^{n}(1-(-1)^{1-\alpha}\mu_j^{-});$$

$$\alpha = \begin{cases} 0; \sum_{i=1}^{m}\mu_i^{+} \geq \sum_{j=1}^{n}\mu_j^{-} \\ 1; \sum_{i=1}^{m}\mu_i^{+} < \sum_{j=1}^{n}\mu_j^{-} \end{cases} \tag{3}$$

Where μ_i^{+} and μ_j^{-} stands for marginal positive and negative causality respectively, while $\mu_{\cup}$ is the total causality; m and n are the number of positive and negative causes linked to the effect, in that order. The α parameter used in (3) helps us to assure that $\mu_{\cup} \in F([0, 1])$.

Improving Interpretation with Linguistic Approximation

Using a simple algorithm for linguistic approximation we can convert fuzzy-numeric results, representing the activation value of concepts, into linguistic sentences that combine the terms selected for causality assessment.

Furthermore, the membership function shape can be analyzed to obtain conclusions about the uncertainty that affects the calculated activation value. For example, disperse membership functions reflect a balance between opposite sign causes. This can also be interpreted as a low degree of consensus, if we are combining FCMs designed by several experts. To perform this analysis we must use a similarity measure based on the fuzzy set shape, as for example the Hamming distance [Klir and Yuan, 1995], in combination with heuristics about significant patterns of the membership function shape.

Conclusions and Further Work

We have presented an extension of the FCM knowledge representation using fuzzy numbers to represent causal strength, and reviewing the fuzzy causal algebra for FCMs and fuzzy numbers. We have also suggested the use of linguistic approximation as an approach that allows us to revert the flow of uncertainty-based information, providing a linguistic description of the results.

FCMs can be used to build causal expert systems in controversial domains, using its graphical representation to acquire knowledge from discussions and collaborative meetings. At the moment we are applying this tool to the acquisition of knowledge and production of diagnoses about shrimp diseases such as the WSSV syndrome. We are also exploring the application of FCMs to decision-making automation, where the use of fuzzy numbers can allow us to connect the FCM with fuzzy IF-THEN rules.

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