Reasoning under inconsistency based on implicitly-specified partial qualitative probability relations: a unified framework

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Abstract

Coherence-based approaches to inconsistency handling proceed by selecting preferred consistent subbases of the belief base according to a predefined method which takes advantage of explicitly stated priorities. We propose here a general framework where the preference relation between subsets of the belief base is induced by a system of constraints directly expressed by the user. Postulates taking their source in the qualitative modelling of uncertainty, either probabilistic or possibilistic, are used for completing the implicit specification of the preference relations. This enables us to define various types of preference relations, including as particular cases several well-known systems such as Brewka's preferred sub-theories or the lexicographical system. Since the number of preferred consistent subbases may be prohibitive, we propose to compile the inconsistent belief base into a new one from which it is easier to select one preferred consistent subbase.

1. Introduction

Inconsistency may appear when a plausible consequence, obtained under incomplete information, has to be revised because further information is available. This issue has been extensively investigated in the nonmonotonic reasoning literature. In this paper we rather view inconsistency as being caused by the use (and the fusion) of multiple sources of information. Coherence-based approaches (Rescher; 1976) (Benferhat et al., 1995a) to inconsistency have two main steps, as shown in Fig. 1: i) build one or several preferred consistent subbases of the belief base **K**, and ii) use classical entailment on these subbases.

A preference relation between subbases is a reflexive and transitive relation $\geq$ on $2^{\mathbf{K}}$. Thus specifying it *explicitly* would need, in the extreme case, $O(2^{2*|\mathbf{K}|})$ space which is not reasonable. It is why we have to *implicitly* specify a "small" set of constraints bearing on sets of formulas, which can be completed into a preference relation over $2^{\mathbf{K}}$ using a set of postulates.

Two kinds of such constraints have been considered:

(a) *priority* constraints, which are qualitative in essence, and which consist in specifying an ordering relation *on the formulas in* ***K***, from which the preference relation on $2^{\mathbf{K}}$ is induced. This is the case with Nebel (1991; 1994)'s syntax-based entailment, Brewka(1989)'s preferred sub-theories, Williams(1996)'s approach to belief revision, Geffner(1992)'s conditional entailment, the lexicographical system (Benferhat et al., 1993), (Lehmann, 1995), etc.

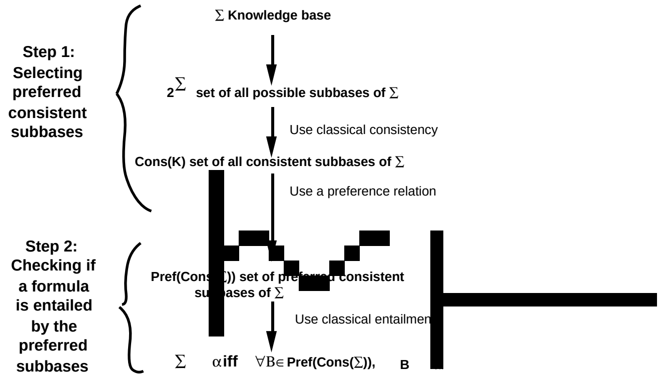

Figure 1

(b) *numerical* constraints, which consist in attaching to each formula a numerical weight which can be a probability degree (infinitesimal or not), a possibility degree (Dubois et al., 1994), a penalty value (Dupin et al., 1994), or an infinitesimal belief degree in belief functions theory (Benferhat et al., 1995b).

However, all these methods for specifying preferences implicitly lack flexibility on the way the preference relation is induced from the constraints, since the set of postulates is fixed once for all and is not a part of the representation language. In the first part of the article (Sections 2-4) we propose a general representation framework for coherence-based reasoning, which encompasses the well-known systems mentioned above. This framework offers a lot of flexibility, and for instance makes it possible to the user to define, a system which lays between Brewka's preferred sub-theories and the lexicographical system.

In the second part of the paper, we deal with the problem of representing the preferred consistent subbases compactly, in order to perform task 2 (see Fig. 1) eff[redacted]y. For this purpose we propose a compilation techn[redacted] which [redacted]ransforms **K** into a new belief bas[redacted] **(K)** ([redacted] additional formulas). This combinatio[redacted] technique is inspired from [redacted]ntly introduced [redacted] the possibilistic logic [redacted]ng (Benferha[redacted] al., 1997[redacted] W[redacted] show that this syn[redacted]c combination of be[redacted]f bases [redacted] a method for t[redacted]ing inconsi[redacted]ency in th[redacted] sense th[redacted] plausible conc[redacted]ion inferred [redacted]om **K** usi[redacted] some e[redacted]tin[redacted] inconsistency-[redacted]lerant consequ[redacted]ce r[redacted]ation, i[redacted]als[redacted] consequence f[redacted] a unique consist[redacted]ubbase of [redacted]**(K)**.

This paper em[redacted]asizes the syntactic aspect[redacted]f t[redacted] approach, while [redacted]on paper (Benferhat [redacted]l., 1998) develops t[redacted] semantical issues and concentr[redacted] preference relati[redacted]

2. Spe[redacted]fyin[redacted] p[redacted]eferences [redacted]mplicit[redacted]

In this pa[redacted]r, we[redacted]nly consider a [redacted]ropositi[redacted] The symbo[redacted] [redacted]epresents th[redacted] classic[redacted]onsequen[redacted] relation, Greek [redacted]tters α,β,[redacted]present formu[redacted] Let ={s_1,…,s[redacted] set of so[redacted]ces (all [redacted] are different). A belief base **K**={φ(s_i) | s[redacted] } as[redacted] is a multiset of the formulas φ provi[redacted]d by [redacted] sources[redacted] For the sake of simplicity, we simply [redacted]rit[redacted] instead [redacted] φ(s_i). The same belief can be present sev[redacted]in [redacted] if it comes from different sources and this explai[redacted] consider **K** as a multiset. However is not a [redacted] but a set since all s_i are differe[redacted] Thus, instea[redacted]of working with $2^{\mathbf{K}}$, which would [redacted]ad to am[redacted] (si[redacted] **K** is a multiset), we prefer t[redacted]ork wit[redacted] ha[redacted] mind tha[redacted]d to φ[redacted]

Def. 1: A partially [redacted]tative [redacted]sitiv[redacted] pr[redacted]ility (PQPP) relation ≥ [redacted] 2[redacted] is a r[redacted] s[redacted]ing th[redacted] following postul[redacted]es: [redacted]X,Y, Z⊆ [redacted]

A1. ≥ is [redacted]ransitiv[redacted]

A2. X Y im[redacted]es X[redacted]Y; (mea[redacted]lusion)

A3. If X[redacted]

X ∪ Y ≥ X ∪ Z ⇔ Y ≥ Z,

where : X>Y means that X≥Y and not [redacted]≥X.

The intuitive mea[redacted] X ≥Y (resp. X[redacted]) is that the set of so[redacted]es in X is at[redacted]least as preferred/prior[redacted]ry/reliab[redacted] as (resp[redacted]strictly more prioritary than)[redacted]e set of sources in Y.[redacted]ote that we do not require ≥ to be[redacted]nnected (≥ is gen[redacted]lly a partial order only), which entail[redacted]hat there[redacted] be incomparable subsets of . The inco[redacted]y relation should not be confused with the equivalence (or indifference) relation defined by: X and Y are equivalent (denoted by X≈Y) iff X≥Y and Y≥X. The relation ≥ is a kind of partially ordered qualitative probability (Lehmann, 1996), where all non-empty events have a "non-null" probability value due to A2. The cancellation property A3 is close to the one of comparative probabilities (Fishburn, 1986), although we remain in the qualitative framework.

Now, what has to be first specified is a set of constraints be[redacted] on the [redacted]nc[redacted] ind[redacted]ed from [redacted] b[redacted] app[redacted] a[redacted]A3[redacted]epeat[redacted] cons[redacted]s of [redacted] ={ X[redacted]>Y_i, i=[redacted]ce s[redacted]cified [redacted] inequali[redacted]aid [redacted] there exist[redacted]a[redacted]vit[redacted]i.e. [redacted]≥ ext[redacted]qualities [redacted]f [redacted]he pr[redacted] closu[redacted] A[redacted]A[redacted] p[redacted] propo[redacted]ti[redacted] s[redacted]each[redacted] co[redacted]exists [redacted]

Pro[redacted]tion 1: [redacted]is [redacted]nt [redacted] there ex[redacted]s a [redacted] deno[redacted] ≥ , su[redacted] tha[redacted]or [redacted]fer[redacted] > [redacted]≥ extend[redacted] ⇔ [redacted]

[redacted] be called[redacted]osure o[redacted]hen [redacted]axio[redacted]A1-A3[redacted] then >[redacted]n be[redacted]n the [redacted]ng way: le[redacted]R ={(I,J[redacted]∈ [redacted] } and R_C[redacted]⊆J}. No[redacted] complete[redacted]∪R_C [redacted]si[redacted] R[redacted](I∪M[redacted]M);[redacted]I[redacted] M disjo[redacted]t s[redacted], (I,J)∈ [redacted]∪R_C}. F[redacted]ally[redacted]e transitive [redacted]su[redacted]of R[redacted]

Example [redacted]Let ={s1, s2, s[redacted]t =[redacted]>{s2,s3[redacted]2}≈{[redacted]}}. The closure of is shown [redacted] 2 (re[redacted]ivity and transitivity are not represented for[redacted]ake [redacted]larity). For instance {s1, s4} > {s2, s3, s4}[redacted]ined from {s1}>{s2,s3} and postulate A3.

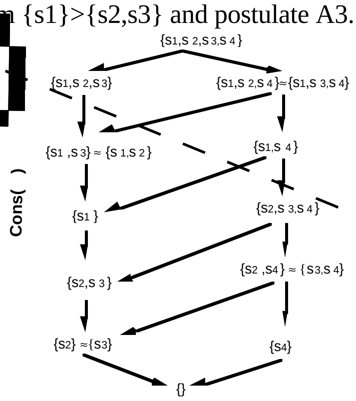

Figure 2

3. PQPP-preference-based entailment

Roughly speaking, > helps selecting preferred subbases of **K** by removing all those which are not maximal. X > Y means that, in order to restore the consistency of **K**, we prefer to maintain the set of beliefs supported by the sources in X rather than to maintain the set of beliefs supported by the sources in Y. In this way, the condition A2 becomes very natural since it corresponds to the idea of minimal change : we try to maintain as many pieces of information from **K** as possible. A3 simply means that only the form[redacted]as[redacted]hich di[redacted] Z sho[redacted]d be taken into acco[redacted]t[redacted]dis[redacted]d Z. Now, coherenc[redacted]ase[redacted]ailm[redacted]t o[redacted]α initial specificat[redacted]is d[redacted] the theories impl[redacted] that are -maxim[redacted]of[redacted]e and a prefere[redacted]ion ≥, we[redacted]> usual way[redacted]Max([redacted]≥[redacted]x The mini[redacted]m o[redacted]ato[redacted]can be[redacted]l[redacted]imal way. For[redacted], [redacted]}[redacted]e conjunctio[redacted]fo[redacted]pro[redacted]es[redacted]

Def. 2: X[redacted]s[redacted]he con[redacted]X) is consistent[redacted]te Cons( )=[redacted]s[redacted]is[redacted]onsi[redacted]set of all co[redacted]

Def. [redacted]**: K,**[redacted] **AND**([redacted]X)

Example [redacted] ={s_1, s_2[redacted]ϕ_2[redacted] ϕ_4=¬a∧c.[redacted]err[redacted] subsets of[redacted]ax[redacted]> {s_2, s_3, s[redacted] **AND**({s_1[redacted]) ≡[redacted] therefore we[redacted]av[redacted]**K**[redacted] **AND**({s_2, s_3[redacted]

Note that is[redacted] w.r.t. **:**

Prop. 2: If '[redacted]α[redacted]

This is due[redacted]o the[redacted] Max(Cons([redacted]≥) is i[redacted] The non-mono[redacted]nicity of[redacted]**K**[redacted]

Example 1 (co[redacted] Let **K'**=**K**∪{φ[redacted]d [redacted] ϕ_5=¬a∧¬[redacted]o[redacted]elief[redacted]E[redacted]Th[redacted]se the same[redacted]th[redacted]one[redacted]mpl[redacted]re explicit c[redacted]nst[redacted]ts [redacted]The[redacted] subsets o[redacted]ar[redacted]M[redacted]= {s_1,s_3}, {s_2, s[redacted]s_5}}.[redacted]can have: **K',** a∨c, since[redacted]s_5})[redacted]a∨

4. Re[redacted]overing co[redacted]eories

This [redacted]ction sho[redacted]the e[redacted]ment based[redacted]P relat[redacted]s allows[redacted]to[redacted]several[redacted] systems[redacted]Coherence[redacted]ased[redacted]ories gene[redacted]ssume a stratifica[redacted]on of[redacted] is[redacted]p} . We note [redacted]th[redacted]urce (acco[redacted]to a[redacted]arbitrary numbering) in i. This stra[redacted]expresses a[redacted]al pre-order between the sources: $\forall s_{ik}, s_{jl} \in$, $s_{ik} > s_{jl}$ [redacted]

[redacted]ex[redacted]ic or[redacted]ng d[redacted]y[redacted]Y iff ∃ such th[redacted]|X∩ i|>|Y∩ i|[redacted] [redacted]Y iff for any i, |X∩ i|=|Y∩[redacted]

The >[redacted]maximal consistent element[redacted]are called preferred [redacted]in (Brewka, 1989) and [redacted]lex-maximal cons[redacted]ent e[redacted]ments are called lexicographically preferred subb[redacted]in [redacted]Benferhat et al., 1993) ([redacted]ehmann, 1995).

[redacted]re are a[redacted]o some other c[redacted]herence theories whic[redacted]ociate po[redacted]ive integer n[redacted]mbers c_i ([redacted]sp. symbo[redacted]wn but ord[redacted]red) to the sources s_i. [redacted]ample[redacted]system which use[redacted]such assignments[redacted]the [redacted]nalty lo[redacted]c proposed[redacted](Dupin et[redacted]1994), [redacted]ra[redacted]ers Co[redacted]s( ) in the fol[redacted]

[redacted]*(sym*[redacted]*al*[redacted]*based ordering* defined by X $>_{pen}$ Y [redacted]resp. X $>_{spe}$[redacted]iff ∑{c_i | s_i∉ X} < ∑{c_j | s_j∉ Y} (resp. [redacted]∑{w_j | s_j∉ Y}).

C[redacted]y[redacted]$>_{spen}$ Y [redacted]plies X $>_{pen}$ Y. Symbolic penalty lo[redacted]c i[redacted]equ[redacted]ent [redacted]an infinitesimal version of belief [redacted]ncti[redacted]nf[redacted]et al., 1995b).

[redacted]e[redacted] cond[redacted]ions for recovering the previous [redacted]e s[redacted]ma[redacted]zed in Fig. 3, see (Benferhat et al., [redacted]).

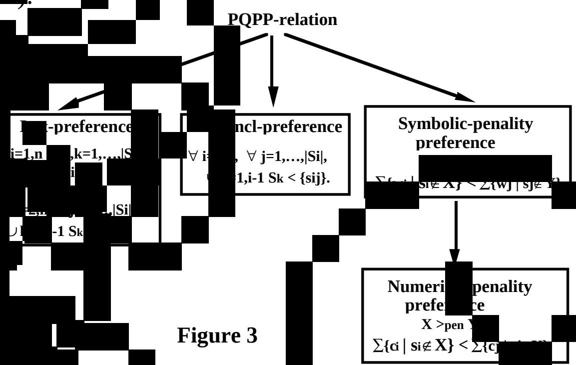

Figure 3

A natural q[redacted]estio[redacted]uggested by th[redacted]icture is: can [redacted]lty lo[redacted]c be[redacted]numerical representati[redacted]PQPP-rel[redacted]Nam[redacted]y, we may wonder if for any[redacted]QPP-relatio[redacted]there[redacted]xists a set of[redacted]hts c_i associated [redacted]the sourc[redacted]s_i su[redacted]h that X >[redacted]Y iff ∑{[redacted]s_i∉ X} < ∑[redacted]c_j | [redacted]The answer is[redacted]o, and we[redacted]can use th[redacted]same cou[redacted]ter-example given [redacted]Kraft, Pratt and Sei[redacted]nberg ([redacted]59) where they show[redacted]hat qualitative p[redacted]bability [redacted]elations cannot be repre[redacted]nted by [redacted]robability distribution. Let ={s_1,s_2,s[redacted] and ε=1. Let c_1=400-ε, c_2=100-ε, c_3=300-ε, c_4=200 and c_5=600. Then define ≥ in the following way:

. for any couple $(X,Y) \neq (\{s_1,s_2,s_3\}, \{s_4,s_5\})$:

$$X \geq Y \quad \text{iff} \quad \sum\{c_i \mid s_i \notin X\} \leq \sum\{c_j \mid s_j \notin Y\}$$

. $\{s_1,s_2,s_3\} > \{s_4,s_5\}$ (although $c_4+c_5 > c_1+c_2+c_3$)

It can be shown that ≥ cannot be represented by any numerical penalty logic. This is due to the following facts:

• Note that the qualitative preference relation ≥ satisfies the following inequalities:
{s1, s4, s5}> {s2,s3,s4,s5} , {s3, s4, s5}>{s1,s2,s5},
{s2,s4,s5}>{s1,s3,s4}, {s1,s2,s3} > {s4,s5},
and that there is no [redacted] integer costs which satisfy the above in[redacted]ualities. Indeed[redacted] c_2+c_3[redacted] c_1[redacted] $c_1+c_2<c_3+c_4$, $c_1+c_3<c_2$[redacted] implies [redacted] $c_3< c_4+c$[redacted]

• The preference rela[redacted]n is a P[redacted] penalty ordering de[redacted]ed[redacted] numbers satisfies [redacted] ms A[redacted]B, and ch[redacted] direction of the lin[redacted] $\{s_4,s_5\}$ has no[redacted] on the satisfacti[redacted] of [redacted]oms since [redacted] $X \neq \{s_1, s_2, s_3$[redacted] $\{s_4, s$[redacted] $\{s_4,s_5\} \geq X \geq \{s_1,s_2,$[redacted]

Apart from the wel[redacted] gen-eral framework, we [redacted]ay defin[redacted] by specifying a set of [redacted]

Example 2: multi-[redacted] + Pare[redacted]

Agent A: [redacted] of constraints [redacted] $\geq_A$.[redacted] Similarly for [redacted] Letting **K**=**K**[redacted]

∀X⊆ [redacted] with $X_A = X$[redacted] global preference[redacted] where the preference [redacted] are not commensur[redacted]

Some system[redacted] partic[redacted]lar s[redacted]ems [redacted]ased on the selection of a[redacted] prefe[redacted]d consi[redacted] cannot be recovere[redacted] xample of such systems i[redacted] he p[redacted]stic logic a[redacted] (Dubois [redacted] al., 1994), wh[redacted]h uses [redacted] ratificati[redacted]d w[redacted]ere the ordering on Cons([redacted] defined[redacted]ing wa[redacted]

- *“best out” o[redacted]ering*: de[redacted] by X $>_{Bo}$ [redacted]f Max{i [redacted] ∃s∈ i and s∉ X[redacted]j / ∃s'∈ j and s'∉ Y[redacted] convention Min∅ = 0.

Possibilistic logic can be recovered using the following postulates: (A1), (A'2) Y⊆X ⇒ X≥Y and (A'3) Y≥Z⇒ X∪Y ≥ X∪Z, instead of A1-A3. A'3 is the main axiom of qualitative possibility theory (Dubois et al., 1994). See (Dubois et al., 1992) for an explicit possibilistic handling of the sources.

5. Compiling inconsistent belief bases

Coherence-based approaches take blindly into account all the beliefs in **K** and compute the set of preferred consistent subsets of **K** although some beliefs are not used in inferences[redacted] belief base **K**={α, ¬α, β} wh[redacted]e all [redacted] beliefs [redacted]e a[redacted]med to be equally reliable. [redacted]is be[redacted]onsistent and admits two prefer[redacted]d c[redacted]sis[redacted]nt subs[redacted]s A={α, β} and B={¬α, β}[redacted] that [redacted] formula ψ, we have both[redacted] B [redacted] This means that we can igno[redacted] eliefs {[redacted]} [redacted] changing the set of [redacted]ausible [redacted]quences [redacted] in the sens[redacted] of preferred co[redacted]ent su[redacted]es inferen[redacted]

Generalizi[redacted] this idea[redacted] ve[redacted]se an a[redacted]rna[redacted] or re[redacted]erence[redacted]ies. [redacted]is is d[redacted]ne i[redacted] wo te[redacted]

• The firs[redacted] be[redacted] oted by[redacted] is ea[redacted]er[redacted] preferred consi[redacted]te[redacted]

• [redacted]n 6, con[redacted]in[redacted]ecti[redacted]g o[redacted] su[redacted] (K[redacted] s[redacted] t[redacted] α[redacted] f[redacted]

The [redacted] by viewing the couple (2 [redacted] belief [redacted]se. W[redacted] (X)=∨[redacted] $s_i \in X\}$ the disjunction of the original formu[redacted]s [redacted]ovided [redacted] the source[redacted] in [redacted] More formally[redacted] n the following way:

Def. 4[redacted] (K) i[redacted] sub[redacted]base [redacted] {[redacted]} [redacted]es n[redacted] c[redacted]tain ne[redacted] tautologies[redacted]

[redacted] 5[redacted] Let [redacted] ⊆2 [redacted], and B=[redacted] fo[redacted]u[redacted] **OR**(Y) is said to be subsu[redacted]d [redacted] ∃**O**[redacted](X)[redacted] B such that X≥Y and **OR**(X) [redacted]**R**(Y).

[redacted] (K) i[redacted] que in general, however [redacted] (K) [redacted] equivale[redacted] t in the sense that in the se[redacted]ond step all t[redacted] selected consistent subbases are classica[redacted]y equivale[redacted]

E[redacted]ample 1 (continue[redacted]: Let **K**={ϕ_1, [redacted] ϕ_3, ϕ_4} [redacted] ϕ_1[redacted]a, ϕ_2=¬[redacted]b, ϕ_3=[redacted]∨¬b, ϕ_4=¬a∧c. A[redacted] ta[redacted]ologies [redacted](X) | X⊆ [redacted] } we get [redacted] ϕ_4, $\phi_1 \vee \phi_4$, [redacted] $\vee\phi_4$, ϕ[redacted]

[redacted] ϕ_1[redacted] ϕ_4≡a∨c[redacted] $\phi_2 \vee \phi_4$≡¬a∨[redacted] ≡¬a∨¬[redacted]

Note th[redacted] ϕ[redacted]s subsu[redacted]ed by ϕ[redacted] si[redacted]e [redacted]∨ϕ[redacted] and {s[redacted] s_4[redacted] > {s_2} [redacted]nd that [redacted]3 i[redacted] also [redacted]subsume[redacted] $\phi_3 \vee \phi_4$ si[redacted]3∨ϕ_4 [redacted] and {s[redacted] s_4[redacted] > {s_3}. The[redacted]

(K) = [redacted], $\phi_1 \vee \phi_4$, [redacted]∨[redacted], $\phi_3 \vee \phi_4$}.

In (Benferhat et al., 1998) an in[redacted]al w[redacted] has been proposed for computing (K).[redacted] pos[redacted]ble by applying an associative and c[redacted]mmutative binary combination operator, denoted by ◊ [redacted] briefly recall the definition of this operator. Let **K**1={**O**[redacted](X_i) | X_i[redacted] i=1,k} and **K**2={**OR**(Y_j) | Y_j⊆ [redacted], j=1,m}. T[redacted] **K**2 is defined by:

$$\mathbf{K}_1 \lozenge \mathbf{K}_2 = \mathbf{K}_1 \cup \mathbf{K}_2 \cup \{\mathbf{OR}(X_i \cup Y_j) \mid X_i \in \mathbf{K}_1, Y_j \in \mathbf{K}_2, \mathbf{OR}(X_i \cup Y_j) \text{ is not a tautology}\}.$$

As it can be[redacted], t[redacted] ope[redacted]tor ◊ intr[redacted]uces new di[redacted] junctions of fo[redacted]as [redacted]hich are[redacted]ot ne[redacted]ssarily subsu[redacted]ed (in the sense [redacted]ef.[redacted]). Let **K**=[redacted]i=1,n} b[redacted]a b[redacted]ef base. Let K_i={ϕ[redacted] one for[redacted]la belief base[redacted] can show that **(K)** is[redacted]K_1◊..◊**K**[redacted]

6. Select[redacted]onsi[redacted]ent su[redacted]ba[redacted] i[redacted] (K)

In the pr[redacted]io[redacted]se[redacted]ti[redacted], [redacted] i[redacted]ent belie[redacted] **K** ha[redacted] been comp[redacted] that all form[redacted]s [redacted](K[redacted] [redacted]**OR**(X[redacted]wit[redacted] X⊆ . In this[redacted]e[redacted]elect a [redacted]ssi[redacted]al con[redacted]stent subset of **(K[redacted]** [redacted]y **BC(K[redacted])**, [redacted]at: **K**, α iff **BC(K)** [redacted]. [redacted]nu[redacted]n[redacted]lled accepted beliefs and roug[redacted]ese[redacted] pieces [redacted]f information in [redacted]K[redacted]) [redacted]ked by[redacted]y minimally inconsi[redacted]**(K)**[redacted] definitions are n[redacted] in [redacted]r[redacted] formally define [redacted] notion of accep[redacted]efs.

Def. 6: A sub[redacted]s[redacted] of **(K)** i[redacted]e mini[redacted]y inconsistent if [redacted]∀**OR**(X)∈[redacted]B[redacted]R(X)}[redacted]⊥.

Let Nogood( ) [redacted]e se[redacted]min[redacted]inconsistent subbases of **(K)**[redacted]

Def. 7: A formul[redacted]R(X) [redacted] **(K)** [redacted]*scape* from a minima[redacted]y i[redacted]ons[redacted]ent [redacted]bb[redacted]**R**(Y)∈ B such that X > [redacted].

Finally, accepte[redacted]bel[redacted]fs are[redacted]efined by:

Def. 8: A form[redacted]**O R**[redacted](X[redacted]o be accepted in **(K)** i[redacted]ogood(**K**), **O**[redacted]X) escape[redacted] from B[redacted]

The [redacted]wing pr[redacted]position[redacted]w[redacted]t[redacted] ca[redacted]e r[redacted]overed u[redacted]ng acc[redacted]ted[redacted]eli[redacted]f[redacted] s[redacted] pro[redacted]ed [redacted](Benferh[redacted] et al.,[redacted]98[redacted]

Pr[redacted]pos[redacted]ion 3: Let [redacted]e [redacted]sub[redacted]f [redacted]**K**[redacted] such[redacted] any for[redacted]ula **OR**[redacted]X)[redacted]**(K)**[redacted] **(K)**. T[redacted] **K** [redacted] iff[redacted]**BC**[redacted]

Fortunately, w[redacted] do not [redacted]ompute[redacted]minimal inconsistent s[redacted]sets of [redacted]K) f[redacted]ing **B**[redacted]W[redacted] will distingui[redacted] two cases:

. If the PQP[redacted]reference[redacted]ion[redacted]ete (i[redacted] ∀X,Y⊆ , we have [redacted]≥ Y [redacted]Y≥[redacted]it enough to compute a subset of , [redacted]noted [redacted]). Then, **OR**(X)∈ **BC(K)** iff X>Incons[redacted]). This ca[redacted] largely developed in (Benferhat et al., 199[redacted]. An effici[redacted] procedure for computing Incons( ) is also pro[redacted]

. If th[redacted]QPP-preference re[redacted]i[redacted]s not complete, w[redacted] need to consider a sub[redacted]e[redacted] the se[redacted] of m[redacted]n[redacted] inconsistent subbases of[redacted]**(K),** d[redacted]ogood[redacted]

This section focuses [redacted]c[redacted]se where[redacted]PQP[redacted] preference relation is n[redacted]mplete[redacted]**C**[redacted]is then m[redacted]re tricky, even [redacted]com[redacted]ll [redacted]minimal [redacted]inconsis[redacted]subbases[redacted] follo[redacted]ing [redacted]efi[redacted]tions[redacted]tools facilit[redacted]ng the [redacted]om[redacted]ng of [redacted]K)[redacted]Th[redacted] improvement is to[redacted]rse[redacted]es only to the mi[redacted]al[redacted]leme[redacted]s[redacted]n[redacted]minim[redacted]y inconsistent subb[redacted]na[redacted]ely:

Proposition [redacted] **C**[redacted]) [redacted] (**[redacted]** iff ∀B∈Nogood([redacted]∃[redacted]R([redacted] [redacted]o tha[redacted]

T[redacted]ake furthe[redacted]en[redacted]e r[redacted]the definiti[redacted] of[redacted]>[redacted]th[redacted]Let A,B∈Nog[redacted](

[redacted]Min(A) [redacted]Min(B) i[redacted]D, s[redacted]

This e[redacted]ension is defi[redacted] compl[redacted]te ≥ (and in this [redacted] to one[redacted]lement) satisfies M[redacted]M[redacted] We w[redacted]ll denote [redacted]N[redacted]) [redacted]n-domin[redacted]ed min[redacted]ally in[redacted]

[redacted]ogood*( )=[redacted]A[redacted]

Such a Nogood[redacted]) is no[redacted]nce[redacted] Nogood( ) [redacted]ains[redacted]ent[redacted](≥ Mi[redacted]and[redacted]th[redacted]n either remove A or B). The fo[redacted]wing pro[redacted]s that in [redacted]er to che[redacted]ula[redacted] acc[redacted]d[redacted]ot it[redacted]enough [redacted]od*( ) i[redacted]of[redacted]

[redacted]opositio[redacted]: **OR**([redacted]n **(K**[redacted] [redacted]OR(X) [redacted]m

[redacted]llowi[redacted]g imp[redacted]ment is [redacted]ortant f[redacted] co[redacted]puting N[redacted]) more easi[redacted]

Proposition [redacted]et A be a minimally i[redacted]sistent [redacted]ase and let **C**[redacted](X) be a belief of **(K)** s[redacted] [redacted]Y[redacted]A, Y≥X. [redacted]hen any minimally inco[redacted]sistent s[redacted]bas[redacted]in[redacted]**OR**(X) is such that Min(A) [redacted]in(B[redacted]

[redacted]ba[redacted] on [redacted]e previous propositions, the following a[redacted]m [redacted]sents a way to compute Nogood*( ). It is [redacted]e efficient as it starts with a minimally inconsis[redacted] subbas[redacted]made of the most important beliefs in **(K)**.[redacted]

Function: Computing [redacted]
Input: ≥ , (K), [redacted]
Begin
• Nogood*( )=Ø;
• Let A a minimal [redacted]
While (A≠Ø) do [redacted]
• Minimize A [redacted]
∃OR(Y[redacted]
• [redacted]
s.t[redacted]
• [redacted]
• [redacted]
[redacted]
• Let [redacted]
(K) s[redacted] wi[redacted]
Min(B)≥ Min(A[redacted]
end
Return (Nogood[redacted]))
end {Function}

Example 1 (continued[redacted] We [redacted] that [redacted] $\phi_1 \vee \phi_4$, $\phi_2 \vee \phi_4$, [redacted]: ϕ_1=a, ϕ_4=¬a∧[redacted], $\phi_1 \vee \phi_4$≡a∨c; ϕ_2[redacted]ϕ_4≡[redacted]a∨[redacted]; [redacted]ϕ_4≡¬a∨¬b. [redacted] compute the c[redacted] **BC(K**[redacted] first computing No[redacted] previous algorithm. Let [redacted] $\phi_3 \vee$[redacted] minimal inconsistent sub[redacted] n[redacted] ϕ_3[redacted] are removed form [redacted] neither {s1}≥{s4} n[redacted]≥{s3[redacted]4}. At [redacted] Nogood*( )=[redacted] and **(K)**=[redacted], $\phi_1 \vee \phi_4$, $\phi_2 \vee \phi_4$, [redacted]. Let[redacted] again the previ[redacted] algorithm. There[redacted] minimal inconsist[redacted] subbase which[redacted] A={[redacted]ϕ_4}. However, A i[redacted] considered since[redacted]

Min ({ϕ[redacted]ϕ_4, $\phi_3 \vee \phi_4$[redacted] ({ϕ[redacted]).

Therefore: N[redacted]*( ) = {{ϕ_1,[redacted]

Lastly the set[redacted]ted beliefs is:

BC(K) =[redacted] ≡ {a∨c}

and we can ea[redacted]y check[redacted]

∀α, **K,** [redacted] α [redacted] **BC(**[redacted]

The propose[redacted] is incremental[redacted] ary coherence base[redacted] hes, in the sense that [redacted] a [redacted] belief ϕ provided [redacted] source s_ϕ is a[redacted]ed to the [redacted]rig[redacted]al belief base **K**, with[redacted] w set [redacted] nstraints ϕ [redacted]e added to , then **(K**∪{[redacted] overed from **(K)**. [redacted] is done by first compu[redacted] **K*=BC(K)** ◊ {ϕ}. The[redacted] if **K*** is consistent then **BC(K∪**{ϕ}**)=K*,** otherwise **BC(K∪** {ϕ}**)** contains accepted beliefs in **K***.

7. Concluding remarks

Several well-known prioritized inconsistency handling methods have been unified into a powerful framework. A [redacted]e treatment of the priorities is provided by an [redacted]licit specification of the priority ordering through user-o[redacted]inated constraints and general postulates. Then a [redacted]owl[redacted]ge "compilation" technique enables us to explicit [redacted] formulas useful for the inference process, which a[redacted]nts to select the consistent subpart of the new belief [redacted]as[redacted] obtained by compilation. This subpart is composed [redacted] all the accepted formulas (i.e., those which escape from [redacted] minimally inconsistent subset of formulas of the [redacted]f base).

8[redacted] References

[redacted]. Benferhat, C. Cayrol, D. Dubois, J. Lang, H. Prade (1993) Inconsistency management and prioritized syntax-based entailment. Proc. of IJCAI'93, pp. 640-645.

S. Benferhat , D. Dubois, H. Prade (1995a) How to infer from inconsistent beliefs without revising?. Proc. of IJCAI'95, 1449-1455.

S. Benferhat, A. Saffiotti and P. Smets (1995b). Belief functions and default reasoning. Proc. of the 11th Conf. on Uncertainty in Artificial Intelligence, pp. 19-26.

S. Benferhat , D. Dubois, H. Prade (1997) From semantic to syntactic approaches to information combination in possibilistic logic. In Aggregation and Fusion of Imperfect Information, (B. Bouchon-Meunier, ed.), Physica Verlag, pp. 141-151.

S. Benferhat, D. Dubois, J. Lang, H. Prade, A. Saffiotti, P. Smets (1998) A general approach for inconsistency handling and merging information in prioritized knowledge bases. To appear in Proc. of KR-98.

G. Brewka (1989) Preferred subtheories: an extended logical framework for default reasoning. Proc. IJCAI'89, 1043-[redacted]

[redacted] J. Lang, H. Prade (1992) Dealing with multi-source [redacted]formation in possibilistic logic. Proc. 10th [redacted] Conf. on Artif. Intelligence (ECAI'92), 38-42.

[redacted], J. Lang, H. Prade (1994) Possibilistic logic. In: Handbook of Logic in Artif. Int. and Logic Progr., Vol. 3 (D.M. Gabbay et al., eds.), Oxford Univ. Press, 439-513.

F. Dupin de St Cyr , J. Lang , T. Schiex (1994) Penalty logic and its link with Dempster-Shafer theory. Proc. of the [redacted]on Uncertainty in Artif. Intellig., 204-211.

C. H. Kraft, J. W. Pratt, A. Seidenberg (1959) Intuitive probability on finite sets. Ann. Math. Statist. 30, 408-419.

P. Fishburn (19[redacted]) The axioms of subjective probabilities. statistical sc[redacted] 1, 335-358.

[redacted]H. Geffner, Def[redacted]t Reasoning:Causal and Conditional Theories. MIT [redacted]ess, Cambridge, MA, 1992.

D. Lehmann (1[redacted]5) Another perspective on default reasoning. [redacted]nnals of Mathematics and Artificial Intelli[redacted], 15, pp. 61-82.

D. [redacted] (1996) Generalized qualitative probability: Savage revisited. In Procs. of 12th Conf. on Uncertainty in Artificial Intelligence (UAI-96), pp. 381-388.

B. Nebel(1991), Belief revision and default reasoning : syntax-based approaches. Proc. of KR'91, 417-428.

B. Nebel (1994) Base revision operator ans schemes: semantics representation and complexity. Proc. 11th European Conf. on AI, 341-345.

N. Rescher (1976) Plausible reasoning. Van Gorcum, Amsterdam.

M.A. Williams (1996). Toward a practical approach to belief revision: reason-based change. Proc. KR'96, 412-420.