Learning Sorting Networks By Grammars

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Definitions and Previous Work

A compare-exchange network, or CMPX-net, is a sequence of operations of the form $[i:j]$, each of which operates on an array, D, of length N. The network is said to have *width* N. The *length* of the network is the number of CMPX's in the network. For each $[i:j]$, we have $i < j$ and $i, j \in [0, N-1]$. To apply a CMPX-net to an array, swap $D[i]$ and $D[j]$ if $D[i] > D[j]$ for each $[i:j]$ in the sequence. A *sorting network*(SNet) is a CMPX-net which will sort D's contents into nondecreasing order no matter how D's contents are ordered initially. A *merging network* (MNet) is a pair containing a CMPX-net of even width, N, and a partition of the indices into two equal-size sets or "sides." If the data on each side of the partition are sorted initially then the output will be sorted. The space of CMPX-nets with n_c CMPX's and width N is large, of size $C(N;2)^{n_c}$. With MNets, the space is still bigger, since we must multiply the number of networks by the number of partitions, $C(N;N/2)$.

We use a genetic algorithm(GA) to search for CMPX-nets which are SNets or MNets. The GA repeatedly samples the space of potential solutions in a series of "generations," each using the relative "fitness" of the previous generation's samples to apportion more samples in promising regions. Mutation and especially cross-over operators are applied to generate similar but novel new sample points; this process is iterated until some stopping criterion is achieved. Hillis has had encouraging success using a GA to evolve sorting networks(Hillis 1991).

In our work, we represent CMPX-nets by grammars which describe CMPX-nets. Terminals define particular CMPX sequences and nonterminals specify ways in which larger networks are built from smaller ones.

Merging Networks — Recent Results

Our most recent experiments have involved MNets for several reasons. MNets can be fully tested in polynomial time using $(N/2+1)^2$ input sequences; exhaustively testing SNets is much more expensive, requiring 2^N input sequences. In addition, our random MNet generation experiments have shown that despite this reduction in the cost of exhaustive testing, the problem is still very difficult. Some MNets are "log-sequential" sorters. That is, if we cycle the output of the network back to its input $log_2 N$ times, then the data will be sorted.

Recent analytic work has produced two log-sequential MNets(Dowd *et al.* 1989; Canfield & Williamson 1991). The network due to Canfield and Williamson is particularly interesting because it is "log spectrum" (The number of distinct $j-i$ over all $[i:j]$ in the network is $O(log_2 N)$) and "log delay" (it can be parallelized to execute in time $O(log_2 N)$). Both of these characteristics can provide critical advantages in hardware implementation.

Using our GA and grammar representation, we have found an interesting network similar to but distinct from the Canfield-Williamson network. The grammar which generates our network requires only two rules and generates an entire family of networks, one for each $N = 2^i, i >= 2$. It appears to be a log-sequential sorter, and has log spectrum and log delay. This network embeds the Canfield-Williamson network of half as many inputs several times and in an overlapping fashion.

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References

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