Subnormal modal logics for knowledge representation

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Abstract

Several widely accepted modal nonmonotonic logics for reasoning about knowledge and beliefs of rational agents with introspection powers are based on strong modal logics such as KD45, S4.4, S4F and S5. In this paper we argue that weak modal logics, without even the axiom K and, therefore, below the range of normal modal logics, also give rise to useful nonmonotonic systems. We study two such logics: the logic N, containing propositional calculus and necessitation but no axiom schemata for manipulating the modality, and the logic NT — the extension of N by the schema T. For the nonmonotonic logics N and NT we develop minimal model semantics. We use it to show that the nonmonotonic logics N and NT are at least as expressive as autoepistemic logic,reflexive autoepistemic logic and default logic. In fact, each can be regarded as a common *generalization* of these classic nonmonotonic systems. We also show that the nonmonotonic logics N and NT have the property of being conservative with respect to adding new definitions, and prove that computationally they are equivalent to autoepistemic and default logics.

Introduction

Several nonmonotonic logics have been proposed as formalisms for common-sense reasoning [Reiter, 1980; McCarthy, 1980; McDermott and Doyle, 1980]. Some of the most interesting and powerful logics (default logic and autoepistemic logics among them) can be obtained from the approach originated by McDermott and Doyle [1980] and further refined by McDermott [1982]. The idea is to introduce a modality L which is read as "is known", "is believed", or 'is provable", and to define a set T of consequences of a given theory in such a way that if a sentence ψ does not belong to T, then the sentence $\neg L\psi$ is in T.

Formally, let I be a set of sentences representing the initial knowledge of an agent, and let $\mathcal{S}$ be a monotonic logic. A set T is called an *$\mathcal{S}$-expansion* of I, if

$$T = \{\varphi : I \cup \{\neg L\psi : \psi \notin T\} \vdash_{\mathcal{S}} \varphi\}, \qquad (1)$$

where $\vdash_{\mathcal{S}}$ denotes derivability in $\mathcal{S}$.

In [McDermott and Doyle, 1980] expansions were defined and investigated with the classical propositional logic in place of $\mathcal{S}$. In this case the notion of expansion has counterintuitive properties. Expansions of I are meant to serve as candidates for a knowledge or belief set based in I. However, a consistent expansion (in the sense of [McDermott and Doyle, 1980]) may contain both ψ and $\neg L\psi$, contrary to the intended interpretation of L. Already in [McDermott and Doyle, 1980], where expansions were first introduced, this undesirable phenomenon was observed. As a way out of the problem, McDermott [1982] suggested to replace in (1) the propositional derivability by the derivability $\vdash_{\mathcal{S}}$ in a modal logic $\mathcal{S}$. If $\mathcal{S}$ contains the inference rule of necessitation, then the counterintuitive properties, like the one indicated above, disappear.

A crucial question is what modal logic $\mathcal{S}$ should be used. McDermott [1982] considered three well-known logics: T, S4 and S5. He noted that, intuitively, all the axioms of S5 seem to be acceptable. On the other hand, he proved that nonmonotonic S5 collapses to the monotonic logic S5 and, hence, cannot be used as a formal description of defeasible reasoning. Logics S4 and T yield "true" nonmonotonic systems adequate for handling some examples of common-sense reasoning [Shvarts, 1990]. But it is hard to justify why we should give up some of the axioms of S5 while retaining the others if all of them seem to be intuitively justified.

It was proved in [McDermott, 1982] that if a logic $\mathcal{S}$ contains the necessitation rule, then each $\mathcal{S}$-expansion contains all theorems of S5. Hence, the presence or absence of some of the axioms of S5 has no effect on the fundamental property of expansions that they are closed under provability in S5. This result may be regarded as an evidence that the search for modal logics which may yield useful nonmonotonic systems should focus on two classes of modal logics: those close to S5, satisfying many of the axioms of S5, and those which satisfy none or few of them.

So far, logics close to S5 have received substantially more attention most notably the modal logics KD45, S4.4, S4F. It has been shown [Shvarts, 1990; Schwarz, 1991a; Truszczyński, 1991b] that if we disregard inconsistent expansions, the nonmonotonic modal logic KD45 is equivalent to the celebrated autoepistemic logic by Moore, logic S4.4 is equivalent to the autoepistemic logic of knowledge introduced in [Schwarz, 1991a], and Reiter's default logic can be naturally embedded in the nonmonotonic logic logic S4F, which, in turn, has a natural interpretation as a logic of minimal (or grounded) knowledge [Schwarz and Truszczyński, 1992].

The logics KD45, S4.4 and S4F share several common features. They are close to S5 and admit natural epistemic interpretation as logics of belief (KD45), true belief (S4.4), knowledge (S4F). Finally, each is maximal with respect to some property of nonmonotonic logics. The logics KD45 and S4.4 are maximal with respect to the property of producing exactly one expansion for modal-free theories [Schwarz, 1991a]. The logic S4F is a maximal logic for which theories without positive modalities (like the theory $\{Lp \supset p\}$ considered by Konolige [1988]) have a unique expansion [Schwarz and Truszczyński, 1992].

Logics close to S5 are normal, that is, they contain the necessitation rule and the modal axiom scheme $L(\varphi \supset \psi) \supset (L\varphi \supset L\psi)$. Normal modal logics possess an elegant and natural semantics, namely, the possible world semantics introduced by Kripke [1963]. Kripke semantics makes it easy to investigate normal modal logics, and was fruitfully exploited in investigations of nonmonotonic logics based on normal modal logics [McDermott, 1982; Moore, 1984; Levesque, 1990; Shvarts, 1990; Marek *et al.*, 1991].

In this paper we will focus our attention on the other end of the spectrum of modal logics. We will consider nonmonotonic logics that correspond to weak modal logics that satisfy none or only few axioms of S5. Namely, we will consider logics *not satisfying the axiom* K. Such logics are not normal. We will refer to them as *subnormal.* Some non-normal logics have been investigated by philosophers [Kripke, 1965; Segerberg, 1971]. (In fact, the first three modal logics introduced, S1, S2 and S3, were not normal; see [Feys, 1965] for a historical survey). Because these logics were aimed at eliminating the so-called "paradoxes of material implication", they do not contain the necessitation rule, but contain the axiom K or some of its weaker versions. Hence, they are different from the subnormal modal logics considered in this paper.

There are at least two reasons why it is important to consider nonmonotonic logics based on weak modal logics. First, according to one of the results of [McDermott, 1982], if $\mathcal{T}$ and $\mathcal{S}$ are two modal logics such that $\mathcal{T} \subseteq \mathcal{S} \subseteq S5$, then each $\mathcal{T}$-expansion is an $\mathcal{S}$-expansion but the converse does not hold in general. In other words, when we replace a logic $\mathcal{S}$ by a weaker one, say $\mathcal{T}$, then often some of the expansions disappear. Hence, using weak modal logics in the schema (1) offers a possible solution to the problem of ungrounded expansions (see Konolige [1988]). Secondly, the assumption that a reasoner has a power of reasoning in a strong modal logic such as KD45, S4.4 or S4F may not be a realistic one. Therefore, it is important to study what types of reasoning can be modeled if weak modal logics are used instead. It turns out that in the nonmonotonic case we do not loose anything by restricting an agent's reasoning capabilities. Namely, and it is perhaps the most surprising result of our work, we show that nonmonotonic modal logics KD45 and S4.4 (that is, essentially, autoepistemic logic and autoepistemic logic of knowledge) can be embedded into nonmonotonic modal logics corresponding to some very weak modal logics containing necessitation. A similar result is also known to hold for default logic [Truszczyński, 1991b].

In this paper we study two modal logics. Both logics are assumed to be closed under the uniform substitution rule and contain propositional calculus and the rule of necessitation. These requirements specify the first of them, the logic N of *pure necessitation.* It was first introduced and investigated in [Fitting *et al.*, 1992] but its nonmonotonic counterpart had been studied earlier in [Marek and Truszczyński, 1990]. The second logic is obtained from N by adding the axiom schema T: $Lp \supset p$. It will be referred to as the logic NT.

The logic N is clearly distinguished among all modal logics contained in S5 and containing the necessitation rule. It is the *weakest* one. It is also the weakest logic without counterintuitive expansions containing ψ and $\neg L\psi$. It is interesting to note that N-expansions were introduced already in the pioneer work of Moore [1985] under the name "modal fixed points". He also noticed that all N-expansions are stable expansions, but not vice versa, and suggests the interpretation of "nonmonotonic N" as a logic of "justified belief". In [Marek and Truszczyński, 1990] nonmonotonic N was studied in detail under the name *strong autoepistemic logic.*

The reasons we are interested in the logic NT are the following. First, while KD45 is quite commonly accepted as a logic of belief, there is no consensus as for the "right" modal logic of knowledge. For example, in the monograph [Lenzen, 1978] among all the modal axioms of S5 only the axiom T is essentially unquestionable (you cannot *know* a false proposition, you can only *believe* that it is true). Hence, NT may be regarded as a part of any reasonable logic of knowledge.

Another reason is more formal in nature. Often modal formulas describing examples of common-sense reasoning have no nested modalities, and no negative occurrences of L. For example, "p is true by default" is usually expressed as $\neg L\neg p \supset p$. For such theories many nonmonotonic logics coincide. For example, it is

proved in [Marek and Truszczyński, 1990] that for theories without negative occurrences of L, $\mathcal{S}$-expansions coincide for all modal logics $\mathcal{S}$ between N and KD45. Similarly, $\mathcal{S}$-expansions coincide for all modal logics $\mathcal{S}$ between NT and S4.4. Recall that KD45 and S4.4 are maximal logics which do not produce "ungrounded" expansions for objective theories. Thus, it seems that T is, probably, the only axiom which makes a difference. Hence, it is important to study the nonmonotonic modal logic corresponding to the weakest modal logic containing the schema T.

The problem with logics like N or NT is, that *they are not closed under the equivalence substitution rule.* For example, $\vdash_{\mathrm{N}} p \equiv \neg\neg p$, but $\not\vdash_{\mathrm{N}} Lp \equiv L\neg\neg p$. This means, that algebraic semantics for such logics are impossible (if we want $\equiv$ to be interpreted as equality in the algebra of truth values). On the other hand, a Kripke-style semantics for N has been found in [Fitting *et al.*, 1992]. The key idea used in this paper was to treat N as a logic with infinitely many modalities, so L in $L\varphi$ and L in $L\psi$ represent two different modalities, if φ and ψ are syntactically different.

For a long time nonmonotonic modal logics, even if an underlying modal logic was normal, lacked an intuitively clear semantics. Recently, such a semantics has been found [Schwarz, 1992]. In this paper we combine ideas from [Fitting *et al.*, 1992] and [Schwarz, 1992] and obtain a possible world semantics for the nonmonotonic logics N and NT. We apply these semantics to prove that the nonmonotonic modal logics KD45 and S4.4 can be embedded into the nonmonotonic logics N and NT. What is more, the embeddings are very simple. For example, to embed the nonmonotonic logic KD45 (that is, the autoepistemic logic) into the nonmonotonic logic N, it is enough to replace each occurrence of L with $\neg L \neg L$. Consequently, the nonmonotonic logic N is at least as expressive as nonmonotonic KD45: the modality $\neg L \neg L$ can be viewed as the "modality of nonmonotonic KD45". In the same time, no faithful embedding of nonmonotonic N (or NT) into nonmonotonic KD45 is known so far. We regard our "embedding" results as the most important results of our paper. They show that subnormal modal logics such as N and NT can easily be used (in the nonmonotonic setting) to simulate other nonmonotonic formalisms and, thus, are viable and powerful knowledge representation tools.

Perhaps even more importantly, the expressive power of nonmonotonic logics N and NT comes at no additional cost in terms of computational complexity, In this paper we study computational properties of the logics N and NT and their nonmonotonic counterparts. It turns out that the logics N and NT behave similarly to logics close to S5 (such as S5 itself, KD45, S4.4 and S4.3.2). Namely, in the monotonic case, it is NP-complete to decide if a theory is consistent in N (or NT) and it is Σ_2^P-complete to decide existence of an N(NT)-expansion. Since the complexity of reasoning with autoepistemic and default logics is also located on the second level of the polynomial hierarchy [Gottlob, 1992], nonmonotonic logics N and NT are computationally equivalent to these two classic nonmonotonic systems. It is worth noting that logics "in the middle" of the spectrum, that is, those containing K but still "far away" from S5, such as K, or T, are much more complex (satisfiability is PSPACE-complete). Hence, our complexity results provide yet another justification for focusing on logics that are either very weak (subnormal) or very strong (close to S5).

Some properties which are quite easy to prove for monotonic modal logics are not at all obvious for nonmonotonic modal logics. For example, adding explicit definitions of the form $q \equiv \varphi$ does not affect the q-free fragment of the set of logical consequences of a theory. Some nonmonotonic logics, for example logic of moderately grounded expansions, do not have this property [Schwarz, 1991b]. It is rather unfortunate since it means that simply introducing a new notation can change the fragment of agent's knowledge that does not involve this new notation at all. In this paper we prove that for every (even subnormal) modal logic $\mathcal{S}$, introducing an explicit definition of the form $q \equiv \varphi$ does not affect the q-free fragments of $\mathcal{S}$-expansions.

Kripke semantics for the logics N and NT

In this section we recall the semantics introduced in [Fitting *et al.*, 1992] and [Truszczyński, 1992] for the logics N and NT.

By a *multi-relational Kripke model* (or, simply, *m-r Kripke model*) we mean a triple

$$\langle M, \{R_\varphi\}_{\varphi \in \mathcal{L}_L}, V \rangle,$$

where M is a nonempty set, commonly referred to as the set of (possible) *worlds* of a model, R_φ, where $\varphi \in \mathcal{L}_L$, is a binary relation on M, and V is a function on M, called a *valuation function*, assigning to each world $\alpha \in M$ a subset $V(\alpha)$ of propositional variables of the language. Hence, the only difference between multi-relational Kripke models and standard Kripke models is that the former have infinitely many accessibility relations while the latter have just one.

The notion of *truth* of a formula φ in a world α of a multi-relational Kripke model $\mathcal{M} = \langle M, \{R_\varphi\}_{\varphi \in \mathcal{L}_L}, V \rangle$, denoted $(\mathcal{M}, \alpha) \models \varphi$, is defined recursively on the length of a formula. The only case where the definition differs from the usual one is the case of modal operator L. If φ is of the form $L\psi$, we define $(\mathcal{M}, \alpha) \models \varphi$ if and only if for every $\beta \in M$ such that $\alpha R_\psi \beta$ we have $(\mathcal{M}, \beta) \models \psi$.

A formula φ is *valid* in a multi-relational Kripke model $\mathcal{M}$ if $(\mathcal{M}, \alpha) \models \varphi$ for every $\alpha \in M$.

Theorem 1 ([Fitting *et al.*, 1992]) *Let $I \subseteq \mathcal{L}_L$ and let $\varphi \in \mathcal{L}_L$. Then $I \vdash_{\mathrm{N}} \varphi$ if and only if φ is valid in every multi-relational Kripke model in which I is valid.*

A multi-relational Kripke model is *reflexive* if each of its accessibility relations is reflexive.

Theorem 2 ([Truszczyński, 1992]) *Let $I \subseteq \mathcal{L}_L$ and let $\varphi \in \mathcal{L}_L$. Then $I \vdash_{\mathrm{NT}} \varphi$ if and only if φ is valid in every reflexive multi-relational Kripke model in which I is valid.*

Minimal model semantics for the nonmonotonic logics N and NT

We will follow the approach of [Schwarz, 1992], where minimal model semantics was proposed for the nonmonotonic logic $\mathcal{S}$ for a wide class of *normal* modal logics. Speaking more precisely, we will consider universal S5-models as special m-r Kripke models (with the same, universal, accessibility relation for every formula) and we will adapt the notion of minimality introduced in [Schwarz, 1992] to the class of all m-r models and the class of all *reflexive* m-r models.

Let $\mathcal{M}' = \langle M', \{R'_\varphi\}_{\varphi \in \mathcal{L}_L}, V' \rangle$ and $\mathcal{M}'' = \langle M'', M'' \times M'', V'' \rangle$ be an m-r Kripke model and Kripke S5-model, respectively. Assume also that M' and M'' are disjoint. By the *concatenation* of $\mathcal{M}'$ and $\mathcal{M}''$, denoted $\mathcal{M}' \circ \mathcal{M}''$, we mean an m-r Kripke model

$$\langle M' \cup M'', \{R_\varphi\}_{\varphi \in \mathcal{L}_L}, V' \cup V'' \rangle,$$

where $R_\varphi = R'_\varphi \cup ((M' \cup M'') \times M'')$.

A Kripke S5-model $\mathcal{M}$ is N-*minimal* (NT-*minimal)* model of I, if $\mathcal{M} \models I$ and there does not exist an m-r Kripke model (reflexive m-r Kripke model) $\mathcal{M}'$ such that

1. $\mathcal{M}' \circ \mathcal{M} \models I$,
2. for some $\beta \in M'$, V'_β differs from V''_α for all $\alpha \in M''$.

Speaking informally, an S5-model $\mathcal{M}''$ is N-minimal, if it is not a common final cluster for all the accessibility relations of any $\mathcal{N}$-model with at least one world different from all the worlds of M''.

This notion of minimality may seem a bit exotic at first. In particular, it is different from the notion of a minimal universal S5-model as discussed by Halpern and Moses [1985] who consider minimality in the class of universal S5-models with respect to the inclusion relation on the sets of worlds of models. On the other hand, a careful examination of our notion of minimality shows that it is very closely related to the notion of minimality used by Moore [1984] and Levesque [1990] in their characterizations of stable expansions in the autoepistemic logic. We refer the reader to [Schwarz, 1992; Schwarz and Truszczyński, 1992] for a more detailed discussion of the minimal knowledge paradigm and comparisons between existing approaches. Similar intuitions behind the notions of minimality studied here will be provided in the full paper.

Theorem 3 *For every theory $I \subseteq \mathcal{L}_L$ and for every consistent theory $T \subseteq \mathcal{L}_L$, T is an* N-*expansion for I if and only if T is the set of all formulas valid in some* N-*minimal model for I.*

Theorem 4 *For every theory $I \subseteq \mathcal{L}_L$ and for every consistent theory $T \subseteq \mathcal{L}_L$, T is an* NT-*expansion for I if and only if T is the set of all formulas valid in some* NT-*minimal model for I.*

These two results show that the semantic notion of N- (NT-) minimal models is in the exact correspondence with the syntactic notion of an N- (NT-) expansion as specified by the equation (1).

Expressive power of nonmonotonic logics N and NT

In the monotonic setting the logics N and NT are very weak. In fact, the logic N is the weakest modal nonmonotonic logic containing necessitation. Despite of that, nonmonotonic logics N and NT are powerful nonmonotonic formalisms. It is known that default logic can be embedded into the nonmonotonic logics N and NT so that there is a *one-to-one* correspondence between extensions of default theories and N- and NT-expansions of the corresponding modal theories [Truszczyński, 1991b; Truszczyński, 1991a]. The main goal of this section is to show that also the autoepistemic logic can be embedded into the nonmonotonic modal logics N and NT.

The autoepistemic logic represents the modality L as the belief modality. The interpretation that logics N and NT give to the operator L is more that of knowledge than belief. Having a formula $L\varphi$ in the set of consequences of a theory I (in either of these logics) means that we are able to provide a very rigorous proof of $L\varphi$ from our initial assumptions I. In the case of logic N, in such a proof we only use propositional calculus and necessitation. Hence, in particular, if I is modal-free, the only way to have a formula $L\varphi$ in the set of consequences of I is to have φ among the propositional consequences of I. In the case of the logic NT the situation is similar. The only difference is that we are also allowed to make use of the axiom schema T ($Lp \supset p$) which seems to be an uncontroversial property of knowledge modality.

For a formula φ, by φ^N we denote the result of simultaneously replacing each occurrence of L in φ by $\neg L \neg L$. For a theory I, we define $I^N = \{\varphi^N : \varphi \in I\}$.

The following theorem shows that under the mapping $I \mapsto I^N$, stable expansions for I are precisely N(NT)-expansions for I^N.

Theorem 5 *Let $T \subseteq \mathcal{L}_L$ be consistent. Let $I \subseteq \mathcal{L}_L$. Then T is an autoepistemic expansion for a theory I if and only if T is an N-expansion for I^N, and if and only if T is an NT-expansion for I^N.*

A similar correspondence between autoepistemic logic and reflexive autoepistemic logic (the nonmonotonic logic S4.4) under the same translation has been discovered earlier in [Schwarz, 1991a]

Reflexive autoepistemic logic of [Schwarz, 1991a] is faithfully embedded into Moore's logic by means of the

translation which rewrites each $L\varphi$ as $\varphi \wedge L\varphi$. Combaining these two translations, we obtain easily the translation of reflexive autoepistemic logic into nonmonotonic N, too.

Algorithmic aspects of reasoning with nonmonotonic logics N and NT

Reasoning in modal logics is often very complex. For example, it is PSPACE-complete to decide whether for a given theory I and a formula φ, $I \vdash_{S4} \varphi$ (where $\vdash_{S4}$ denotes the provability operator in the logic S4). For stronger logics such as KD45, S4.4, S4F and S5 the problem of deciding whether a formula φ is a consequence of a theory I becomes easier namely, NP-complete (assuming PSPACE does not collapse to the first level of the polynomial hierarchy). In this section, we will study the complexity of reasoning in the logics N and NT.

Theorem 6 *Problems of deciding for a given finite set I of formulas, if I is consistent with logic N, or with logic NT, are NP-complete (in the length of I). Problems of deciding, for given finite set I and formula φ, if $I \vdash_{\mathcal{S}} \varphi$, is co-NP-complete, for $\mathcal{S}$ being N or NT.*

Our complexity results have direct implications of on the complexity of reasoning in nonmonotonic logics N and NT. The results for the case of the logic N have been obtained by Gottlob [1992]. Therefore, we derive here only the results for the nonmonotonic logic NT. As the main tool in our argument we will use a syntactic characterization of NT-expansions which follows from general results given in [Shvarts, 1990; Marek *et al.*, 1991]. Adapting this characterization to the case of logic NT, we obtain complexity results for the following algorithmic problems associated with the nonmonotonic logic NT:

EXISTENCE Given a finite theory $A \subseteq \mathcal{L}_K$, decide if A has an NT-expansion;

IN-SOME Given a finite theory $A \subseteq \mathcal{L}_K$ and a formula $\varphi \in \mathcal{L}_K$, decide if φ is in some NT-expansion of A;

NOT-IN-ALL Given a finite theory $A \subseteq \mathcal{L}_K$ and a formula $\varphi \in \mathcal{L}_K$, decide if there is an NT-expansion for A not containing φ;

IN-ALL Given a finite theory $A \subseteq \mathcal{L}_K$ and a formula $\varphi \in \mathcal{L}_K$, decide if φ is in all NT-expansions of A.

Theorem 7 *Problems* EXISTENCE, IN-SOME *and* NOT-IN-ALL *are Σ_2^P-com-plete. Problem* IN-ALL *is Π_2^P-complete.*

For the case of the logic N the same complexity results have been obtained by Gottlob [1992]. Since reasoning in default and autoepistemic logics is also Σ_2^P- or Π_2^P-complete (depending on the type of question) [Gottlob, 1992], it follows that computationally our nonmonotonic logics are equivalent to these two "classic" nonmonotonic formalisms.

Explicit definitions

All standard logics are conservative with respect to adding explicit definitions. Speaking informally, naming a formula by a *new* propositional symbol does not change the set of theorems in the original language. Since logics are supposed to model general principles of reasoning and be applicable in a wide spectrum of domains, the property that the set of conclusions is (essentially) invariant under new names seems to be natural and desirable to have.

It is known (see [Schwarz and Truszczyński, 1992]) that those nonmonotonic logics in the McDermott and Doyle's family which are based on normal modal logics are conservative with respect to adding new names. That is, after a new name is introduced, each expansion of the resulting theory is a conservative extension of an expansion of the original theory. In this paper we will show, using different means than in [Schwarz and Truszczyński, 1992] (the proof given there does not carry over to the case of subnormal modal logics) that every modal nonmonotonic logic in the family of McDermott and Doyle, in particular the nonmonotonic logics N and NT, share this desirable property.

For a theory $I \subseteq \mathcal{L}_L$, we define

$$I^{q,\eta} = I \cup \{q \equiv \eta\}.$$

Let p be a propositional variable and let ψ and φ be formulas. we write $\psi(p/\varphi)$ to denote the result of substituting φ for p uniformly in ψ. Clearly, if p does not occur in ψ then $\psi(p/\varphi) = \psi$. Let q be an atom not in $\mathcal{L}_L$ and let η be a formula from $\mathcal{L}_L$. For a theory $T \subseteq \mathcal{L}_L$ define

$$T^{q,\eta} = \{\psi \in \mathcal{L}_L^q : \psi(q/\eta) \in T\},$$

where $\mathcal{L}_L^q$ denotes the extension of the language $\mathcal{L}_L$ by q.

The following theorem states that nonmonotonic N and NT admit explicit definitions.

Theorem 8 *Let $\mathcal{S}$ be a modal logic. Let $I \subseteq \mathcal{L}_L$. A theory S is an $\mathcal{S}$-expansion for $I^{q,\eta}$ if and only if $S = T^{q,\eta}$ for some $\mathcal{S}$-expansion T of I. Moreover, S is a conservative extension of T, that is, T is exactly the q-free part of S.*

Conclusions

In this paper we have studied the nonmonotonic logics that can be obtained by the method of McDermott and Doyle from two very weak modal monotonic logics: the logic N of pure necessitation and its extensions, the logic NT. We argued that despite the fact that these logics lack several of the axiom schemata that characterize properties of knowledge and belief, the resulting nonmonotonic systems are at least as suitable for representing these notions as are autoepistemic, reflexive autoepistemic and default logics.

We developed a minimal model semantics for the nonmonotonic logics N and NT. We proved that autoepistemic logics can be embedded into either of these

two logics (the same result for the default logics has been already known earlier). We showed that both our nonmonotonic logics have a desirable property of being conservative with respect to adding new names. We also showed that reasoning with monotonic logics N and NT is computationally equivalent to reasoning in propositional logic, and established the complexity of reasoning with nonmonotonic logics N and NT. Our results show that the complexity of reasoning with nonmonotonic logics N and NT is located on the second level of the polynomial hierarchy and, thus, is the same as the complexity of reasoning with autoepistemic, reflexive autoepistemic and default logics.

Finally, nonmonotonic logics N and NT are conservative with respect to adding explicit definitions. All these results indicate that nonmonotonic logics N and NT are viable and powerful nonmonotonic formalisms.

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References

Feys, R. 1965. *Modal Logics.* Louvain E. Nauwelaerts, Paris.

Fitting, M. C.; Marek, W.; and Truszczynski, M. 1992. Logic of necessitation. *Journal of Logic and Computation* 2:349–373.

Gottlob, G. 1992. Complexity results for nonmonotonic logics. *Journal of Logic and Computation* 2:397–425.

Halpern, J.Y. and Moses, Y. 1985. Towards a theory of knowledge and ignorance: preliminary report. In Apt, K., editor 1985, *Logics and Models of Concurrent Systems.* Springer-Verlag. 459 – 476.

Konolige, K. 1988. On the relation between default and autoepistemic logic. *Artificial Intelligence* 35:343–382.

Kripke, S. 1963. Semantical analysis of modal logic I: Normal modal propositional calculi. *Zeitschrift für mathematische Logic and Grundlagen der Mathematik* 67 – 96.

Kripke, S. 1965. Semantical analysis of modal logic II: Non-normal modal propositional calculi. In Addison, J.W.; Henkin, L.; and Tarski, A., editors 1965, *The Theory of Models*, volume 9. North-Holland, Amsterdam. 206 – 220.

Lenzen, W. 1978. *Recent Work in Epistemic Logic*, volume 30 of *Acta Philosophica Fennica.* North-Holland, Amsterdam.

Levesque, H. J. 1990. All I know: a study in autoepistemic logic. *Artificial Intelligence* 42:263–309.

Marek, W. and Truszczyński, M. 1990. Modal logic for default reasoning. *Annals of Mathematics and Artificial Intelligence* 1:275 – 302.

Marek, W.; Shvarts, G.F.; and Truszczyński, M. 1991. Modal nonmonotonic logics: ranges, characterization, computation. In *Second International Conference on Principles of Knowledge Representation and Reasoning, KR '91*, San Mateo, CA. Morgan Kaufmann. 395–404. An extended version of this article will appear in the Journal of the ACM.

McCarthy, J. 1980. Circumscription — a form of nonmonotonic reasoning. *Artificial Intelligence* 13:27–39.

McDermott, D. and Doyle, J. 1980. Nonmonotonic logic I. *Artificial Intelligence* 13:41–72.

McDermott, D. 1982. Nonmonotonic logic II: Nonmonotonic modal theories. *Journal of the ACM* 29:33–57.

Moore, R.C. 1984. Possible-world semantics for autoepistemic logic. In Reiter, R., editor 1984, *Proceedings of the workshop on non-monotonic reasoning.* 344–354. (Reprinted in: M.Ginsberg, editor, *Readings on nonmonotonic reasoning.* pages 137 – 142, 1990, Morgan Kaufmann.).

Moore, R.C. 1985. Semantical considerations on nonmonotonic logic. *Artificial Intelligence* 25:75–94.

Reiter, R. 1980. A logic for default reasoning. *Artificial Intelligence* 13:81–132.

Schwarz, G.F. and Truszczyński, M. 1992. Modal logic **S4F** and the minimal knowledge paradigm. In *Proceedings of TARK 1992*, San Matco, CA. Morgan Kaufmann.

Schwarz, G. 1991a. Autoepistemic logic of knowledge. In Nerode, A.; Marek, W.; and Subrahmanian, V.S., editors 1991a, *Logic Programming and Non-monotonic Reasoning.* MIT Press. 260–274.

Schwarz, G.F. 1991b. Bounding introspection in nonmonotonic logics. In *Third International Conference on Principles of Knowledge Representation and Reasoning, KR '92*, Cambridge, MA.

Schwarz, G.F. 1992. Minimal model semantics for nonmonotonic modal logics. In *Proceedings of LICS-92.*

Segerberg, K. 1971. *An essay in classical modal logic.* Uppsala University, Filosofiska Studier, 13.

Shvarts, G.F. 1990. Autoepistemic modal logics. In Parikh, R., editor 1990, *Proceedings of TARK 1990*, San Mateo, CA. Morgan Kaufmann. 97–109.

Truszczyński, M. 1991a. Embedding default logic into modal nonmonotoninc logics. In Nerode, A.; Marek, W.; and Subrahmanian, V.S., editors 1991a, *Logic Programming and Non-monotonic Reasoning.* MIT Press. 151–165.

Truszczyński, M. 1991b. Modal interpretations of default logic. In *Proceedings of IJCAI-91*, San Mateo, CA. Morgan Kaufmann. 393–398.

Truszczyński, M. 1992. A generalization of Kripke semantics to the case of logics without axiom K. A manuscript.