The P-Systems: A Systematic Classification of Logics of Nonmonotonicity

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Abstract

In the last years many logics of nonmonotonicity have been developed using various different formalisms and axiomatizations which makes them very difficult to compare. We develop a classification scheme for these logics using only a few simple concepts and axioms based on conditional logics, properties of partial preorders of possible states (worlds) and centering assumptions. Our framework (the *P-Systems*) allows us to discuss the similarities, main differences and possible extensions of these logics in a simple and natural way.

1 Introduction

A large number of papers on semantic and axiomatic characterizations of logics of nonmonotonicity has appeared in recent years, leaving the reader sometimes with the impression of very interesting but in many cases incomparable approaches. Even when comparisons are made, authors tend to focus more on the differences between these logics than their similarities, treat similarities as surprising results or are simply not aware of similarities to related work.

The goal of this paper is to provide a common semantic and axiomatic framework for this work and to focus on the similarities of the various approaches. The result is a rather simple framework (the *P-Systems*) which nevertheless serves well to characterize the principles of various logics and to understand their similarities and main differences. Depending on the constraints used in formulating such a logic, these logics of nonmonotonicity can be used to answer *what-if* questions ("If kangaroos had no tails, would they topple over?"), to formalize defaults ("Normally kangaroos have tails."), to incorporate inconsistent updates ("Kangaroos have no tails.") and other.

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We will classify the various kinds of logics of nonmonotonicity using constraints on the relation between different possible situations and centering assumptions, leading to a unified view of a large number of seemingly different formalisms and logics. Our work extends formalizations based on conditional logic discussed in [17] and [4] and several results scattered through the literature in various formalisms. Among others, we discuss the relationship between operators and principles used for belief revision ([12], [25], [6]), conditional logics of normality and modal logics ([9], [10], [3], conditional logics for counterfactuals ([22], [17], [20], [14]), nonmonotonic consequence relations ([15]) and general conditional logics of nonmonotonicity ([1]).

Due to space constraints equivalence proofs are omitted. They can be found in a longer version of this paper ([18]).

2 Conditional Logics

2.1 Preliminaries

Conditional logics have been used in various disguises to formalize nonmonotonic reasoning. The underlying idea is that a conditional $a \Rightarrow b$ is true iff $a \wedge \neg b$ is either impossible or at least a *less plausible* possibility, in some sense, than $a \wedge b$. This allows us to formalize *what-if* statements (counterfactuals), as well as statements of normality (defaults). We will use $\Rightarrow$ to denote the conditional implication, $\supset$ to denote material implication, and $\equiv$ to denote logical equivalence.

The first work on this subject has been [22] and [17] who formalized the notion of counterfactuals. Various conditional logics have been analyzed in later papers and compared to each other (e.g. [4]). Starting with the paper of [13] the use of counterfactuals and conditional logics has been discussed also for artificial intelligence applications with a lot of work appearing in the last years.

For sake of simplicity we discuss only the case of propositional logics, although some of the logics have been extended to first-order. Additionally, we concentrate on theorems containing only unnested occurrences of $\Rightarrow$ as most logics can only handle this case.

We neglect small differences between two logics if they are only caused by the different formalisms used although the logics share the same model-theoretic principles.[1] We think this approach is justified in order to stress the common properties of these logics which have not been obvious in many recent papers.

2.2 Normal Conditional Logics

We start by describing a very basic conditional logic called CK which has been axiomatized in [5]. Although all our remaining logics are more restricted than CK, CK is useful to characterize the common properties of the logics of nonmonotonicity discussed in section 3. All logic systems we discuss obey additional constraints and are formalized simply by extending the axioms valid in CK.

For CK we consider a semantic model $\mathcal{M} = \langle W, f, P\rangle$, where f is a mapping that selects a set of worlds $f(w, A)$ for each world $w \in W$ and proposition $A \in P$. For such a wff A, $||A||^{\mathcal{M}}$ stands for the set of worlds in $\mathcal{M}$ in which A is true. To evaluate a conditional $A \Rightarrow B$ at a world w in a model $\mathcal{M}$ we use

$$\models_w^{\mathcal{M}} A \Rightarrow B \text{ iff } f(w, A) \subseteq ||B||^{\mathcal{M}}$$

We do not include any constraints on $f(w, A)$, except that it only selects worlds in which A is true, i.e.

$$f(w, A) \subseteq ||A||^{\mathcal{M}}$$

Assuming that this definition of f defines a notion of plausibility and selects the ***most plausible*** worlds in which A is true, we may paraphrase it by

> $A \Rightarrow B$ is true at w if B holds in all most plausible A-worlds for w as selected by f.

This very basic logic CK can be defined by the following two basic inference rules:

$$(RCEA)\ \frac{A \equiv B}{(A \Rightarrow C) \equiv (B \Rightarrow C)}$$

$$(RCK)\ \frac{(B_1 \wedge \cdots \wedge B_n) \supset C}{((A \Rightarrow B_1) \wedge \cdots \wedge (A \Rightarrow B_n)) \supset (A \Rightarrow C)}$$

Similar rules have been used in axiomatizations of other conditional logics compared in this paper. They can be derived in CK:

$$(RCEC)\ \frac{B \equiv C}{(A \Rightarrow B) \equiv (A \Rightarrow C)}$$

$$(RCM)\ \frac{B \supset C}{(A \Rightarrow B) \supset (A \Rightarrow C)}$$

$$(RCR)\ \frac{(B \wedge C) \Rightarrow D}{((A \Rightarrow B) \wedge (A \Rightarrow C)) \supset (A \Rightarrow D)}$$

In [15], $RCEA$ corresponds to ***left logical equivalence***, RCK to ***right weakening***.

Similarly, some axioms used in various logics of nonmonotonicity are already theorems in CK.

$$\begin{array}{ll} CC & (A \Rightarrow B) \wedge (A \Rightarrow C) \supset (A \Rightarrow B \wedge C) \\ CM & (A \Rightarrow (B \wedge C)) \supset ((A \Rightarrow B) \wedge (A \Rightarrow C)) \end{array}$$

In [15], CC corresponds to ***and***. As all logics in [15] already contain $RCEA$ and RCK, ***and*** is redundant in their axiomatization. Logics including only RCM instead of RCK have to include CC as axiom. If we use $RCEC$ instead of RCM, we also have to include CM. In [4] CC corresponds to *A1* and CM to *A2*.

Additionally, the rule *Mp* (modus ponens) is valid for $\supset$, as well as all truth-functional tautologies of propositional calculus (PC).

It is possible (and in some cases useful) to define conditional logics which do not satisfy RCK (e.g. the logic from [11], as axiomatized as the logic G in [19]). CC and CM are not theorems of G, and the plausibility relation is dependent on both antecedent and consequent of the conditional $A \Rightarrow B$. Although this is an interesting possibility, we will not discuss such logics in our paper.

3 The P-Systems

3.1 System_P

The previous section has left open the specification of the selection function f and thus the notion of how to determine the plausible worlds. Building on [4], we proceed by introducing a ternary ***plausibility*** relation defining a partial pre-order on those worlds with respect to the current world.

We understand under a semantic model $\mathcal{M}$ the set of all pairs $\langle W, R\rangle$, with W an nonempty set of possible states (or worlds) and R an ternary relation on it. For $x \in W$, $W_x = \{y \mid \exists z Rxyz\}$.

Each state is labeled with a propositional model describing the facts at this state. Several states can be labeled by the same model (see [15] for an example), although this will usually not be the case. The relation

[1] An example would be the difference in expressiveness between the logics described in [3] and those in [15].

$Rxyz$ may be interpreted by saying that from the point of view of x, y is at least as plausible as z.

The minimal requirements we place on the relation R are *reflexivity* and *transitivity*:

$$\forall x \in W\ \forall y \in W_x\ Rxyy$$
$$\forall x \in W\ \forall yzw \in W_x\ (Rxyz \wedge Rxzw \supset Rxyw)$$

R defines therefore a *partial pre-order* on the states in W (which is local to each $x \in W$). The selection function $f(x, A)$ can be defined as selecting the most plausible worlds according to this partial pre-order R, which is expressed by

$$f(x,A) = \begin{array}{l}\{y \in W_x \mid y \in \|A\|^{\mathcal{M}} \wedge \\ (\forall z \in W_x \cap \|A\|^{\mathcal{M}}\ Rxzy \supset Rxyz)\}\end{array}$$

Our definition from section 2.2 is then equivalent to the one used in [4], where $A \Rightarrow B$ is true with respect to a world $x \in W$ iff

$$\begin{array}{l}\forall y \in W_x \cap \|A\|^{\mathcal{M}}\ \exists z \in W_x \cap \|A\|^{\mathcal{M}} : \\ (Rxzy \wedge \forall t \in W_x \cap \|A\|^{\mathcal{M}}\ (Rxtz \supset t \in \|B\|^{\mathcal{M}}))\end{array}$$

and can be paraphrased as

> $A \Rightarrow B$ is true at x if B holds in all most plausible A-worlds for x according to R.

For simplicity's sake, we assume the limit assumption as defined in [22] and [17] for infinite numbers of worlds. This assumption basically forbids an infinite sequence of ever more plausible worlds (i.e. *well-foundedness* for partial pre-orders).[2] For a finite number of worlds this assumption is obviously valid. The smoothness condition defined in [15] is more general, but equivalent to the limit assumption in our case (R being reflexive and transitive).

To get a logic system corresponding to this definition, we have to add the following three axioms to CK. We call the resulting conditional logic *System_P*.

$$\begin{array}{ll} ID & A \Rightarrow A \\ ASC & (A \Rightarrow B) \wedge (A \Rightarrow C) \supset (A \wedge B \Rightarrow C) \\ CA & (A \Rightarrow C) \wedge (B \Rightarrow C) \supset (A \vee B \Rightarrow C) \end{array}$$

ID needs reflexivity whereas ASC needs transitivity. CA reflects the fact, that each state is labeled exactly with one model (in contrast to weaker logics defined in [15]).

System_P is equivalent to

- the minimal counterfactual logic $\mathcal{S}$ defined in [4].

[2]We will not present a constraint on R, as well-foundedness is only second-order definable.

In [15], ID corresponds to *reflexivity*, ASC to *cautious monotony* and CA to *or*. The axioms correspond to *A0, A3, A4* in [4]. In [1], CA has been named AD.

Such a logic might be suitable for reasoning about morality or obligation, where we neither assume an absolute pre-order valid for all worlds nor that our world is the most preferred one.

In the following sections we will add additional constraints to *System_P* which will be reflected in the subscripts of these logics (i.e. the name *System_*P_{MC} will be used for a logic extending *System_P* by modularity and centering axioms).

3.2 System_$\mathbf{P}_C$

We have said nothing about the actual world so far. Indeed, this is the crucial difference between conditional logics used for evaluating counterfactuals and conditional logics of normality (or obligation).

Counterfactuals are evaluated with respect to the actual world. This is represented by so-called *centering* axioms (see [17]). Assuming *weak centering* the actual world w is among the most plausible worlds for x, while under *strong centering* w alone is the most plausible world.

The difference between strong and weak centering has been analyzed in [19]. Basically, strong centering is assumed in minimal change theories, where only minimal changes for accommodating a certain fact are assumed. In contrast, weak centering is used in small change theories, which additionally allow small, non-minimal changes, if the difference to minimal changes is negligible. A good example are probability-based systems using a cutoff based on the relative difference to the most probable hypothesis (i.e. the cutoff criterion used in the model-based diagnosis system Sherlock described in [8]).

Logics of normality (like *System_*P_A, which will be discussed in section 3.3) include these centering assumptions as plausible defaults, but do not enforce them.

Strong centering is represented by the following constraint on the plausibility relation R:

$$\forall x \in W\ (x \in W_x \wedge \forall y \in W_x\ (x \neq y \supset Rxxy \wedge \neg Rxyx))$$

Weak centering is represented by the similar constraint

$$\forall x \in W\ (x \in W_x \wedge \forall y \in W_x\ Rxxy)$$

The corresponding axioms included in *System_*P_C are

$$\begin{array}{ll} MP & (A \Rightarrow B) \supset (A \supset B) \\ CS & (A \wedge B) \supset (A \Rightarrow B) \end{array}$$

where MP corresponds to weak centering, CS to strong centering.

Conditional logics for evaluating counterfactuals equivalent to $System\_P_C$ are based on the notion of minimal change. A prominent example is

- the counterfactual logic SS defined in [20].

The logic P from [15] is mistakenly compared to SS in [16]. This is not true, as P lacks the axioms for centering. Let us note, however, that the presence or absence of the centering axioms does not make any difference if we are only concerned with assertions of the form $A \Rightarrow B$.

3.3 System_P$_A$

Up to now, we have considered partial pre-orders of states with respect to specific states. We might include the assumption that the partial pre-order of states is absolute (i.e. the same with respect to all worlds), represented by the following constraint:

$$\forall xw\ (Rxyz \supset Rwyz)$$

Having an absolute plausibility measure simplifies things as we can use a global binary relation $\leq$ to express preferences between different situations. Absolute pre-orders are therefore useful for reasoning about defaults and normality.

As we consider only non-nested theorems in this paper, we do not have to add additional axioms for absoluteness (see the following section 3.3.1). Thus, the theorems of $System\_P_A$ are the same as of $System\_P$. This is a welcome result, as a binary relation is easier to handle and is also the basis for the Kripke semantics used in modal logic.

Using this connection, we can show that $System\_P_A$ corresponds to

- the minimal logic of normality $CT4$ equivalent to the modal logic $S4$ defined in [3], and to
- the logic of preferential consequence relations P defined in [15].

In $CT4$ each world w can access all worlds x which are at least as plausible as w, i.e.

$$x \leq w \supset wR_a x$$

where R_a is the accessibility relation of modal logic. This leads at once to the following definition of $A \Rightarrow B$ in terms of necessity and possibility used in [3]:

$$\Box(\Box\neg A \vee \Diamond(A \wedge \Box(A \supset B)))$$

The definition formalizes the idea, that either A is false in each plausible world, or A is true in some plausible world w and in each world at least as plausible as w, $A \supset B$ is valid. This definition is therefore equivalent to the one we used for defining the truth-value of the conditional $A \Rightarrow B$.

The following weaker definition

$$\Box\neg A \vee \Diamond(A \wedge \Box(A \supset B))$$

which is mentioned in [3] amounts to checking this formula only in the current world x. It is therefore sufficient, that in some world w_i reachable from x $(A \wedge \Box(A \supset B))$ is true, while it is false in another reachable world w_j.

So if the worlds accessible from x form two sets which are not connected, we may get both $(A \wedge \Box(A \supset B))$ and $(A \wedge \Box(A \supset \neg B))$. This is why we can get absurd results such as both $A \Rightarrow B$ and $A \Rightarrow \neg B$ are true (each being supported by another set).

3.3.1 Nested Theorems

As indicated in the last section, we do not need to add additional axioms representing absoluteness, if we only consider theorems, which do not contain nested occurrences of $\Rightarrow$. We can express this by the following theorem, which can easily be proved given the definition of $\Rightarrow$.

Theorem 3.1 All theorems containing unnested occurrences of $\Rightarrow$, which can be expressed by a relational principle on the ternary relation R can also be expressed by a relational principle on the binary global relation $\leq$.

Special nested theorems have to be expressed by *index principles* relating the view of different worlds. An example for a nested theorem depending on such an index principle can be found in [24] (formulated there for a strict partial order). The absorption law

$$(A \Rightarrow (B \Rightarrow C)) \supset ((A \wedge B) \Rightarrow C)$$

is equivalent to the index principle

$$\forall xyz\ (\neg Rxzy \supset \forall u\ Ryzu)$$

Unfortunately, this constraint is too strong to be of much use.

Nested theorems of another sort (containing $\Rightarrow$ as the primary operator) are mentioned in [3]. $CT4$ (and indeed all extensions of $System\_P$ containing no index principles) include the theorems:

$$(A \wedge (A \Rightarrow B)) \Rightarrow B$$
$$(A \Rightarrow C) \Rightarrow ((A \wedge B) \Rightarrow C)$$

The first axiom corresponds to the axiom for weak centering and basically introduces *Mp* (modus ponens) for $\Rightarrow$. It is easy to show that it is equivalent to

$$(A \Rightarrow B) \Rightarrow (A \supset B)$$

which corresponds to a default assumption stating that our world is as normal as we can possibly assume.

Strong centering can be transformed similarly into

$$(A \wedge B) \Rightarrow (A \Rightarrow B)$$

which expresses the fact, that we tend to generalize $\Rightarrow$ relations as much as possible using induction from existing facts.

The second axiom mentioned in [3] corresponds to the thesis *strengthening antecedents* valid for classical logic. Indeed, even the versions of *transitivity*

$$(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$$

and *contraposition*

$$(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$$

seem to be valid. So all these additional theses really tell us is that "normally" we use classical logic. We conclude, that nested theses of the form discussed in [3] do not seem to contain much new information, but are interesting for making these normality assumptions explicit.

3.4 System_P$_{CA}$

Assuming both centering and absoluteness, we get *System_P$_{CA}$* equivalent to

- the conditional logic C, presented as logic of nonmonotonicity in [1].

However, it seems to us, that a general logic of nonmonotonicity should not include centering axioms. These are indeed basically made useless by the construction used in [1], where theorems are defined with respect to minimal worlds. Therefore any logic corresponding to *System_P$_A$* would probably be more suitable for this purpose than C.

3.5 System_P$_{DC}$

Another possibility, which has not been explored much, is to require a *directed* partial pre-order, where any two elements have an upper bound.

Directedness can be expressed by the following constraint on R:

$$\forall x \in W\ \forall yz \in W_x\ \exists u\ (Rxyu \wedge Rxzu)$$

The corresponding axiom is

$$CV' \quad (A \Rightarrow C) \wedge \neg(A \Rightarrow \neg B) \wedge \neg(\top \Rightarrow \neg A) \supset (A \wedge B \Rightarrow C)$$

$\top \Rightarrow A$ can be read as "normally A" (as discussed in [3]). Under strong centering $\top \Rightarrow A$ is true, iff A is true in the actual world w.

If we require directedness and centering, we get logics suitable for update semantics computing results of changes to the actual world using set-theoretic minimality (which respects directedness), such as

- the update operator for reasoning about action described in [25].

3.6 System_P$_{DA}$

Directedness has been first considered in [3], where it has been combined with the constraint of absoluteness. *System_P$_{DA}$* is therefore equivalent to

- *CT4G*, a logic of normality based on *S4.2*, which is mentioned in [3].

Model-based diagnosis systems using minimality in a set-theoretic sense are special versions of the logic described by *System_P$_{DA}$*, such as

- MBD systems using the definition of diagnosis described in [7] and [21].

In this case, the "most normal" situation is the empty diagnosis (everything is correct), so the axiom CV' does not help us much, if we want to use conditionals to describe the effects of faults (which are not valid in the most normal state of affairs).

3.7 System_P$_{MC}$

A higher degree of monotony than in our previous logics follows from the assumption that the plausibility relation ranks the possible worlds in disjunct levels of plausibility. This leads to a plausibility relation, where all reachable situations are comparable, but where equally plausible situations may exist.

This corresponds to the constraint of *almost-connectedness* of the relation *Rxyz*:

$$\forall x \in W\ \forall yz \in W_x\ (Rxyz \vee Rxzy)$$

It is equivalent to the partial pre-order being *modular* (as recognized in [13]), which we can also formalize by

$$\forall xyz\ (x \leq y) \wedge (y \leq x) \wedge (z < x) \supset (z < y)$$

where $(a < b) \equiv (a \leq b) \wedge (b \not\leq a)$.

This logic is strictly stronger than $System\_P_{DC}$, as any modular pre-order is directed.

The corresponding axiom to almost-connectedness is the familiar axiom

$$CV \quad (A \Rightarrow C) \wedge \neg (A \Rightarrow \neg B) \supset (A \wedge B \Rightarrow C)$$

It is called *rational monotonicity* in [15].

If we assume almost-connectedness plus centering we get some familiar conditional logics used for counterfactuals, as well as some other rational logics. Assuming strong centering, we get

- the prototype of counterfactual logic, *VC* of [17],
- the Gärdenfors rationality axioms from [12], which are equivalent to *VC*,

and as special versions

- the counterfactual construction described in [14], and
- the update operator of [6].

If we assume only weak centering we get the logic

- *VW*, which also has been defined in [17].

$System\_P_{MC}$ is one of the most monotonic system we can get including the assumptions of almost-connectedness and centering. It corresponds to a system of plausibility spheres ordered around the actual world. We cannot add absoluteness to this system, as this would collapse the partial pre-order into one equivalence class.[3]

This is in accordance with the result in [17], that the system VCA (which corresponds to $System\_P_{MCA}$) is ordinary truth-functional logic in disguise. Similarly the monotonic logic *M* from [15] is defined by one equivalence class representing all plausible models.

3.8 System_$\mathbf{P}_{MA}$

If we combine almost-connectedness and absoluteness, we get the following *logics of normality*:

- *N* as defined in [9] and [10],
- *CT4D*, equivalent to *S4.3*, as defined in [3],
- *R*, the logic of rational consequence relations, as defined in [16].

Interestingly enough, *N* lacks the axiom *ASC* (a fact which has also been mentioned in [16] and [3]) which it should include, being based on an *S4.3*-like semantic model.

[3] We might however add the weaker condition of *universality* (where each world is reachable from each other, see [17]) to get Lewis' logic *VCU*.

3.9 System_$\mathbf{P}_{MCX}$

There is one more assumption we can add, and that is the constraint of *linearity* of the plausibility relation, which excludes ties between possible situations.

We then have

$$\forall x \in W\ \forall yz \in W_x \quad \begin{array}{l}(Rxyz \vee Rxzy) \wedge \\ (Rxyz \wedge Rxzy \supset y = z)\end{array}$$

Alternatively we might express this constraint by almost-connectedness and *antisymmetry*.

The corresponding partial order is *simple* or *total*, and is also called a *chain*:

$$\forall xy\ (x = y) \vee (x < y) \vee (y < x)$$

The corresponding axiom is

$$CEM \quad (A \Rightarrow B) \vee (A \Rightarrow \neg B)$$

validating the rule of "conditional excluded middle" well known from monotonic reasoning.

This logic is equivalent to

- the first formally defined logic for counterfactuals, *C2*, which is discussed in [22].

Slightly modifying a theorem from [23], we can show, that no more new universal constraints can be added to $System\_P_{MCX}$. The only universal additional restrictions are restrictions to finite cardinalities. In this sense $System\_P_{MCX}$ is the nonmonotonic system which retains most of monotonic inferences.

4 Conclusion and Further Work

We have shown how most logics of nonmonotonicity including many counterfactual logics, logics of normality, as well as belief revision and update semantics can be classified using a simple framework built upon normal conditional logics, properties of partial pre-orders between possible situations and centering assumptions. The resulting framework of *P-Systems* makes it much easier to compare the various logics and to understand their similarities, differences and possible extensions.

An interesting avenue for research, which has only been mentioned in this paper, is to generalize this framework by investigating the correspondence of index principles and nested theorems for $\Rightarrow$. We are also considering to extend our framework by including non-normal conditional logics weaker than *CK*.

Comparing hypothetical reasoning using intuitionistic logic (e.g. [2]) within the framework described in this paper could prove fruitful given the fact, that intuitionistic logic can be transformed into the modal logic *S4*.

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