A Skeptical Theory of Inheritance in Nonmonotonic Semantic Networks

John F. Horty
Philosophy Department
University of Maryland
College Park, MD 20742

Richmond H. Thomason
Linguistics Department
University of Pittsburgh
Pittsburgh, PA 15260

David S. Touretzky
Computer Science Department
Carnegie Mellon University
Pittsburgh, PA 15213

Abstract: This paper describes a new approach to inheritance reasoning in semantic networks allowing for multiple inheritance with exceptions. The approach leads to a definition of inheritance that is both theoretically sound and intuitively attractive: it yields unambiguous results applied to any acyclic semantic net, and these results conform to our own intuitions in the cases in which the intuitions themselves are firm and unambiguous. Since, however, the definition provided here is based on an alternative, *skeptical* view of inheritance reasoning, it does not always agree with previous definitions when it is applied to nets about which our intuitions are unsettled, or in which different reasoning strategies could naturally be expected to yield distinct results.

1. Introduction

This paper describes a new approach to inheritance reasoning in semantic networks allowing for multiple inheritance with exceptions. Like the previous approaches of [Touretzky, 1986] and [Etherington, 1987], but unlike many others, such as [Roberts and Goldstein, 1977] or [Fahlman, 1979], the approach presented here leads to a definition of inheritance which is both theoretically sound and intuitively attractive: it yields unambiguous results applied to any acyclic semantic net, and the results conform to our intuitions in the cases in which our intuitions themselves are firm and unambiguous. Since, however, the definition provided here is based on an alternative, skeptical view of inheritance reasoning, it does not always agree with these previous definitions when it is applied to nets about which intuitions are unsettled, or in which different reasoning strategies could naturally be expected to yield distinct results.

We do not attempt in this paper to provide any systematic comparison of our approach to nonmonotonic inheritance either with those of [Touretzky, 1986] and [Etherington, 1987], or with other similar approaches to nonmonotonic reasoning. This project of comparison and evaluation is begun in [Touretzky *et. al*, 1987a] and [Touretzky *et al.*, 1987b], where we set out a partial design space for the classification of inheritance systems and investigate the consequences of various design decisions. However, we will note here that while the *credulous* reasoners of Touretzky and Etherington may produce an exponential number of extensions from a single network, the kind of skeptical reasoner we describe always produce a unique extension. Skepticism may therefore prove to be more practical in some applications.

2. Notation

Letters from the beginning of the alphabet (a, b, c) will represent *objects*, and letters from the middle of the alphabet (p, q, r) will represent *kinds* of objects. We use letters from the end of the alphabet (u, v, w, x, y, z) to range over both objects and kinds.

An *assertion* will have the form $x \rightarrow y$ or $x \not\rightarrow y$, where y is a kind. If x is an object, such an assertion should be interpreted as an ordinary atomic statement: $a \rightarrow p$ and $b \not\rightarrow p$, for instance, are analogous to Pa and $\sim Pb$ in logic; they might represent statements like 'Tweety is a bird' and 'Jumbo isn't a bird'. If x is a kind, these assertions should be interpreted as *generic* statements: $p \rightarrow q$ and $r \not\rightarrow q$, for example, might represent the statements 'Birds fly' and 'Mammals don't fly'. There is nothing in ordinary logic very close in meaning to generic statements like these, since they can be true even in the presence of exceptions. In particular, 'Birds fly' can't be interpreted to mean $\forall x[Px \supset Qx]$, and 'Mammals don't fly' doesn't mean anything like $\forall x[Rx \supset \sim Qx]$; for detailed argumentation on this point, with supporting linguistic evidence, see [Carlson, 1982].

Capital Greek letters will represents *nets*, where a net consists of a set I of individuals and a set K of kinds, together with a set of positive links and a set of negative links, both subsets of $(I \times K) \cup (K \times K)$. We identify the positive and negative links in a net with our positive and negative assertions.

Lower case Greek letters will range over sequences of links, among which we single out for special consideration the *paths*, defined inductively as follows: each assertion is a path; and if $\sigma \rightarrow p$ is a path, then both $\sigma \rightarrow p \rightarrow q$ and $\sigma \rightarrow p \not\rightarrow q$ are paths. As this notation indicates, paths are special kinds of link sequences—joined, in the sense that the end node of any link in a path is identical with the initial node of the next link. It follows from their definition that paths are subject also to two further constraints. First, a negative link can occur in a path, if at all, only at the very end: $a \rightarrow p \not\rightarrow q$ is a path, but $a \not\rightarrow p \rightarrow q$ isn't. Second, an individual can occur only as the initial node of a path: $p \rightarrow a \not\rightarrow q$ isn't a path.

Paths will be said to *enable* assertions, or statements, much in the way that proofs enable their conclusions: a path of the form $x \rightarrow \sigma \rightarrow y$ is said to enable the assertion $x \rightarrow y$, and likewise, a path of the form $x \rightarrow \sigma \not\rightarrow y$ is said to enable the assertion $x \not\rightarrow y$. As this suggests, it is often natural to understand a path—like a proof—as representing a particular chain of reasoning behind the assertion it enables. The path $a \rightarrow p \rightarrow q$, for example, might enable the assertion 'Tweety flies', while representing an argument like "Tweety flies because he is a bird and birds fly."

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3. Inheritance

Since we identify the links in a net with assertions, a net can be viewed as a set of hypotheses, or axioms. Let us say that an assertion A is ***supported*** by a net Γ if we can reasonably conclude that A is true whenever all the links in Γ are true—if the information contained in Γ would naturally lead to the conclusion that A. We want to know what we can conclude from a given net; so our object is to define the general conditions under which a net Γ supports an assertion A.

In the context of ordinary deductive logic, we often find ourselves in a similar situation, when we want to know what statements are ***deducible*** from a given set of hypotheses. There, it is a common practice to approach the question in a roundabout way. Instead of defining the relation of deducibility directly, one first characterizes the deductions—sequences of statements representing certain kinds of arguments, or chains of reasoning—and then defines a statement as deducible from a set of hypotheses if those hypotheses permit a deduction of that statement.

Of course, the process of drawing conclusions from a set of hypotheses through inheritance reasoning is quite different from the process of drawing conclusions through deduction. Still, we find it helpful in the case of inheritance to follow a similar kind of roundabout strategy in describing the consequences of a set of hypotheses. Instead of trying to specify directly the statements supported by a given net, we first characterize the arguments or chains of reasoning—represented, now, by paths—that are ***permitted*** by a net. As in the case of ordinary deducibility, this relation between sets of hypotheses and the chains of reasoning they permit is really the central idea; and it will be the primary focus of our attention. Once we have identified the paths that a net permits, it is natural to describe the statements supported by a net by stipulating that a net ***supports*** a statement just in case it ***permits*** a path ***enabling*** that statement.

4. Motivation

In this section we examine several simple examples of nets and the paths they should permit, in order to illustrate the principles underlying our general characterization of the permission relation, which is then presented in Section 5.

Consider, first, the simplest kind of case imaginable, a linear net Γ_1 (Figure 1). Just to fix an interpretation, let a = Tweety, p = Canaries, q = Birds, and r = Flying Things. Γ_1 explicitly contains the information, then, that Tweety is a canary, that canaries are birds, and that birds fly. Now given just this information, we would certainly want to allow a chain of reasoning along the lines of "Since Tweety is a canary, a kind of bird, and birds fly, Tweety flies"—so we want the net Γ_1 to permit the compound path $a \rightarrow p \rightarrow q \rightarrow r$, representing this argument. In just the same way, we want the net Γ_2 (Figure 2), with b = Jumbo, s = Royal Elephants, t = Elephants, and u = Flying Things, to permit the path $b \rightarrow s \rightarrow t \not\rightarrow u$, which represents an argument something like "Jumbo is a royal elephant, a kind of elephant, and elephants don't fly; so Jumbo doesn't fly."

These examples illustrate some of the compound reasoning paths that can be constructed by assembling the direct links contained in a net, but they don't yet tell us, when we think of the construction as proceeding inductively, ***how*** these paths are to be assembled. There are, of course, two natural options for assembling compound paths from direct links: roughly, top-down and bottom-up. Most treatments of inheritance reasoning, including that of [Touretzky, 1986], presume the top-down approach. They are guided, more or less explicitly, by a picture of inheritance according to which properties are imagined to flow downward through the semantic net, from more general to more specific kinds and then finally to individuals, unless the flow is interrupted, somehow, by an exception. Formally, this "property flow" picture leads to the construction of compound paths through the process of ***backward chaining***, according to which, at the inductive step, a compound path of the form $x \rightarrow y \rightarrow \sigma$ is assembled by adding the direct link $x \rightarrow y$ to the path $y \rightarrow \sigma$.

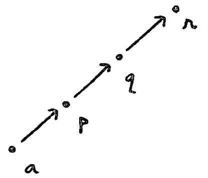

Figure 1: Γ_1

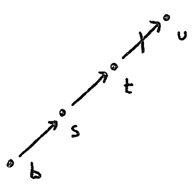

Figure 2: Γ_2.

The present treatment, on the other hand, is intended to capture a kind of bottom-up approach to inheritance reasoning. This approach seems especially natural when one wants to push the analogy, as we do, between paths and arguments—since arguments, at least as they are usually represented (say, by proof sequences), tend to move from the beginning forward. Formally, the bottom-up approach leads to the construction of compound paths through the process of ***forward chaining***: at the inductive step, the compound path $\sigma \rightarrow x \rightarrow y$ is assembled by adding the direct link $x \rightarrow y$ to the path $\sigma \rightarrow x$; and likewise, the compound path $\sigma \rightarrow x \not\rightarrow y$ is assembled by adding the direct link $x \not\rightarrow y$ to the path $\sigma \rightarrow x$. This adherence to forward chaining is one of the central principles guiding our approach. Not only does it embody a different metaphor for inheritance reasoning ("argument construction" instead of "property flow"), but it leads also to different technical results, as illustrated by our discussion of the net Γ_7 in Section 6, below.

In our approach, then, compound permitted paths are assembled through forward chaining, but of course, not every path constructible through forward chaining from the materials in a given net should be permitted by that net. Conflicts can interfere, as in the net Γ_3 (Figure 3). This net has come to be known as the Nixon Diamond, because of the interpretation under which a = Nixon, q = Quakers, r = Republicans, and p = Pacifists. What Γ_3 tells us explicitly, under this interpretation, is that Nixon is both a Quaker and a Republican, that Quakers are pacifists, and that Republicans are not pacifists. Unrestricted forward chaining would allow us to construct from this information both the paths $a \rightarrow q \rightarrow p$ and $a \rightarrow r \not\rightarrow p$. But since these two paths conflict, enabling the contradictory statements $a \rightarrow p$ and $a \not\rightarrow p$, we don't want Γ_3 to permit both these paths at once. Given just the information contained in Γ_3, we wouldn't want to conclude both that Nixon is a pacifist and that he isn't.

What you say about inheritance depends crucially on your treatment of nets, like this Nixon Diamond, which contain compound conflicting paths. One option is to suppose, although you can't permit both of two such paths, that it is always reasonable to permit one or the other. In the case of the Nixon Diamond, for example, this strategy would lead us to the conclusion that ***either*** the path $a \rightarrow q \rightarrow p$ ***or*** the path $a \rightarrow r \not\rightarrow p$ should be

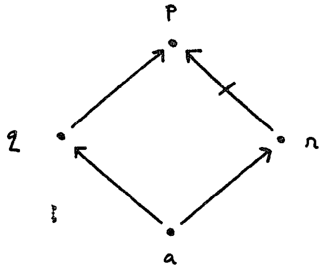

Figure 3: Γ_3

permitted. What lies behind this strategy is a kind of *credulity* or *belief-hunger*—the idea that it's best to draw as many conclusions as possible from a given net, even at the cost of making arbitrary choices among conflicting arguments. As developed in [Touretzky, 1986], this strategy involves associating with each net containing compound conflicting paths a number of consistent extensions, reminiscent of the "fixed points" of [McDermott and Doyle, 1980], or the "extensions" of [Reiter, 1980]. For this reason, because they can consistently be associated with a number of different extensions, nets like these are often described as "ambiguous."

We take a different point of view. Rather than supposing that an inheritance reasoner should try to conclude as much as possible from a given net, we adopt a broadly *skeptical* attitude, according to which conflicting arguments tend to neutralize each other. We begin with the idea, which will have to be explained in more detail, that *a compound argument is neutralized by any conflicting argument which is not itself preempted.* Given just the information in the Nixon Diamond, for example, our inheritance reasoner won't conclude either that Nixon is a pacifist or that he isn't. It won't conclude that he is a pacifist, since the information contained in the net provides the materials for constructing an argument to the contrary; it won't conclude that he isn't a pacifist, since the net also provides the materials for constructing an argument that he is.

Although our approach is based, generally, on the skeptical idea that such paths tend to neutralize each other, the special brand of skepticism we adopt here is restricted in two ways. First, we suppose that only *compound* paths can be neutralized; and second, that paths can be neutralized only by conflicting paths which are not themselves *preempted.* Both of these restrictions are important; we examine them in turn.

As an example of a net containing non-compound conflicting paths, consider Γ_4 (Figure 4). (Again, take a = Nixon and p = Pacifists.) According to the definition we provide, Γ_4 will permit *both* the conflicting paths $a \rightarrow p$ and $a \not\rightarrow p$: our reasoner will conclude from Γ_4 both that Nixon is a pacifist and that he isn't. This may seem odd, especially in light of our cautious, skeptical approach to Γ_3. It may appear, from a certain point of view, that Γ_4 presents us with nothing but a limiting case of the phenomenon found in Γ_3—so that consistency of principle should lead us to conclude, if Γ_3 doesn't permit either the path $a \rightarrow q \rightarrow p$ or the path $a \rightarrow r \not\rightarrow p$, that Γ_4, likewise, shouldn't permit either of the paths $a \rightarrow p$ or $a \not\rightarrow p$. But it is also possible to isolate a point of view from which our different treatment of the conflicting paths in Γ_3 and Γ_4 seems just right.

Remember, we are talking about the design of an inheritance reasoner, a mechanism for drawing conclusions from a certain kind of database—a set of statements that can be represented as the set of links in a net. Now when we think of the net Γ_3 as a database, it is, of course, consistent: in fact, under the Nixon interpretation, all of the statements contained in Γ_3 are true. Obviously, no one would want a reasoning mechanism to draw inconsistent conclusions from consistent information; so it follows at once that Γ_3 *can't* permit both the paths $a \rightarrow q \rightarrow p$ and $a \rightarrow r \not\rightarrow p$, since these two paths enable the contradictory statements that Nixon is a pacifist ($a \rightarrow p$) and that he isn't ($a \not\rightarrow p$). On the other hand, when you look at Γ_4 as a database, it already *contains* both of these statements; so in this case, we are faced with the problem of drawing the appropriate conclusions from information that is already inconsistent.

This is a notoriously difficult problem, but we find that it is both possible and useful to adopt in the context of *inheritance* reasoning a proposal that was originally formulated, in [Belnap, 1977a] and [Belnap, 1977b], as a guide for *deductive* reasoning in the presence of inconsistency. As a general principle, then, we propose that a reasoner ought to be able to conclude from a set of statements every statement actually contained in that set, at least—even if the set is inconsistent. It follows, of course, that if our inheritance reasoner were actually provided with the information contained in Γ_4—that Nixon both is and isn't a pacifist—then it ought to *conclude* from this information both that Nixon is a pacifist and that he isn't. Thinking of deductive reasoning, Belnap argues that the presence of inconsistent information shouldn't enable a mechanical reasoner to derive arbitrary conclusions, as it would in the case of a theorem prover using classical logic. We have shown in [Thomason *et al.*, 1986], however, that this much of the motivation behind relevance logic is already built into inheritance reasoning, even in the simple case of monotonic inheritance. Thus, the reasoner we describe here will conclude from Γ_4 both that Nixon is a pacifist and that he isn't, but it won't then go on to draw irrelevant conclusions from this contradiction: it won't conclude, for instance, that Nixon is a Democrat.

The second restriction on our broadly skeptical outlook is the idea that even compound arguments are neutralized only by those conflicting arguments that are not themselves preempted. This idea—that certain compound arguments can be, as we say, *preempted* by others—really lies at the heart of our approach, allowing us to transform a simplistic and dogmatic skepticism into something much more interesting.

Again, we begin with an example, the net Γ_5 (Figure 5). This net results from adding the link $p \not\rightarrow r$ to Γ_1, and the interpretations of these two nets will overlap as well. Just as before, we take a = Tweety, q = Birds, and r = Flying Things; but now let's shift the earlier interpretation so that p = Penguins, giving some plausibility to the new link $p \not\rightarrow r$. If things are like this, what should we conclude about Tweety: does he fly or not? Well, there are two paths to consider: $a \rightarrow p \rightarrow q \rightarrow r$, which enables the conclusion that Tweety flies, and $a \rightarrow p \not\rightarrow r$, which enables the opposite conclusion. Since both of these paths are compound, and they enable conflicting conclusions, simple skepticism would bar us from reaching any conclusion at all. But evidently, in this case, we *should* reach a conclusion: we should conclude, in fact, that Tweety doesn't fly—since he is a penguin, and penguins don't fly. The reason we are able to conclude here that Tweety doesn't fly—even though he is a bird, and birds fly—is that penguins happen to be a specific kind of bird, so that, in case of conflicts, the information we have about Tweety in virtue of his being a penguin should override whatever we would otherwise suppose to be true of him simply because he is a bird.

This illustrates the central intuition behind preemption: *that*

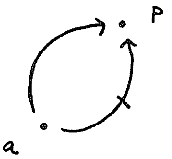

Figure 4: Γ_4

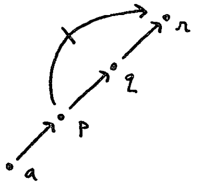

Figure 5: Γ_5

information about specific kinds should be allowed to override information about more general kinds. As we define it, a path will be preempted in a net, roughly, when the net provides the materials for constructing a conflicting argument based on more specific information. In the case of Γ_5, for example, we will want to say that the path $a \to p \to q \to r$ (telling us that Tweety flies because he is a bird) is preempted, since: (i) the net permits the path $a \to p$ (telling us that Tweety is a penguin), (ii) p's are a specific kind of q (penguins are a specific kind of bird), and (iii) the net contains the direct link $p \not\to r$ (telling us directly that penguins don't fly). Focusing on (ii), it is easy to see in terms of the net topology that what makes p more specific than q, according to Γ_5, is simply the fact that this net permits a path (a direct link, in this case) from p to q. So, restating in a way that combines (i) and (ii), we can say that the path $a \to p \to q \to r$ is preempted in Γ_5 precisely because there is a certain kind, p (penguins), such that Γ_5 both permits the path $a \to p \to q$ (telling us that Tweety is a penguin *and* that penguins are a specific kind of bird) and contains the direct link $p \not\to r$.

In this form, the idea of preemption can easily be generalized to apply to arbitrary nets and paths. We will say that a path of the form $x \to \tau \to v \to y$ (telling us that x's, as v's, are y's) is preempted in a net Γ just in case there is a node z $(z \neq v)$ such that Γ both permits a path of the form $x \to \tau_1 \to z \to \tau_2 \to v$ (telling us that x's are z's, a more specific kind of v's) and contains the link $z \not\to y$ (telling us that z's, in particular, are not y's). With exact symmetry, we will say also that a path of the form $x \to \tau \to v \not\to y$ is preempted in Γ if there is a node z $(z \neq v)$ such that Γ both permits a path of the form $x \to \tau_1 \to z \to \tau_2 \to v$ and contains the link $z \to y$.

5. The definition

Let's use the symbol '$\models$' to stand for the permission relation, so that '$\Gamma \models \sigma$' means that the net Γ permits the path σ. We have now considered the central principles underlying our approach to this idea—*forward chaining*, along with a certain kind of *restricted skepticism*. It remains only to organize these principles into a rigorous definition. Our adoption of forward chaining suggests that a bottom-up, inductive definition should be possible. In order to frame such a definition, however, we need to be able to associate with each path σ some measure of its "complexity" in a given net Γ, in such a way that it can be decided whether $\Gamma \models \sigma$ once it is known whether $\Gamma \models \sigma'$ for each path σ' less complex in Γ than σ itself.

The natural thing to think is that we might be able to identify the complexity of a path, in this sense, with its length—but this won't work, since shorter paths can be neutralized by longer, conflicting paths. To see what will work, we first introduce an auxiliary idea. As we recall from Section 2, a path is a joined sequence of links containing a negative link, if at all, only at the very end. Let's say, now, that a *generalized path* is a sequence of links joined like an ordinary path, except that it can contain negative links anywhere, and perhaps more than one. Formally, we can catch this idea by specifying that each assertion is a generalized path, and that, if σ is a generalized path, then both $\sigma \to y$ and $\sigma \not\to y$ are generalized paths. (It follows, of course, that the generalized paths include the ordinary paths.) Using this auxiliary concept of a generalized path, we now define the *degree of a path σ in a net* Γ—written, $\deg_\Gamma(\sigma)$—as the length of the longest generalized path in Γ from the initial node of σ to its end node.

As it turns out, this idea of degree provides just the right notion of path "complexity" for an inductive definition of $\models$, the permission relation between nets and paths: it can be decided whether $\Gamma \models \sigma$ entirely on the basis of information regarding paths whose degree in Γ is less than that of σ, along with information about the direct links contained in Γ itself. On the other hand, in order to assure that $\deg_\Gamma(\sigma)$ should always be well-defined, we need to restrict our attention to nets which are *acyclic*, in the sense that they contain no generalized paths whose initial nodes are identical with their end nodes. (This is a common restriction; much of the analysis in [Touretzky, 1986], for instance, also applies only to acyclic nets.) Given this idea of degree, then, and restricting ourselves to acyclic nets, we can now present our definition of the permission relation.

Although the definition is inductive at heart, it has the overall structure of a definition by cases: it deals separately with compound paths and direct links (non-compound paths). Only in the case of compound paths is there any need to resort to induction; direct links can be handled all at once, as follows.

Case I: σ is a direct link. Then $\Gamma \models \sigma$ iff $\sigma \in \Gamma$.

It is important to note that even if σ is a direct link, it could easily turn out that $\deg_\Gamma(\sigma) > 1$, since Γ might contain a compound generalized path from the initial node of σ to its end node. On the other hand, if $\deg_\Gamma(\sigma) = 1$, then the path σ has to be a direct link. Thus, in addition to taking care of all the direct links at once, whatever their degree, Case I serves also as the basis clause for the induction on degree which extends the permission relation from direct links to compound paths. The inductive clause is as follows.

Case II: σ is a compound path with, say, $\deg_\Gamma(\sigma) = n$. As an inductive hypothesis, we can suppose it is settled whether $\Gamma \models \sigma'$ whenever $\deg_\Gamma(\sigma') < n$. There are then two subcases to consider, depending on the form of σ.

1. σ is a positive path, of the form $x \to \sigma_1 \to u \to y$. Then $\Gamma \models \sigma$ iff
   - (a) $\Gamma \models x \to \sigma_1 \to u$,
   - (b) $u \to y \in \Gamma$,
   - (c) $x \not\to y \notin \Gamma$,
   - (d) For all v such that $\Gamma \models x \to \tau \to v$ with $v \not\to y \in \Gamma$, there exists z $(z \neq v)$ such that $\Gamma \models x \to \tau_1 \to z \to \tau_2 \to v$ and $z \to y \in \Gamma$.
2. σ is a negative path, of the form $x \to \sigma_1 \to u \not\to y$. Then $\Gamma \models \sigma$ iff
   - (a) $\Gamma \models x \to \sigma_1 \to u$,
   - (b) $u \not\to y \in \Gamma$,
   - (c) $x \to y \notin \Gamma$,

(d) For all v such that $\Gamma \models x \rightarrow \tau \rightarrow v$ with $v \rightarrow y \in \Gamma$, there exists z ($z \neq v$) such that $\Gamma \models x \rightarrow \tau_1 \rightarrow z \rightarrow \tau_2 \rightarrow v$ and $z \not\rightarrow y \in \Gamma$.

It should be clear that this definition of the permission relation accurately represents the general approach to inheritance reasoning described in Section 4. Case I tells us that any statement actually contained in a net should be permitted by that net. The two subcases of Case II, dealing respectively with positive and negative compound paths, are perfectly symmetric. In each subcase, the clauses (a) and (b) capture the idea of forward chaining: compound paths are permitted by a net only if they can be constructed by adding direct links from the net to initial permitted segments of those paths. The clauses (c) and (d) take care of conflicts. What (d) says is that, even if a compound path is constructible through forward chaining, it can be permitted only if each potentially conflicting compound path is preempted. Of course, only compound conflicting paths can actually be preempted, since preemption involves the intermediate nodes of path, and direct links have no intermediate nodes; but if, for skeptical reasons, we don't want a path to be permitted which conflicts with an unpreempted compound path, we certainly don't want to permit a path that conflicts with a direct link. This is the force of the clause (c).

Both the clauses (a) and (d) in the inductive step refer to other paths of a certain form permitted by the net; but this is no problem, because at any step in the induction, paths of this form will always have a degree less than that of the path being considered.

6. Some examples

This definition of the permission relation yields the advertised results applied to the nets Γ_1 through Γ_5 from Section 4. In order to highlight some of the interesting features of our definition, we consider here the paths permitted by a couple of more complicated nets.

We mentioned in Section 4 that credulous (or belief-hungry) inheritance reasoners would tend to associate with nets containing compound conflicting paths a number of different consistent extensions, or fixed points. It is tempting, therefore, to suppose that the set of paths permitted by a given net under the present skeptical analysis might simply be the ***intersection*** of the various extensions associated with that net according to the credulous analysis provided by [Touretzky, 1986]. However, nets like Γ_6 (Figure 6)—which have the topology of ***nested*** Nixon Diamonds—show that this is not so. In this case, we have $\Gamma_6 \models a \rightarrow p \not\rightarrow q$ (the potentially conflicting path $a \rightarrow s \rightarrow t \rightarrow q$ poses no problem; this path is not permitted, since its initial segment $a \rightarrow s \rightarrow t$ is itself neutralized by the path $a \rightarrow r \not\rightarrow t$). But the path $a \rightarrow p \not\rightarrow q$ isn't contained in all the Touretzky extensions associated with this net; some contain instead the path $a \rightarrow s \rightarrow t \rightarrow q$.

The net Γ_7 (Figure 7) illustrates a different feature of our definition, resulting not so much from our particular brand of skepticism as from our adherence to forward chaining. Here, we have $\Gamma_7 \models a \rightarrow p \rightarrow q \rightarrow s$. The potentially conflicting path $a \rightarrow p \rightarrow r \not\rightarrow s$ poses no problem since its compound initial segment $a \rightarrow p \rightarrow r$ conflicts with the direct link $a \not\rightarrow r$. On the other hand, though Γ_7 permits $a \rightarrow p \rightarrow q \rightarrow s$, and so supports the statement $a \rightarrow s$, the net does not permit the path $p \rightarrow q \rightarrow s$, and indeed does not support the statement $p \rightarrow s$.

This kind of situation can seem a bit anomalous if one's ideas about inheritance reasoning are conditioned by the top-down or "property flow" approach, according to which individuals are supposed to inherit their properties strictly in virtue of belonging to certain classes of things—their ancestors in the network—which possess those properties. The problem is that, while Γ_7 supports the statement that the individual a is an s, it is unclear how a could have ***inherited*** this property. After all, the only immediate ancestor of a in the network is the node p. According to the top-down approach, then, a must have inherited all the positive properties it does inherit simply in virtue of being a p; if it possess any particular property, such as being an s, this could only be due to the fact that p's possess that property. But as we have seen, Γ_7 ***doesn't*** support the statement that p's are s's.

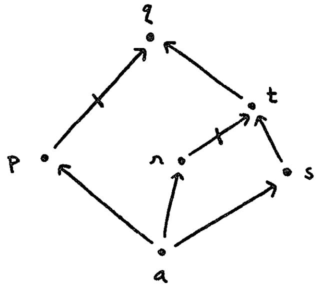

Figure 6: Γ_6

Against the background of the bottom-up or "argument construction" view of inheritance reasoning, however, the situation presented by this example is perfectly coherent. Since Γ_7 contains the materials for constructing unpreempted, compound arguments enabling both the conclusion that p's are s's and the conclusion that p's are not s's, our broadly skeptical point of view forces us to withhold judgment, endorsing neither of these conclusions. The individual a, though, is a ***particular*** p for which the general kind of argument enabling the conclusion that p's are not s's is blocked: that argument depends on the information that p's are r's, but Γ_7 tells us explicitly that a is not an r. Since the general argument that p's are not s's is explicitly blocked for this particular individual, then, it cannot conflict in the case of a with the argument that p's are s's; so we conclude that a is an s.

7. Implementations

The theory described here has been implemented as a Common Lisp program. The algorithm is a line-by-line translation of the definition in Section 5, except that, for reasons of efficiency, the degree $\deg_\Gamma(\sigma)$ of each path σ in Γ is not actually computed. Instead, the program performs a topological sort on the graph and orders potential paths according to the number $T(x)$ which the topological sort assigned to the last node x of each path. It is easily shown that if $\sigma_1 = x_1 \rightarrow \tau_1 \rightarrow y_1$ and $\sigma_2 = x_2 \rightarrow \tau_2 \rightarrow y_2$ and $T(y_1) < T(y_2)$, then either $\deg_\Gamma(\sigma_1) < \deg_\Gamma(\sigma_2)$, or there is no generalized path from x_1 through y_2 to y_1. Therefore, a program whose notion of path complexity is based on topological order will always produce results in agreement with our definition. It may consider paths in a different order than a definition based on degree, but it will only do so in situations where this cannot affect the result.

In addition, we have been exploring parallel marker propagation inheritance algorithms. Purely parallel nonmonotonic

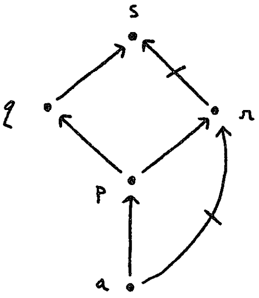

Figure 7: Γ_7

inheritance—skeptical or credulous—is not possible on a marker propagation machine due to the necessity of handling preemption. (By "purely parallel" we mean in time bounded by a constant times the depth of the graph.) However, a marker propagation machine can quickly find all relevant paths and make the uncontested inferences; it can then fall back on a serial algorithm to handle the difficult cases. We have developed a hybrid (parallel-serial) inference algorithm for answering particular queries about whether a net Γ supports statements of the form $x \rightarrow y$ or $x \not\rightarrow y$. This algorithm runs in time proportional to $D(x,y) \cdot [1 + N_C(x,y)]$, where $D(x,y)$ is the depth of the query (the length of the longest path between x and y) and $N_C(x,y)$ is the number of nodes contested with respect to the query. (A contested node is any node z on a path from x to y such that paths $x \rightarrow r_1 \rightarrow z$ and $x \rightarrow r_2 \not\rightarrow z$ exist; they need not be permitted paths.) The algorithm will be described in detail, and its correctness proved, in [Horty *et al.*, 1987], a more complete version of this paper.

8. Conclusion

We have presented in this paper a new, skeptical theory of inheritance reasoning in nonmonotonic semantic networks. As far as we know, this theory represents the first significant alternative to the analysis of nonmonotonic inheritance reasoning presented in [Touretzky, 1986]. (A less radical alternative is described in [Sandewall, 1986]; although it differs in some ways from Touretzky's, Sandewall's is nevertheless a credulous theory.) The fact that there should be distinct but, perhaps, equally well-motivated accounts of correct reasoning in this context comes as something of a surprise; it is reminiscent of the situation in philosophical logic, where there are rival logics embodying distinct conceptions of correct deductive reasoning.

In the context of inheritance reasoning, the existence of these distinct approaches has a number of theoretical consequences, which we are exploring in our current research. Much of this research is focused more or less directly on inheritance theory: we are studying the relations among the different analyses of nonmonotonic inheritance reasoning [Touretzky *et. al*, 1987a], [Touretzky *et. al.*, 1987b], and working to extend some of these analyses to more expressive nonmonotonic network languages. However, it is also possible that this research will shed some light on more general treatments of nonmonotonic reasoning. It has been shown in [Etherington, 1987], for example, that the default logic of [Reiter, 1980] can be used to provide a specification for correct inheritance reasoning in nonmonotonic semantic networks: Etherington establishes a close correspondence between these networks and certain kinds of default theories ("network default theories"). But these results, linking default logic to nonmonotonic inheritance, presuppose a *credulous* analysis of inheritance reasoning; this bias towards a credulous approach to nonmonotonic reasoning is in fact built into both Reiter's default logic and the nonmonotonic logic of [McDermott and Doyle, 1980]. Since, as we have shown, there turns out to be an equally well-motivated *skeptical* theory of nonmonotonic reasoning, at least in the case of semantic networks, it might be useful at this point to seek a weaker version of default or nonmonotonic logic, exhibiting instead a bias toward skepticism—or perhaps a more general logic that is neutral between the credulous and skeptical approaches.

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