QUALITATIVE REASONING WITH HIGHER-ORDER DERIVATIVES

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ABSTRACT

The goals of qualitative physics are to identify the distinctions and laws which govern qualitative behavior of devices such that it is possible to predict and explain the behavior of physical devices without recourse to quantitative methods. Although qualitative analysis lacks quantitative information, it predicts significant characteristics of device functioning such as feedback, ringing, oscillation, etc. This paper defines higher-order qualitative derivatives and uses them to formulate six fundamental laws which govern the gross-time behavior of physical devices. These qualitative laws are based on the Mean Value Theorem and Taylor's Expansion of the quantitative calculus. They substitute for what often requires sophisticated problem-solving. We claim they are the best that can be achieved relying on qualitative information.

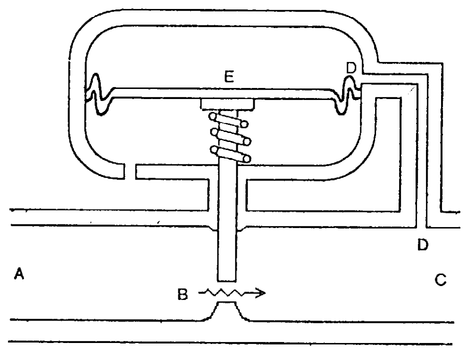

Figure 1 : Pressure Regulator

INTRODUCTION

Considerable progress has been made in qualitative reasoning about physical systems (de Kleer and Brown, 1984) (de Kleer and Brown, 1982) (Forbus, 1982) (Hayes, 1979) (Kuipers, 1982a) (Williams, 1984a) (Williams, 1984b). Description, explanation and prediction of events which occur over short time intervals is fairly well understood. However, when enough time passes the fundamental mode of behavior of the device may change. Discovering laws which govern this gross scale time behavior has proven illusive. At first sight it appears that the inherent ambiguity of qualitative analysis makes it impossible to formulate powerful laws. This is not the case, and we have identified six fundamental laws which govern the gross-time behavior of a device *which rely on qualitative information alone.*

We have built a computer program based on these laws and tested it out on many examples. For purposes of explanation, we draw all our examples from a simple fluid-mechanical pressure-regulator illustrated in Figure 1.

(de Kleer and Brown, 1982) presents a theory of causal analysis applicable for small time scales using the pressure regulator as an example. Using that theory it is possible to determine the direction of change for all device quantities, causal explanations for their change, and identification of the negative feedback. In this paper we address pressure regulator events occurring over a longer time scale. If the input pressure rises indefinitely, will the valve eventually completely close? Does the valve oscillate when a sudden input is applied?

QUALITATIVE MODELING

Qualitative calculus uses alternating qualitative values consisting of intervals separated by points (from (Williams, 1984a) (Williams, 1984b)). The points are landmark values where transitions of interest occur. The qualitative value of x is denoted $[x]$. One quantity space (Forbus, 1982) of particular interest is $[x] = +$ iff $x > 0$, $[x] = 0$ iff $x = 0$, and $[x] = -$ iff $x < 0$. (Notice that to have a landmark of k we use the qualitative space of $[x - k]$.)

The behavior of device components is described by qualitative equations. Arithmetic is straightforward, except for the case of addition of opposite signs when the result is ambiguous. The quantitative law that flow through a constriction is proportional to the pressure across it (i.e., $F = kP$) is represented qualitatively as $[F] = [P]$. k drops out as it is always positive.

The valve of the pressure regulator (Figure 1) has three operating regions each characterized by different equations. When the valve closes, the qualitative equation $[F] = [P]$ no longer holds as the area available for flow ($[A] = 0$) and flow is zero ($[F] = 0$) no matter what the pressure across the valve ($[P]$). Analogously, when the valve is completely open there is no longer any restriction to fluid flow $[F]$ but the pressure across the valve is zero ($[P] = 0$).

OPEN:$[A = A_{max}], [P] = 0$
WORKING:$[0 < A < A_{max}], [F] = [P]$
CLOSED:$[A = 0], [Q] = 0$

The other important component of the pressure-regulator is the sensor which measures the output pressure to set the size of the valve opening. The sensor acts by converting the output pressure to a force with diaphragm E, and pushing down on the spring. So increased pressure causes decreased valve area. The diaphragm moves to a position such that spring force (kx) balances the force exerted by the pressure on the diaphragm ($PA_{diaphragm}$). Thus, the distance the spring compresses (x) is proportional to the pressure (P). The area obstructed by the valve $A_{obstructed}$ is proportional to x. The area available for flow (A) is $A_{max} - A_{obstructed}$, thus qualitatively $[P] = [A_{max} - A] = [A_{max}] - [A] = [+] - [A]$. (We usually use $+$ as a qualitative value, but in ambiguous contexts such as this equation we use "$[+]$" instead.)

The above models determine the relationships between qualitative values of the pressures and flows of the pressure regulator. We are interested in developing a model to predict how a change in any one of these quantities causes changes in the others. For this we need to define a qualitative derivative of a quantity. Just as we write $[x]$ for the qualitative value of x we write $[\frac{dx}{dt}]$ for the qualitative value of $\frac{dx}{dt}$ or abbreviated, ∂x.

In the WORKING mode of the pressure regulator, the qualitative differential equations for the valve and sensor are $\partial F = \partial P + \partial A$ and $\partial A = -\partial P_{out}$. These equations are both derived from the quantitative equations relating these variables. The form of the quantitative equation is $F = A\sqrt{P}$, thus

$$\frac{dF}{dt} = \frac{dA}{dt}\sqrt{P} + \frac{A}{\sqrt{P}}\frac{dP}{dt}, P > 0.$$

As A and P are always positive, this reduces to the simpler qualitative equation

$$\partial F = \partial A + \partial P.$$

Notice that if we tried to derive the qualitative relation of the derivatives from the previously given qualitative equation relating flow and pressure ($[F] = [P]$) and differentiated, we would get $\partial F = \partial P$ which is incorrect.

By definition of pressure $\partial P = \partial P_{in} - \partial P_{out}$. Presumably the pressure-regulator delivers an output to a load which demands more flow as pressure increases, since $\partial P_{out} = \partial F$. Using these it is possible to determine the qualitative response to an input pressure rise $\partial P_{in} = +$: $\partial P_{in} = +$, $\partial P_{out} = +$, $\partial P = +$, $\partial F = +$, $\partial A = -$. This solution indicates that although the output pressure rises, the drop across the valve increases and the area available for flow decreases thereby reducing the amount of the output rise. However, the qualitative solution neither addresses how the pressure-regulator achieves this behavior nor its gross time behavior such as whether it completely closes, opens or oscillates. See (de Kleer and Brown, 1984) for a discussion of causal explanation; here we provide a framework for reasoning about its gross time behavior.

SIMULATION

As the input pressure rises, the output continues to rise and the area available for flow continues to drop. Conversely, if the pressure drops, the area increases. If enough time passes the valve may completely CLOSE ($\partial A = -$ causes A to reach threshold 0) or OPEN ($\partial A = +$ causes A to reach threshold A_{max}), the qualitative equations change and hence the behavior change. The basic simulation loop analyzes the behavior over time as follows:

(1) Start with some initial state.

(2) Solve for the qualitative changes in each quantity.

(3) Identify those quantities which are moving to their thresholds.

(4) Construct a set of the possible next states from these transitions.

(5) For each next state not yet analyzed, recursively go to (2).

This nondeterministic simulation algorithm identifies all of the states reachable from the initial state and all possible transitions between them. The device can be in each state for an interval of time. So the time-line of the device is a simply a sequence of intervals, each associated with some state.

Step 4 is expanded into the following generate and test sequence.

(4a) Construct a partial description of succeeding (continuous) states from threshold information (Rule 0); the plausible next states are generated using the qualitative integration equation for each significant[1] device quantity $[x_{next}] = [x_{current}] + \partial x_{current}$.

(4b) Generate noncontradictory states (Rule 1) which match the partial descriptions generated in step 4a.

(4c) Check all transitions from the current state to potential successors using rules 2 through 6.

We summarize the rules for generation and testing here and explain and exemplify each more extensively afterwards.

- Rule (0) *Value continuity.* Values must change continuously over a transition.
- Rule (1) *Contradiction avoidance.* The system cannot transition to a state which is inconsistent with respect to the qualitative equations.
- Rule (2) *Instant change rule.* Changes from zero happen instantaneously and no other changes can happen at an instant.
- Rule (3) *Derivative continuity.* Rule 0 also applies to derivatives.
- Rule (4) *Derivative instant change rule.* Rule 2 also applies to derivatives.
- Rule (5) *Higher-order derivatives.* Rules 0 and 2 apply to all orders of derivatives.
- Rule (6) *Change to all zero derivatives is impossible.* A quantity which is non-zero at some instant cannot ever become identically zero.

We list out rules 3 through 5 separately from 0 and 2 to give examples of different levels of analysis.

RULE(0) : VALUE CONTINUITY

We define continuity for qualitative variables. A change is continuous if the value goes from an interval to its bounding point(s), or from a point to one of its two neighboring intervals. In the quantity

[1] A significant quantity defines a component operating region or is an independent state variable.

space used here the continuous changes are between 0 and + or — (in either direction), but not between + and —.

The continuity rule is: no quantity may change discontinuously in any transition between states.

RULE(1): CONTRADICTION AVOIDANCE

Step 4 of the simulation algorithm gives a partial description of potential next states. The qualitative equations determine the values of the remaining quantities. In many cases there are no possible values which are consistent with both the qualitative equations and the partial description generated by step 4. This eliminates potential transitions generated in step 4a. This often avoids having to decide which possible transition occurs first ((Williams, 1984a) uses transition-ordering in his analysis instead).

We can use this rule to prove that the valve can't close, i.e., even though the area available for flow is decreasing it will never reach zero. If the valve is closed then $[A] = 0$; then by the valve equation $[F] = 0$. The load is passive $[F] = [P_{out}]$ so $[P_{out}] = 0$. Substituting into the sensor equation $[A] = -[P_{out}] + [+]$ we get $[0] = -[0] + [+]$ a contradiction.

In the pressure regulator it is possible to argue that the valve cannot close in the following way. Every increment in input pressure causes smaller and smaller decrements in valve area; therefore the area approaches zero asymptotically (i.e., becomes arbitrarily close to zero but never reaches zero). This asymptotic argument is unnecessary if one sees that the closed state is inconsistent. Thus, contradiction avoidance substitutes for all sorts of sophisticated reasoning.

An alternative to the simulation algorithm. the envisioning algorithm provides a computationally more elegant method of eliminating transitions to inconsistent states. As a precursor to the loop, the envisioning algorithm identifies all possible legal device states. Then step 4 only considers transitions to legal device states. Another advantage to generating all states is that when all legal transitions have been identified, one can easily notice unreachable sets of states and orphaned singlets.

QUALITATIVE AMBIGUITY

The power of the remaining rules are illustrated by examining the diaphragm-spring-stem fragment of the pressure regulator. If the input pressure increases, the output pressure increase, producing a force on the diaphragm. This force acts against the spring force and friction. The valve slowly gains velocity as it closes; however, by the time it reaches the position where the force exerted by the pressure balances the restoring force of the spring, the valve has built up a momentum causing it to move past its equilibrium position, thus reducing the pressure below what it should be. As it has overshot its equilibrium the spring pushes it back; but by the same reasoning the valve overshoots again, thereby producing ringing or oscillation. Figure 2 illustrates the essential details: a mass situated on a spring and shock absorber (i.e., friction).

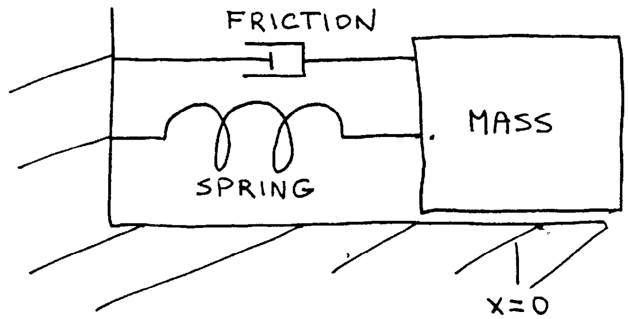

Figure 2 : Mass-Spring-Friction System

The behavior of the mass is described by Newton's Law $F = ma$ or qualitatively $[F] = \partial v$. Hooke's Law for the spring $F = -kx$ becomes $\partial F = -[v]$. The resistance of the shock absorber is modeled by $[F] = -[v]$ and $\partial F = -\partial v$. For simplicity sake, define $x = 0$ as the mass position with the spring at equilibrium, and $x > 0$ to be to the right. The net force on the mass is provided by the spring and shock absorber: $F_{mass} = F_{spring} + F_{friction}$ or qualitatively $[F_{mass}] = [F_{spring}] + [F_{friction}]$. This system of four qualitative equations has thirteen possible solutions (interpretations) (see Table 1).

	1	2	3	4	5	6	7	8	9	10	11	12	13
$[F_{mass}] =$	0	—	—	—	0	+	+	+	+	+	0	—	—
$[F_{friction}] =$	0	—	0	+	+	+	+	+	0	—	—	—	—
$[F_{spring}] =$	0	—	—	—	—	—	0	+	+	+	+	+	0
$[v] =$	0	+	0	—	—	—	—	—	0	+	+	+	+

Table 1 : Solutions to Mass-Spring Equations

	1	2	3	4	5	6	7	8	9	10	11	12	13
	1	123	1	1	1	123	123	123	1	1	1	123	123
$\partial F_{mass} =$	0	+0—	+	+	+	+0—	+0—	+0—	—	—	—	+0—	+0—
$\partial F_{friction} =$	0	+	+	+	0	—	—	—	—	—	0	+	+
$\partial F_{spring} =$	0	—	0	+	+	+	+	+	0	—	—	—	—
$\partial v =$	0	—	—	—	0	+	+	+	+	+	0	—	—

Table 2 : State Splitting By Derivatives

In many cases of qualitative reasoning, one of the interpretations is correct, the remaining are theoretically possible but unintended modes of operation (de Kleer, 1984). However, the mass-spring system oscillates by moving between these interpretations. Movement between interpretations is governed by the derivatives of the quantities which are determined by the equations: $\partial v = [F_{mass}]$, $\partial F_{spring} = -[v]$, $\partial F_{friction} = -\partial v = -[F_{mass}]$, and $\partial F_{mass} = \partial F_{friction} + \partial F_{spring} = [F_{mass}] - [v]$. Table 2 gives the values of the derivatives. Note that the derivatives themselves are sometimes ambiguous.

Table 2 illustrates how much work we get from the contradiction avoidance rule. For example, state 6's derivative equations have three interpretations which we notate 6–1, 6–2, and 6–3. Derivative interpretations are only ambiguous in ∂F_{mass}, so state 6–3 refers to the state in which $\partial F_{mass} = -$. In state 6–3, every quantity is approaching its zero threshold, since $[x] = -\partial x$ for all quantities. As we have no information about which can happen first, or happen together, all possible combinations of transitions need to be considered. As there are 4 possible transitions, there are $2^4 - 1$ possible choices. Only 3 of those 15 possibilities are realizable because 12 of the resulting states are contradictory.

This simple rule eliminates the need for more sophisticated rules often used for transition ordering. For example, (Williams, 1984a) (Williams, 1984b) uses the rule: if x and y are heading for a threshold, and $x = f(y)$ holds at the threshold as well, transitions in x and y co-occur. All applications of this specialized rule as well as many others are covered by the contradiction avoidance rule.

Figure 3 illustrates some of the possible states and all state transitions generated by the algorithm using just rules 0 and 1. As we don't have any information about the 2nd order derivatives, we first assume all transitions between first order solutions are possible.

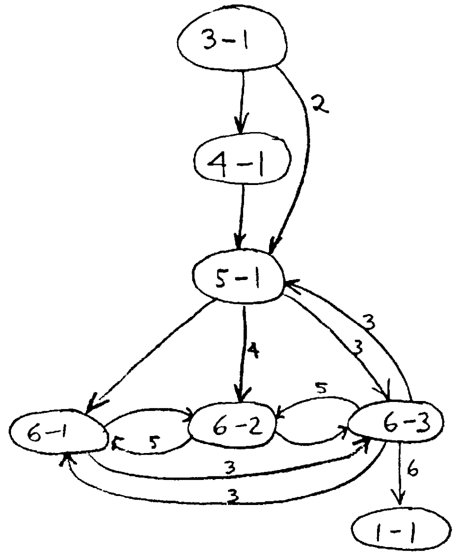

Figure 3 : State Transitions of the Spring-Mass System

After applying contradiction avoidance rule, there are still a large number of impossible transitions shown in this graph that can be eliminated by rules 2 through 6. Each numbered arc is impossible, the number indicates which rule eliminates it.

RULE(2): INSTANT CHANGE RULE

In state 3, the mass is not moving, but the force of the spring is pulling to the left. Thus, the mass has moved as far as possible to the right. Envisioning predicts states 4 and 5 as possible successors. The transition from state 3 to state 5 is impossible. In state 4, the mass has started moving to the left in response to the spring pulling it towards the wall. In state 5 the mass has a velocity to the left, but there is no net force on the mass. Thus the mass must have moved close to its equilibrium position where the weakened spring force perfectly balances friction. To transition from 3 to 5 the mass would have to have moved close to its equilibrium position at the same instant it began to move.

More formally, any change in any quantity from zero happens before any change of a quantity to zero. Consider two quantities (at some time) $[x] = +, \partial x = -$ and $[y] = 0, \partial y = +$. As $[x] = +$, $x = k > 0$, thus if $\frac{dx}{dt} > -\infty$, it will take some time for x to drop for zero. However, y becomes greater than 0 in an arbitrarily short time period, so this happens first. In the case of state 3, $[v] = 0, \partial v = -$ and $[F_{mass}] = -, \partial F_{mass} = +$ so $[v] = -$ occurs first. Thus, state 3 can transition to 4 but not 5. This is equivalent to case (a) of the equality change rule of qualitative process theory (Forbus, 1982).

RULE(3): DERIVATIVE CONTINUITY

All derivatives[2] must be continuous in a continuous system with well-behaved inputs. This rule has consequences even if the derivatives are not computed. Although the derivatives may be unknown, the quantities must still vary continuously.

This rule has consequences both within interpretations and between interpretations. All transitions between states labeled n–1 and n–3 are impossible because ∂F_{mass} cannot continuously change between $+$ and $-$. More interestingly, this rule imposes a sense of direction on the state diagram. State 5 can transition to state 6–1, but not vice versa. For state 5 to transition to state 6, ∂F_{mass} must be $+$ so that $[F_{mass}]$ can change from 0 to $+$. For $[F_{mass}]$ to change back to zero, ∂F_{mass} must be $-$ (i.e., system must be in state 6–3. For, 6–3 to transition to 5, ∂F_{mass} must change from $-$ to $+$ which is ruled out by the rule. As a consequence of this rule it is possible to prove that oscillation between two states is not possible unless both of their derivatives are ambiguous.

RULE(4) : DERIVATIVE INSTANT CHANGE RULE

All quantities must obey rule 2, even if their derivatives are unknown. Thus transitions between a situation where $\partial x = 0, \partial y = +$ and $\partial x = +, \partial y = 0$ are impossible. As a consequence transitions between states 5 and 6–2 are impossible. This contradicts case (b) of the equality change law of qualitative process theory (Forbus, 1982), and thus we produce a different analysis than he does.

By rules 1–4 it is possible to prove that oscillation requires a minimum of 8 states.

INSTANTS

Any state in which a quantity is constant and its derivative non-zero is momentary (e.g., $[x] = 0, \partial x = +$). More generally, if *any* zero quantity changes, the state is momentary. As a consequence the ontology for time is expanded to instants (corresponding to momentary states) and intervals. If more than one zero quantity has a non-zero derivative, we can either think of them changing one at a time or all at once. By modeling what happens as a series of instants we get an intuitively satisfying sense of causality; by grouping these instants in a single instant we get consistent transitions with simultaneous changes from instant to following time interval.

As a consequence of rules 2 and 4, if $[x]$ changes from 0, no other ∂y can change back to 0, so any tendencies to change will persist,

[2]Some transitions corresponding to operating region shifts (not ambiguities) need to be handled with some care. For example, a piece-wise linear model has undefined derivatives at the joints.

and the ultimate effect remains the same. In fact, it is interesting to note that if some non-zero quantity has a non-zero derivative, neither the quantity nor its derivative can change in the instant(s), and the transition is considered in the following interval.

Unfortunately, the qualitative integration equation $[x_{next}] = [x_{current}] + \partial x_{current}$ is invalid for instants (it can be proved for intervals using the Mean Value Theorem). Suppose one drops a ball. At the moment the ball is released, it can't be moving, but immediately thereafter it is. At the moment of release it cannot have moved, has zero velocity, and negative acceleration. Qualitatively, $[x] = 0$, $\partial x = 0$, and $\partial^2 x = -$. So $[x]$ becomes $-$ even though $\partial x = 0$. The correct qualitative integration for instants is $[x_{next}] = [x_{current}] + \partial^n x_{current}$ where $\partial^n x$ is the first non-zero derivative. This result can be proven using the Taylor expansion of $x(t)$.

The difficulty with applying this rule is that higher-order derivatives may not be known. Fortunately, it is often easy to tell what order n is necessary. n is the qualitative order of the system which can be determined directly from the variables mentioned in the equations. The spring-mass equations only referenced forces and velocities thus no information about instants is to be gained from second derivatives. The dropping ball example, mentions three orders of derivatives and thus requires solving for second derivatives. Notice that as the spring-mass system is a second order system we are guaranteed that if the system is in state 1, it cannot move out by itself.

An alternate solution suggested by (Williams, 1984a) (Williams, 1984b)is to rewrite the integration rule for instants as $[x_{next}] = [x_{current}] + \partial x_{next}$ (which can be proven from the Mean Value Theorem). If and only if there is any non-zero $\partial^n x$ at the instant, $x_{next}, \partial x_{next}, ..., \partial^{n-1} x_{next}$ will be non-zero in the following interval (by integration). The two problems with Williams' formulation are: first, it requires knowing what happens next to know what happens next; second, it is consequently difficult to tell whether the current state is momentary or not. He avoids the second problem by an axiom requiring that intervals and instants must alternate. Therefore it is always possible to tell whether the current state is momentary. By rules 2 and 4, if ∂x is non-zero at an instant, it is non-zero in the interval after, so the only difficult case occurs if $[x] = \partial x = 0$. This case is handled by considering all states that satisfy $[x_{next}] = \partial x_{next}$ as possible next states.

RULE (5): HIGHER-ORDER DERIVATIVES

Rules 0 through 4 apply to all derivative orders. Recall that higher-order qualitative derivatives are not defined in terms of lower order qualitative derivatives as is done in conventional calculus. $\partial(\partial x))$ makes no sense. This was illustrated for the valve equation. The higher-order qualitative derivative must be defined in terms of the quantitative derivative. $\partial^n x = [\frac{d^n x}{dt^n}]$. For brevity we sometimes use $\partial^0 x = [x]$.

For linear systems, computing higher-order derivatives is easy. Differentiating a linear equation produces a linear equations so the form of the equations does not change. As there are finitely many solutions to these equations, it is easy to represent in a finite structure all higher order derivatives.

As the mass-spring system is linear, differentiating the models does not change their essential form: $\partial^{n+1} v = \partial^n F_{mass}$, $\partial^{n+1} F_{spring} = -\partial^n v$, $\partial^{n+1} F_{friction} = -\partial^{n+1} v = -\partial^n F_{mass}$, and $\partial^{n+1} F_{mass} = \partial^{n+1} F_{friction} + \partial^{n+1} F_{spring} = \partial^n F_{mass} - \partial^n v$

Table 3 summarizes the solutions for state 6.

	1	2	3	3	3
$\partial v =$	+	+	+	+	+
$\partial F_{friction} =$	−	−	−	−	−
$\partial F_{spring} =$	+	+	+	+	+
$\partial F_{mass} =$	+	0	+	0	−
	1	1	1	3	3
$\partial^2 v =$	+	0	−	−	−
$\partial^2 F_{friction} =$	−	0	+	+	+
$\partial^2 F_{spring} =$	−	−	−	−	−
$\partial^2 F_{mass} =$	−	−	+	0	−

Table 3 : Higher-Order Derivatives of State 6

These second-order derivatives show that state 6–1–1 can transition to state 6–2–1, but that 6–2–1 cannot transition back.

In terms of higher-order derivatives the rules can be summarized succinctly:

(0,3) $\partial^n x_{next} = \partial^n x_{current} + \partial^{n+m} x_{current}$, m first non-zero derivative.

(1) Avoid contradictions at all derivative orders.

(2,4) Any change from zero happens first.

Two states are different if they differ in any known $\partial^n x$. A state is momentary iff $\partial^n x = 0$ and $\partial^{n+1} x \neq 0$, where $n+1$ is the qualitative order of the system.

RULE(6): NO CHANGE TO ALL ZERO DERIVATIVES

A transition (subject to the same caveats as rule 3) cannot go from a state where a quantity is non-zero to one where it and *all* of its derivatives are zero. This rule eliminates the transitions from states 6–3 and 12–1 to state 1, because all $\partial^n x$ are zero in state 1. This rule is justified by the Taylor expansion. Take for example $[v]$. $v(t)$ and all its derivatives are continuous over all the states (there is no change in operating region so there is no possible way for a discontinuity to occur). In state 1, v and all its derivatives are zero. However, we can write v as a Taylor expansion around some time point when the device is in state 1. As all the derivatives of v are zero, v must necessarily be zero everywhere. Thus, if the device is in state 1 it will always remain in state 1 and *has always been in state 1*. Therefore the transition from 6–3 to 1 is impossible as v is non-zero in state 6 and zero in state 1.

QUALITATIVE vs. QUANTITATIVE

The quantitative solution to the spring-mass system is of the form $e^{-kt} sin(wt)$, i.e., a damped sine wave (Figure 4a). Figure 4b illustrates the qualitative state diagram after all the rules have been applied. Qualitative reasoning obtains a qualitative description of the behavior of the mass-spring system without recourse to quantitative methods.

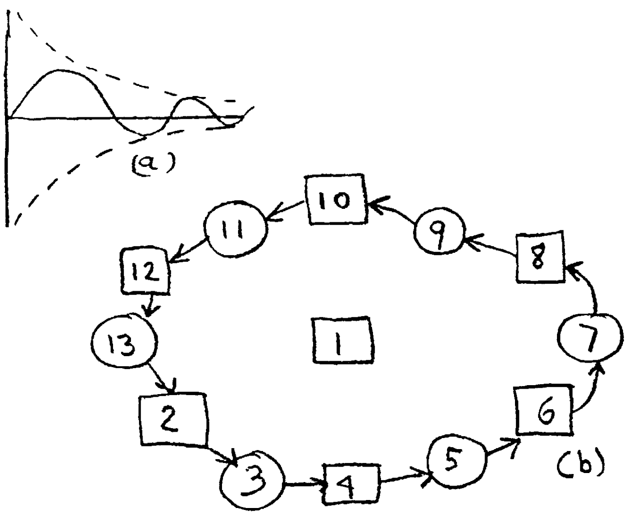

Figure 4 : Qualitative and Quantitative Behavior

Below we give an English description of the states indicated in Figure 4. If the system starts in State 1 it remains there, and if the system starts in any other state it cycles through states 2 through 13 ("*" indicate instants).

(1) A quiescent state which the system cannot leave.

(2) The mass is to the right of equilibrium and decelerating.

(3) The whole system is stationary at the extreme right end of motion.*

(4) The spring pulls back the the mass towards equilibrium.

(5) Near equilibrium spring force has become weak, equaling friction.*

(6) Friction dominates spring force.

(7) Mass reaches equilibrium position, but momentum carries it past.*

(8) Spring begins to compress, system decelerates.

(9) Spring is completely compressed to the left, mass stationary.*

(10) System begins rightward movement towards equilibrium.

(11) Near equilibrium spring force has become weak, equalling friction.*

(12) Friction dominates spring force.

(13) Mass reaches equilibrium, but momentum carries it past to the right.*

OPEN PROBLEMS

We presented the fundamental laws of time-like behavior: qualitative integration/continuity, contradiction avoidance, moving off instants and moving to zeros. These simple, but general and powerful laws capture what would otherwise require sophisticated inference techniques.

Figure 4b does not include possible transitions to quiescence. This is technically correct (using Newton's Law, Hooke's Law, and Friction) — the exponential decay in oscillation amplitude approaches zero asymptotically. However, common-sense tells us that the oscillation must eventually halt. What kind of qualitative equations correctly model the common-sense physics that a transition towards quiescence is possible: perhaps a model of Coulomb friction, or some sort of qualitative "quantum" mechanics? (Forbus, 1982) and (Williams, 1984a) define this problem out of existence by assuming an axiom that all approached thresholds are eventually reached.

Although the momentary states of Figure 4b must end, there is no guarantee that any particular interval will end. The ambiguity of qualitative analysis does not allow us to deduce that state 2 ends. For example, we could design a spring whose restoring force rapidly damped out to zero asymptotically with time. Such a spring still obeys the qualitative Hooke's Law, but the system might never stop moving to the right (i.e., the velocity would approach 0 asymptotically producing no oscillation). Of course, if we knew the spring constant was greater than some fixed landmark (true for non-pathological springs) there would be enough information to determine that oscillation is mandatory — this requires a more sophisticated qualitative physics.

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