STABILITY OF AUTOMATIC GUIDANCE FOR A MOBILE ROBOT

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ABSTRACT

A control law for the automatic guidance is proposed in this paper as if each obstacle exerts upon the mobile robot a repulsion, which varies inversely with the distance between the robot and the obstacle, and becomes infinite as the robot approaches the obstacle. Stability of the automatic guidance is studied when the mobile robot deviates from the ideal path. The results lead to criteria of selecting control parameters for better guidance.

I. INTRODUCTION

Development of mobile robots, equipped with microprocessors and sensory devices, has attracted great attention in current research [1-4]. The need for automatic guidance and control of such vehicles emerges from being powerful and potential means for those tasks which are repetitious, dangerous or unattainable, such as industrial conveyance, handling of radioactive or expolsive materials, fire-fighting, underwater or interplanetary missions, blind aids, and many other purposes. Several types of automatic guidance system were proposed, and a number of mobile robots have constructed for research and development. The mobile robot, considered in this paper, is aimed to travel in a plane universe, avoiding collisions with fixed obstacles and wall on route. It consists of a platform, moved by two driving wheels and balanced by two castors.

II. CONTROL LAW

To guide automatically the robot through a collision-free passage in an environment of obstacles, a control law may be devised as if each obstacle exerts upon the robot a repulsion, which varies inversely with the distance between the robot and the obstacle, and becomes infinite as the robot approaches the obstacle. Mathematically we propose

$$\frac{\omega_R - \omega_L}{\omega_R + \omega_L} e_z = \Gamma = \int e_v \times \nabla \phi \, d\alpha, \qquad (1)$$

$$\phi(r) = C(r/r_o - 1)^{-n}, \qquad (2)$$

where the robot, as shown in Fig. 1, of a nominal

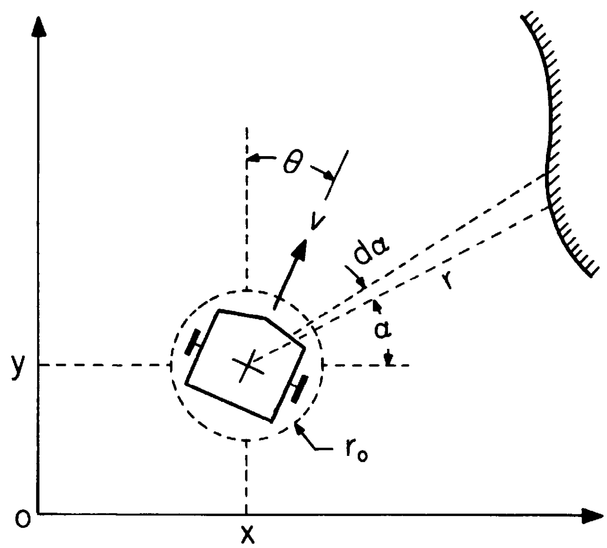

Fig. 1. Interaction of the mobile robot and an obstacle

radius r_o (defined as the external limit for collision) travels in a potential field ϕ with a velocity v; ω_R and ω_L denote the angular speeds of right and left wheels; e denotes the unit vector, and (r, α) specify the position of an obstacle relative to the robot. Γ indicates the turning moment about the vertical axis; negative Γ for right turns and positive Γ for left turns. There are two control parameters, both of positive values: n and K, where K is related to C by $K = nC/r_o$ for mathematical convenience. Large values of n intend to reduce sensitivity of the information collected on distant obstacles and thus make the robot "near-sighted", while large values of K lead to sharp turns.

Let the position of robot be specified by the coordinates (x, y) in a reference frame fixed to the environment. From the geometry of path, as shown in Fig. 2, we write

$$dx = ds \sin \Theta, \quad dy = ds \cos \Theta. \qquad (3)$$

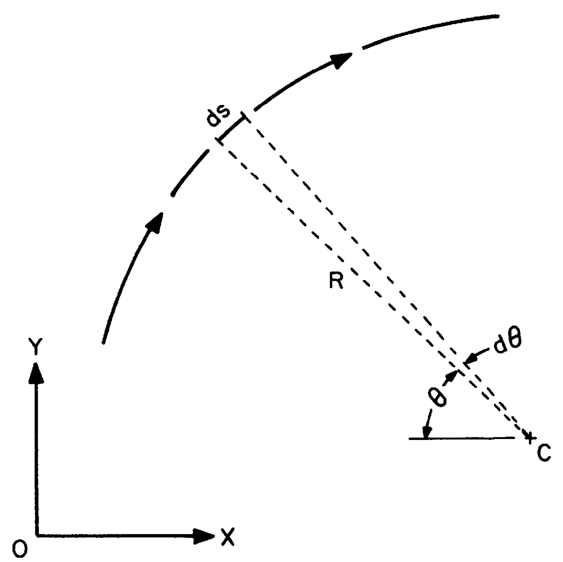

Fig. 2. Geometry of the path

If the radius of curvature, R, of the path is defined to be positive for left turns and negative for right turns, then

$$\omega_R/(R + \tfrac{1}{2}b) = \omega_L/(R - \tfrac{1}{2}b),$$

where b denotes the distance between two driving wheels. Substituting this expression into equation (1), we have

$$\Gamma = \tfrac{1}{2}\, b/R. \qquad (4)$$

But the infinitesimal length of the path may be expressed as $ds = -R\, d\Theta$, where ds is positive in the direction of robotic motion. Substituting this relation into equation(4) to eliminate R, we obtain

$$d\Theta/ds = -(2/b)\Gamma. \qquad (5)$$

To determine geometry of the path, equations (3) and (5) should be solved together with expressions (1) and (2).

III. STABILITY OF GUIDANCE

Let us consider a straight passage bounded by two infinitely long walls. Apparently the ideal path is on the centerline of passage. Suppose that for some reason the robot deviates from the ideal path in both position and direction, in this section we shall study how the robot is guided back to the ideal path.

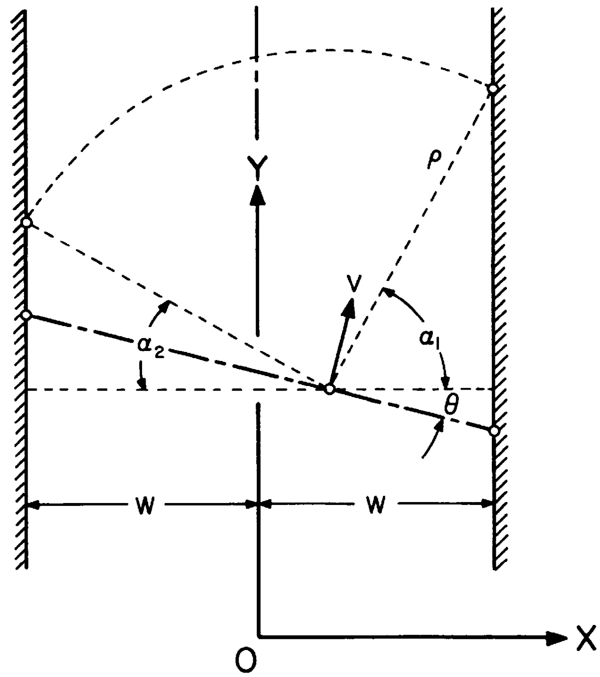

Fig. 3. Deviation from the ideal path

As shown in Fig. 3, without loss of generality the reference frame may be fixed in such a way that the Y-axis coincides with the centerline of the passage with y = 0 initially. Let Θ denote the angular deviation of robot, ρ the range of sensor, and 2w the perpendicular distance between two walls. The angular limits of two walls α_1 and α_2 are defined by

$$\cos \alpha_1 = (w - x)/\rho, \quad \cos \alpha_2 = (w + x)/\rho. \qquad (6)$$

Following expressions (1) and (2), we write

$$\Gamma = K \int_{-\Theta}^{\alpha 1} \left(\frac{w - x}{r_o \cos \alpha} - 1\right)^{-(n+1)} \cos(\alpha + \Theta)\, d\alpha$$
$$- K \int_{\Theta}^{\alpha 2} \left(\frac{w + x}{r_o \cos \alpha'} - 1\right)^{-(n+1)} \cos(\alpha' - \Theta)\, d\alpha'. \qquad (7)$$

For small deviations, we assume that x/w, Θ, r_o/w, and w/ρ are small quantities of the same order. Applying the technique of small perturbations to expression (4), an approximate expression of the first order may be written as

$$\Gamma = 2(n+1)K\left(\frac{r_o}{w}\right)^{n+1}\left[\frac{\sin\Theta}{n+2} + \Lambda(n+2)\frac{x}{w}\right]\cos\Theta \tag{8}$$

where the function $\Lambda(n)$ is defined as

$$\Lambda(n) = \int_0^{\pi/2} \cos^n\alpha \, d\alpha.$$

For the environment under consideration, it is more convenient to use y as the independent variable. We may combine equations (3) and (5) to eliminate ds, and with the substitution of expression (8) we write

$$dx/dy = \tan\Theta, \tag{9}$$

$$\frac{d\Theta}{dy} = -\frac{4}{b}(n+1)K\left(\frac{r_o}{w}\right)^{n+1}\left[\frac{\sin\Theta}{n+2} + \Lambda(n+2)\frac{x}{w}\right]. \tag{10}$$

Equations (9) and (10) may be solved to determine the geometry of path. For $\Theta \ll 1$ as assumed, the linearized solutions may be obtained as follows:

$$x = \exp(-\lambda\tfrac{y}{w})\left[x_o\cosh(k\tfrac{y}{w}) + \frac{\lambda x_o + w\Theta_o}{k}\sinh(k\tfrac{y}{w})\right], \tag{11}$$

$$\Theta = \exp(-\lambda\tfrac{y}{w})\left[\Theta_o\cosh(k\tfrac{y}{w}) - \frac{\lambda\Theta_o + (\lambda^2 - k^2)x_o/w}{k} \times \sinh(k\tfrac{y}{w})\right], \tag{12}$$

where the parameters λ and k are defined as

$$\lambda = 2\,\frac{n+1}{n+2}\,K\,\frac{r_o}{b}\left(\frac{r_o}{w}\right)^n, \tag{13}$$

$$k^2 = \lambda^2 - 2(n+1)\Lambda(n)\lambda. \tag{14}$$

Equation (14) indicates that k is imaginary when

$$K(r_o/b)(r_o/w)^n < (n+2)\Lambda(n). \tag{15}$$

IV. NUMERICAL RESULTS AND CONCLUSIONS

As a numerical illustration to solution (11), we choose the initial conditions $x_o/w = 0.1$ and $\Theta_o = 10$ degrees. Let us assume that $n = 1$; for this value $\Lambda(n) = 1$. Equation (14) indicates that when $\Lambda = 4$, k is zero; in this case the path is represented by the dotted curve in Fig. 4. Two other paths for $\lambda = 8$ and 2, corresponding to a real and an imaginary k respectively, are also depicted in this figure by the solid curves. The figure reveals that an imaginary k provides poor guidance.

Next, we shall consider the cases of large deviations while linearization and small perturbation methods are not applicable. Equations (3) and (5) are solved numerically with Γ given by expression (7) for the initial conditions $x_o/w = \frac{1}{2}$ and $\Theta_o = 20$ degrees. The results for $n = 1$ and $K\,r_o/w = 9$, as shown in Fig. 5, agree with those from the small-perturbation method in general pattern.

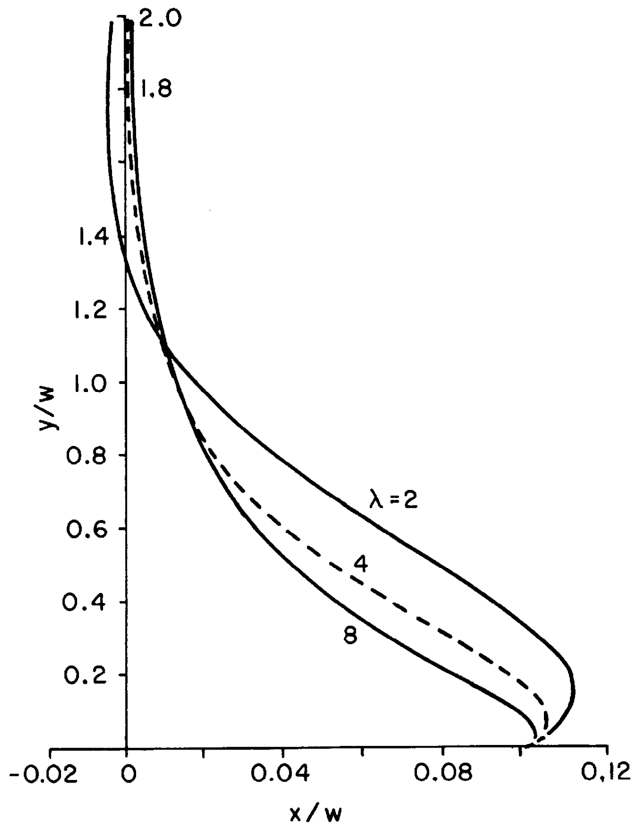

Fig. 4. Returning paths to the ideal path indicate stability of guidance for $n = 1$ and various λ

Conclusions may be drawn from the results obtained that the control parameters should be selected based on the following criteria:

(a) To avoid short sight of the robot, small n is desirable.
(b) To avoid enormous centrifugal force resulted from a sharp turn, small K is desirable.
(c) To ensure better guidance the imaginary k should be avoided; this defines the minimum K which is set by the value of n as shown by inequality (15). This in turn requires a small n.
(d) In order to reduce processing time which is essential for a smooth movement, simplicity of the mathematical evaluation is highly desirable. Thus, in normal conditions we should set k to be zero.

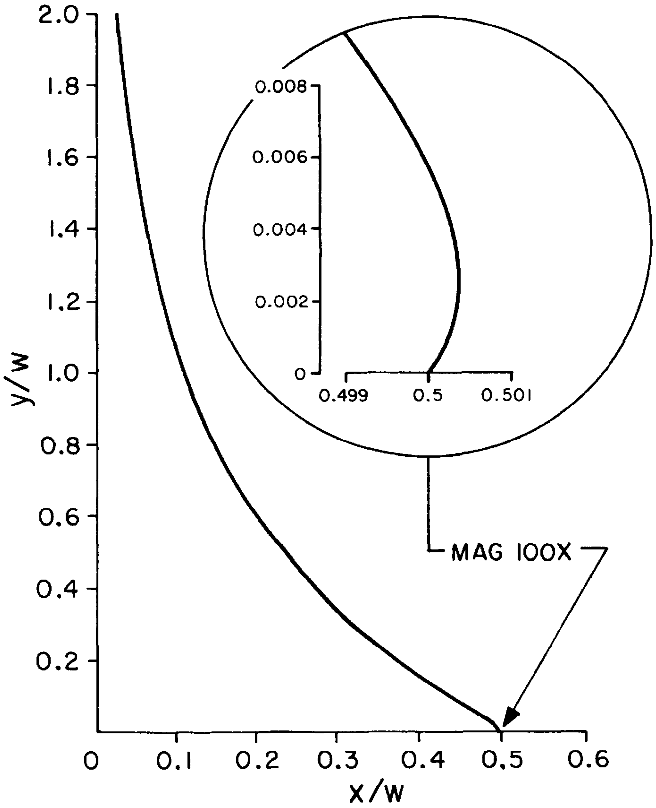

Fig. 5. Returning path to the ideal path for $n = 1$ and $Kr_0/w = 9$

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