Modeling Motion with Qualitative Process Theory

Kenneth D. Forbus
The Artificial Intelligence Laboratory
Massachusetts Institute of Technology
545 Technology Square
Cambridge, Mass. 02139 USA

Abstract

Representing motion is an important part of Naive Physics. Previous qualitative models of motion were centered around the idea of Qualitative States. This paper discusses an alternative representation in terms of Qualitative Process Theory. Viewing motion as a process has several advantages, notably the ability to make more detailed inferences about dynamics and the ability to combine process descriptions to model more complex systems. After examining the relationship between Qualitative State and QP theory representations of motion, the utility of the QP representations are illustrated by analyzing an oscillator.

1. Introduction

Representing motion is an important part of Naive Physics [Hayes, 1979a]. Previous qualitative models of motion were based the idea of Qualitative States and qualitative "simulation rules" to represent changes of state ([deKleer, 1975][Forbus, 1981a]). This paper examines an alternate way of representing motion based on Qualitative Process Theory[Forbus, 1981b][Forbus, 1982a] and compares it with Qualitative State representations. The power of the Qualitative Process theory (QP) descriptions is illustrated by an analysis of a simple oscillator to determine the existence of a limit cycle. The example is drawn from [Forbus, 1982a], which contains more details.

2. Qualitative State Representation

The Qualitative State representation is based on the notion of state in classical mechanics. Certain parts of the classical state are represented abstractly (typically position is represented by a piece of space, and velocity by a symbolic heading) and the type of activity, which classically is implicit in the choice of descriptive equations, is made explict. Qualitative states are linked by qualitative simulation rules that map a qualitative state into the qualitative states that can occur next. These rules are usually run to closure from some initial state, producing a description of all the possible states called the envisionment. The envisionment can be used to answer simple questions directly, assimilate certain global assumptions about motion, and plan solutions to more complex questions.

While envisioning is useful, it cannot deal with many complicated domains and questions.[1] Domains where moving objects continually interact, including mechanisms such as clocks, are hard to model with qualitative states because it is hard to build qualitative simulation rules for the motion of a compound object from simulation rules for the motions of its parts. More importantly, qualitative reasoning is more than just simulation. An example that will be examined below is determining whether or not pumping an oscillator will result in stable behavior. Such questions require richer notions of quantity, process, and time than qualitative state representations provide.

3. Qualitative Process Theory - Basics

Qualitative Process theory (QP) extends the ontology of common sense physical models by adding the notion of a physical process. Processes are things like flowing, boiling, and stretching that cause changes in physical situations. QP theory provides a language for specifying processes and their effects in a way that induces a natural qualitative representation for quantities and allows both the deduction of what processes occur in a situation and how they might change. Space permits only a brief sketch of the theory; its present status is described in [Forbus, 1982b].

A situation is composed of objects and relationships between them. The continous parameters of an object, such as temperature and pressure, are represented by quantities. A quantity consists of two parts, an amount and a derivative, each of which has parts sign and magnitude (denoted A_s, A_m, D_s, and D_m respectively). When we wish to refer to a quantity or some part of it at a particular time (either instant or interval), we write:

(M Q t)

which means "the value of Q measured at t". The derivative of a quantity is determined by the sum of the influences on it. A principle tenet of QP theory is that only processes cause changes, so only processes impose influences. Processes also can induce functional dependences between quantities, and

($\propto_Q$ Q R)

means "there exists an increasing monotonic function induced by a process such that Q is functionally dependent on at least R". $\propto_Q$- signifies the same, but with the implicit function being decreasing monotonic. In basic QP theory, the value of a quantity is defined in terms of the inequalities that hold between it and its Quantity Space - a partially ordered collection of numbers and quantities mainly determined by the vocabulary of processes for the domain.

A process is specified by five parts:

- Individuals: descriptions of the entities the process acts between.
- preconditions: statements that must be true for the process to act, but not deducible solely within QP theory.
- quantity conditions: statements that must be true for the process to act, but are deducible within QP theory.
- Relations: the relationships between the individuals which hold when the process is active.
- Influences: descriptions of what quantities are affected by the process

A process acts between any collection of individuals it matches, whenever both the preconditions and quantity conditions are true. Preconditions are distinct from quantity conditions because some factors are external to the physics of a domain - a purely physical theory cannot predict whether or not someone will walk by and turn on a stove, for instance, although it can predict that a result of this action will be that the water sitting in the kettle on top of it will soon boil. Object descriptions peculiar to a domain, such as viewing a spring as a Hooke's law device, are specified in the same way as processes except there can be no influences.

QP theory can be viewed as providing a language for representing portions of physical theories. In this language objects

1. [Forbus, 1982b] discusses these limitations in more detail.

and simple processes are primitives, with shared parameters and sequential occurance providing the means of combination. Abstraction is provided both by naming combinations and by a type hiearchy (although the notational conventions for the type hiearchy have not been worked out). General laws, such as energy conservation and Newton's laws, can be viewed as constraints on what processes are possible. Let us see how these ideas can be used to represent motion.

4. A Vocabulary for Motion

A simple vocabulary for abstract one dimensional motion will serve as an illustration.[1] Figure 1 contains the process specifications for motion and acceleration.

The motion description says that motion will occur when a movable object is free in the direction of its velocity, and that velocity is non-zero. Motion is a positive influence on the position quantity of an object, in that if the velocity is positive the position will "increase" and if negative, the position will "decrease". Acceleration occurs when a movable object has a non-zero net force in a free direction, and the influence it provides on velocity is qualitatively proportional to the net force and inversely proportional to the mass of the object. Friction occurs when there is surface contact, and produces a force on the object which is qualitatively proportional to the normal force and acts in a direction opposite that of the motion (encoded by I- instead of I+). While this description is Newtonian, Aristotelian and Impetus theories[2] could also be described - QP theory constrains the <u>form</u> of physical theories, not their content.

Fig. 1. Process Descriptions of Motion and Acceleration

```
Motion(B,dir)
individuals:(movable-object B)
Preconditions: Free-direction(B, dir)
QuantityConditions: (greater-than Am[Vel(B)]
                                   zero)
Influences: (I+ Pos(B) Vel(B))

Acceleration(B,dir)
individuals:(movable-object B)
Preconditions: free-direction(B,dir)
QuantityConditions: (greater-than Am[Fnet(B)]
                                   zero)
Relations: Let Acc be a number
         (∝Q Acc Fnet(B))
         (∝Q- Acc Mass(B))
         (correspondence (Acc zero)
                         (Fnet(B) zero))
Influences: (I+ Vel(B) Acc)

Moving-Friction(B,S)
individuals:(movable-object B)
          (surface S)
Preconditions:Sliding-Contact(B,S)
QuantityConditions:Motion(B,along(S))
Relations: Let fr be a number
         (∝Q fr Fnormal(B,S))
Influences: (I- Falong(B,S) fr)
```

1. More detailed representations are the target of work in the Mechanism World, which concerns simple devices such as clocks. Much work remains, especially in the geometric descriptions required.

2. [McClosky, 1982] argues naive theories of motion in our culture correspond to Impetus theories, not aristotelian theories.

Collisions are complicated. The simplest version just involves a reversal of velocity, as illustrated in figure 2. Here `direction-towards(C,B,dir)` asserts that the object is moving in direction `dir` from `C` to `B`, `start`, `end`, `during` and `duration` define the temporal aspects of an episode in a process history that corresponds to this process occuring. Even our more complicated models of collisions appear to use such behavioral descriptions, such as a compound process consisting of contacting the surface, compression, expansion, and finally breaking contact. The type of collision which occurs can be specified by refering to the theory of materials of the objects involved.

The process vocabulary for motion presented above is quite abstract. The particular kind of motion - flying, sliding, rolling, or swinging - is not mentioned. These motions would be specializations of the motion process considered above, defined by additional preconditions and relations (sliding and rolling require surface contact and could involve friction, for instance). The advantage of having the abstract description as well as the more detailed ones is that weak conclusions can be drawn even with little information. If we kick something and it isn't blocked, for instance, then it will move.

Now we can examine the relationship between this representation of motion and the Qualitative State representation. If we assume motion and acceleration are the only processes that occur, then the limit analysis for a moving object will only include the possibilities raised by dynamics. To include the possible changes in process caused by kinematics (i.e., hitting something) the relevant geometry of the situation must be removed from the preconditions and mapped into a Quantity Space. This requires describing space by a place vocabulary,[1] and using the elements in the place vocabulary as the elements in the position Quantity Space. To induce an ordering between the elements for motion in two and three dimensions a direction must also be included in the process description, since partial orders are only well-defined for one dimension. The ambiguity due to dimensionality and symbolic heading can be encoded by the lack of ordering between the Quantity Space elements. This also means the place must be encoded in the motion process, which in turn means that an instance of a motion process in this vocabulary will look like a Qualitative State for the same collection of places and type of motion. The qualitative simulation rules correspond to a compilation of the limit analysis on this new motion vocabulary.

From this perspective we can see the relative strengths of the two representations. For evolving motion descriptions the qualitative state representation makes sense, since kinematic

Fig. 2. Collision Specification

```
Collide(B,C,dir)
Individuals: (movable object B)
             (immobile object C)
Precondition: (and contact(B,C) direction-towards(B,C,dir))
QuantityCondition: Motion(B,dir)
Relations: (= (M Vel(B) start) (- (M Vel(B) end)))
           (= (M Vel(B) during) zero)
           (= duration zero)
           (T direction-towards(C,B,dir) end)
           (T contact(B,C) end)
; (T <statement> <time>) means
; "<statement> is true during <time>"
```

1. [Forbus, 1981a] describes the principles involved and defines a place vocabulary for motion through space in a simple domain.

constraints are essential to motion. Its "compiled" nature makes qualitative states inappropriate for very simple deductions (where only part of a qualitative state is known) and more complex questions involving dynamics or compound systems. The next section illustrates the kind of detailed analysis made possible by the QP description of motion.

5. An Oscillator

Consider the block B connected to the spring S in figure 3. Suppose that the block is pulled back so that the spring is extended. Assume also that the contact between the block and the floor is frictionless. What happens?

First, the spring object includes:

```
Relations:
Ds(Lrest(s))=0 ;i.e., Lrest is constant
Let Disp(s)=(- A(L(s)) Lrest(s))
(αQ- Fi(s) Disp(s))
(correspondence (Fi(s) zero) (Disp(s) zero))
```

where F_i is the internal force due to the composition of the spring. Since `Disp(s)` is greater than zero, the spring will exert a force. Because the block is rigidly connected to the spring, the net force on the block will be negative and since the block is free to move in the direction of the force, an accelleration will occur. The acceleration will in turn cause the velocity to move from zero, which will in turn cause $D_s(Pos(B))=-1$. By rigid contact, $D_s(L(S))=-1$ and by the α_Q relation with displacement, $D_s(F_{net}(B))=1$. The processes occuring are motion(B, -), relaxing(S, -), acceleration(B, -). The next process limit occurs when $L(s)=L_{rest}(s)$, ending the relaxing. The correspondence tells us the force on the block becomes zero, so the acceleration will end as well. However, the motion does not. Setting aside the details, the next set of processes are Motion(B, -), compress(S), and acceleration(B, +). The only limit point in the quantity spaces that are changing is the zero velocity point (assuming the spring is unbreakable), so the motion will continue until the velocity is zero. The conclusion that the next set of processes are Motion(B, +), relaxing(S, +), acceleration(B, +) and then Motion(B, +), stretching(S, +), acceleration(B, -) follows in the same way. At the end event of the last set of processes, the orderings on the quantity spaces and the processes evoked are the same as the initial instant. Thus we can conclude that an oscillation is occuring. Note that the processes need to be the same, because the preconditions might have changed. Figure 4 illustrates the process history for the oscillator.

An additional complexity is introduced if we allow for properties of materials, such as the spring being breakable. The relevant effects of material composition can be modelled by introducing elements into the force quantity space for the spring corresponding to the occurances of processes such as breaking and crushing, in addition to those for stretching and compressing. It appears that an assumption is needed to rule out crushing at t_3, but breaking can be ruled out by an energy argument (essentially, energy considerations lead to the conclusion that the position of the block at t_5 is no greater than the position at t_1, so that if it didn't break then it won't break later). The details can be found in [Forbus, 1982b].

To further analyze this system, we must treat the collection of objects as a system and the processes that occur as a compound process. Representing the combination allows the explicit representation of properties belonging to the collection, such as the energy of the system, and properties defined over a cycle of the combination, such as energy lost and maximum displacement. We can then determine the consequences of perturbing the situation in various ways. In particular, the relations for the compound process include:

```
(αQ MaxDisp(Obj) E(System))
(correspondence (MaxDisp(Obj) zero) (E(system) zero))
```

This relationship makes it possible to deduce that if friction were introduced (i.e., $D_s(E(system))=-1$) the oscillation process will eventually stop, and that if the system is pumped so that its energy increases (i.e., $D_s(E(system))=1$), that the materials involved in the

Fig. 3. Sliding Block With Friction

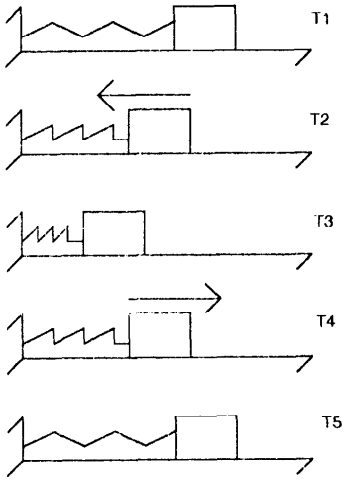
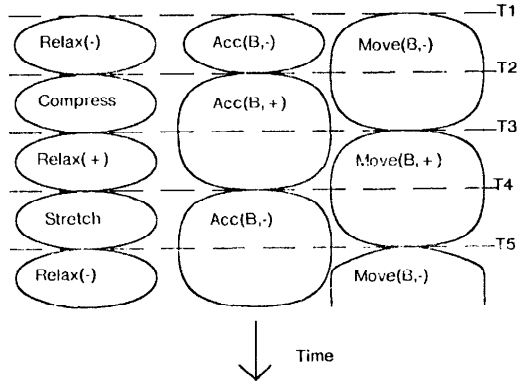


Fig. 4. Process history for the oscillator



oscillator may break in some way.[1] Suppose for example the oscillator is subject to friction, but we pump it with some fixed amount of energy per cycle, as would happen in a mechanism such as a clock. Is such a system stable? The only things we will assume about the friction process in the system is that

```
Relations:(α0 E(loss) E(System))
        (correspondence (E(loss) zero) (E(System) zero))
Influences:(I- E(System) E(loss))
```

where `E(loss)` is the net energy lost due to friction over a cycle of the oscillator process. The loss being qualitatively proportional to the energy is based on the fact that the energy lost by friction is proportional to the distance travelled, which in turn is proportional to the maximum displacement, which itself is qualitatively proportional to the energy of the system, as stated above.

The lower bound for the energy of the system is zero, and an upper bound for energy is implicit in the possibility of the parts breaking. The result, via the $\propto_0$ statement above, is a set of limits on the quantity space for `E(loss)`. If we assume `E(pump)`, the energy which is added to the system over a cycle, is within this boundary then there will be a value for `E(System)`, call it `E(stable)`, such that:

```
∀ t ∈ intervals
                (implies (= (M E(System) t) (M E(stable) t))
                         (= (M E(loss) t) (M E(pump) t)))
```

Note that `E(stable)` is unique because $\propto_0$ is monotonic. If the energy of the system is at this point, the influences of friction and pumping will cancel and the system will stay at this energy. Suppose

```
(> (M E(System) t) (M E(stable) t))
```

over some cycle. Then because the loss is qualitatively proportional to the energy, the energy loss will be greater than the energy gained by pumping, i.e., D_s`(E(System))=-1`, and the energy will drop until it reaches `E(stable)`. Similarly, if `E(System)` is less than `E(stable)` the influence of friction on the energy will be less than that of the pumping, thus D_s`(E(System))=1`. This will continue until the energy of the system is again equal to `E(stable)`. Therefore for any particular pumping energy there will be a stable oscillation point. This is a qualitative version of the proof of the existence and stability of limit cycles in the solution of non-linear differential equations.

6. Conclusions

This paper has illustrated how motion can be represented using Qualitative Process theory. As the example indicates, the notions of quantity and process it provides allows useful deductions about systems involving motion to be made. The previous Qualitative State representation for motion can be viewed as a simplified process vocabulary where kinematic information has been included, and qualitative simulation rules can be viewed as a compilation of the limit analysis on this vocabulary. This suggests that for some purposes Qualitative States will be more useful, in that the usual QP theory limit analysis will only encode changes due to dynamics, not kinematics. It should be possible to smoothly merge the two representations, using the QP description to decide on the type of motion, the Qualitative State representation to determine the motions possible, and the QP description to provide more subtle analysis to choose between the alternatives of the qualitative simulation as well as examine other kinds of questions. This of course is a topic for future consideration.

7. References

de Kleer, Johan "Qualitative and Quantitative Knowledge in Classical Mechanics" TR-352, MIT AI Lab, Cambridge, Massachusetts, 1975

Forbus, K. "A Study of Qualitative and Geometric Knowledge in Reasoning about Motion" TR-615, MIT AI Lab, Cambridge, Massachusetts, February, 1981

Forbus, K. "Qualitative Reasoning about Physical Processes" Proceedings of IJCAI-7, 1981

Forbus, K. "Qualitative Process Theory" MIT AI Lab Memo No. 664, February 1982

Forbus, K. "Qualitative Reasoning about Space and Motion" to appear in Mental Models, D. Gentner and A. Stevens, editors.

Hayes, Patrick J. "The Naive Physics Manifesto" in Expert Systems in the Microelectronic age, edited by D. Michie, Edinburgh University Press, May 1979

Hayes, Patrick J. "Naive Physics 1 - Ontology for Liquids" Memo, Centre pour les etudes Semantiques et Cognitives, Geneva, 1979

McCloskey, M. "Naive Theories of Motion" to appear in Mental Models, D. Gentner and A. Stevens, editors.

1. The Tacoma Narrows bridge phenomena, something every engineer should know about.